New Constructions of Partitionable Sets and Almost Partitionable Sets

Authors: Zhou, Junling, Chang, Yanxun, Chang, Yanxun

Date: 2022-04-19T00:00:00+00:00

Abstract

Partitionable Sets (PS) and Almost Partitionable Sets (APS) are two important classes of combinatorial configurations in combinatorial design theory, which have close connections with many other combinatorial structures, such as Z-cyclic Whist tournament designs, cyclic difference arrays, cyclic balanced sampling plans excluding adjacent units, disjoint difference families, and optical orthogonal codes. Due to the rather stringent requirements of partitionable sets and almost partitionable sets, their existence problems remain far from resolved to date. This paper establishes new construction methods for partitionable sets of order p2 and almost partitionable sets of order p for the case where p 7 (mod 8) is a prime, and presents several new results on the existence of these two classes of combinatorial configurations. In particular, for primes p with p 7 (mod 8), this paper determines the existence of PS of order p2 for the vast majority of cases with p<30000, provides the existence and asymptotic existence of APS of order p under specific conditions, and obtains the existence of APS of order p for p<50000 with 16 possible exceptions.

Full Text

New Constructions for Partitionable Sets and Almost Partitionable Sets

Junling Zhou, Yanxun Chang
Institute of Mathematics, Beijing Jiaotong University, Beijing 100044, P. R. China
jlzhou@bjtu.edu.cn, yxchang@bjtu.edu.cn

Abstract

Partitionable sets (PS) and almost partitionable sets (APS) represent two important classes of combinatorial configurations in design theory. These structures are intimately connected to various other combinatorial objects, including Z-cyclic patterned starter whist tournaments, cyclic difference matrices, cyclic balanced sampling plans excluding contiguous units, disjoint difference families, and optical orthogonal codes. The existence problems for PS and APS remain largely unresolved due to their stringent structural requirements. This paper focuses on the case where p 7 (mod 8) is a prime and develops new constructions for partitionable sets of order p^2 and almost partitionable sets of order p. Specifically, for primes p 7 (mod 8), we establish the existence of PS of order p^2 for a substantial proportion of primes $p < 30000$, demonstrate both the existence and asymptotic existence of APS under certain conditions, and prove the existence of APS of order p for all primes $p < 50000$ with at most sixteen possible exceptions.

MSC: 05B05
Keywords: partitionable set, almost partitionable set, whist tournament, cyclotomic class

The paper entitled "New constructions for partitionable sets and almost partitionable sets" (in Chinese) will be published in *Sci Sin Math*, 2022 (52). Below we present several tables that provide supporting data for some of the proofs in this paper.

Table I: Data for Theorem 2.1
(cid:24)

Table II: Data for Lemma 3.4
(cid:24)
(b1 b2 b3 b4)
(3 507 277 62)
(17 987 1076 891)
(7 1107 1990 74)
(11 463 62 1772)
(13 1379 2513 529)
(3 2821 3148 1595)
(11 2026 2469 4406)
(7 1388 1063 5732)
(5 7461 4192 5690)
(6 1511 1810 6621)
(11 803 4116 5561)
(3 393 8679 6259)
(19 6181 7645 7804)
(3 2477 9934 2036)
(11 7491 4941 9631)
(11 10800 5770 10210)
(13 650 13758 5218)
(6 2538 15791 15008)
(3 5074 16418 2495)
(13 4312 7782 5762)

(12 20960 16446 7163)
(3 10819 4447 21684)
(7 5146 605 1496)
(3 15972 21590 7937)
(3 16945 9028 19470)
(3 19091 22030 13596)
(3 13661 25511 14078)
(3 27068 25629 4984)
(3 20617 16992 26872)
(3 26019 8673 19188)
(3 2974 31447 3770)
(3 29054 17233 20904)
(3 17491 23613 35386)
(3 110 9706 3374)
(6 9884 40445 43592)
(5 8149 8289 40046)
(cid:24)
(b1 b2 b3 b4)
(14 1003 813 654)
(3 49 838 1128)
(3 1311 2586 778)
(3 2304 784 25)
(12 2879 1999 4081)
(7 14 2725 2579)
(11 3657 111 4685)
(3 2362 3101 2974)
(6 3101 5435 4112)
(17 1266 202 1974)
(3 4366 5079 6450)
(7 7899 1297 4289)
(3 9214 6057 2823)
(13 9655 5500 9379)
(11 12638 5989 11254)
(7 7606 2366 10493)
(19 14307 7026 95)
(6 9002 16240 1791)
(6 14812 10846 5700)
(11 16524 3453 4917)
(7 19975 9149 19175)
(6 5687 3682 16741)
(5 5951 12183 8719)
(3 1489 12952 21587)
(11 19343 21777 13599)
(11 21888 21707 26409)
(3 1881 23120 27446)
(14 198 473 27798)

(13 28982 20236 28565)
(5 10653 14500 7992)
(13 10409 34649 24289)
(17 33256 31081 4071)
(5 17042 20835 35147)
(3 39338 6076 7739)
(3 47717 22681 2054)

Table III (1): Data for Lemma 3.5 with t = 3
(cid:24)
(x1 x2 bl)
(13 38 378)
(53 88 797)
(49 76 767)
(51 130 1817)
(37 198 1124)
(11 66 2429)
(11 40 1010)
(35 70 149)
(69 256 3013)
(19 48 3181)
(15 198 4449)
(43 204 3280)
(37 448 4116)
(3 144 1276)
(19 280 4372)
(5 224 3078)
(11 630 7090)
(25 142 6506)
(11 236 4675)
(57 716 7594)
(7 74 4934)
(43 636 4640)
(7 314 4418)
(9 642 2970)
(3 554 4937)
(3 502 1570)
(7 742 1121)
(33 248 9406)
(7 416 4461)
(13 1228 144)
(15 880 1675)
(25 1272 4265)
(3 438 4135)
(5 1150 10010)
(17 1394 14898)

