Postprint: Multi-Depot Semi-Open Vehicle Routing Problem with Splittable Pickup and Delivery Demands

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Abstract

For the multi-depot semi-open vehicle routing problem with split delivery and pickup demands, a mathematical model is constructed with the objective of minimizing total vehicle travel distance. A large-mutation neighborhood genetic algorithm is designed for solution, employing two-dimensional chromosome encoding and order crossover strategy, while simultaneously utilizing large mutation strategy and neighborhood search strategy to enhance the algorithm' s global and local optimization capabilities, and the effectiveness of the proposed model and algorithm is verified through comparative case studies. Experimental results demonstrate that the large-