(23 168 13024)
(27 796 15047)
(77 582 15614)
(25 134 11714)
(3 244 7141)
(7 1476 10704)
(3 274 1910)
(5 1228 851)
(11 1576 8223)
(9 1660 21981)
(51 150 4853)
(15 518 4241)
(29 940 14215)
(7 1866 3882)
(15 1228 8364)
(5 1396 7972)
(3 1484 26339)
(31 806 18469)
(37 2468 26167)
(15 480 3053)
(5 1418 13436)
(3 844 9004)
(7 1520 4521)
(25 1224 3672)
(23 1938 4194)
(21 1552 25947)
(5 1322 15151)
(5 2678 15570)
(41 1036 31904)
(cid:24)
(x1 x2 bl)
(5 10 461)
(25 46 89)
(87 98 69)
(21 118 2041)
(57 114 1576)
(7 164 202)
(15 74 2181)
(45 124 3529)
(21 98 685)
(19 130 1885)
(39 112 3491)
(11 284 1202)
(13 354 5621)
(49 244 2573)
(35 42 6038)

(3 696 7841)
(3 304 7522)
(17 746 369)
(3 498 8430)
(21 670 4100)
(15 216 485)
(3 130 2447)
(45 614 5275)
(3 504 3814)
(17 706 6344)
(9 182 1906)
(21 278 5716)
(7 416 187)
(13 734 7066)
(5 1012 2236)
(21 286 11236)
(3 1196 15989)
(21 970 4058)
(47 482 8851)
(9 154 9778)
(29 1218 5178)
(31 1178 16355)
(15 1404 5788)
(37 382 18873)
(9 640 18662)
(49 762 16093)
(3 1078 9089)
(3 1390 17130)
(63 656 7981)
(7 642 9598)
(7 762 22649)
(37 874 19814)
(29 796 9557)
(21 446 7066)
(11 1622 9414)
(7 1938 9877)
(9 298 8131)
(27 182 19210)
(45 770 8530)
(35 230 4720)
(19 816 29320)
(35 1364 18712)
(49 974 13030)
(19 260 28834)
(5 1416 32871)
(11 1984 3624)

(5 264 16545)
(17 1576 27744)
(21 2628 9599)

Table III (2): Data for Lemma 3.5 with t = 3
(cid:24)
(x1 x2 bl)
(27 424 3829)
(7 2194 34978)
(17 1204 20531)
(83 2772 27397)
(17 2978 13664)
(13 2372 35560)
(13 1600 2469)
(13 1630 10757)
(45 1650 37110)
(5 764 8736)
(7 66 28708)
(31 184 8972)
(25 1286 35439)
(7 2334 19611)
(35 2052 27423)
(29 2992 5942)
(7 50 19121)
(27 660 13916)
(5 874 8073)
(11 2356 36401)
(25 1400 29004)
(39 1258 3622)
(13 1914 872)
(17 1404 31464)
(7 92 28591)
(13 1952 28716)
(11 2910 46233)
(cid:24)
(x1 x2 bl)
(5 2882 15185)
(23 796 35597)
(15 2336 32993)
(19 68 10563)
(7 1706 10589)
(3 334 17870)
(17 2296 32453)
(25 3106 33390)
(49 2048 28444)
(9 2458 790)

(7 1944 33883)
(11 294 20539)
(11 174 34699)
(21 2192 38277)
(9 2138 12312)
(9 2548 19517)
(7 3694 43562)
(3 1712 34593)
(7 3466 4523)
(23 300 41500)
(57 1118 38780)
(37 3068 26110)
(7 2798 16464)
(7 242 43710)
(27 1378 10974)
(31 2758 11256)

Table VI: Data for Lemma 3.5 with t ≥ 7
(cid:24)
(y z u v)
(343 3198 1461 3)
(125 7876 5688 168)
(27 1140 4875 5467)
(343 8111 10034 9430)
(125 2580 9241 6049)
(5185 8104 6516 3353)
(27 10643 9958 10427)
(27 7946 11021 12865)
(27 5589 6227 10849)
(125 4 5225 16676)
(216 6579 10968 21534)
(125 16677 19820 14537)
(9261 21751 14040 6591)
(27 4483 13920 12189)
(125 12775 18811 1638)
(343 2383 23740 24916)
(343 26972 22113 13861)
(1000 4365 15003 23904)
(125 2355 9015 18220)
(343 21163 12410 15596)
(27 14997 33228 101)
(3375 31446 5234 1813)
(27 6853 33992 20623)
(216 23199 33164 23369)
(216 6199 30753 34540)
(125 2365 25033 18445)

(125 9674 23059 23700)
(27 39615 10781 2891)
(1728 24726 25517 6226)
(27 33348 30207 18584)
(x1 x2 bl)
(53 82 3166)
(81 146 710)
(17 132 8360)
(275 348 1964)
(91 224 2763)
(187 378 2800)
(49 124 5002)
(147 340 16570)
(69 446 912)
(95 678 7086)
(171 272 598)
(253 384 1456)
(133 376 3084)
(17 308 15313)
(263 284 915)
(163 912 2150)
(125 636 25546)
(51 1058 24203)
(87 872 18453)
(127 528 5573)
(89 136 18522)
(101 370 21515)
(113 564 34736)
(5 762 22589)
(393 600 26042)
(39 1244 32621)
(85 544 25750)
(87 130 3892)
(331 606 13836)
(33 1314 37917)

Note: Figure translations are in progress. See original paper for figures.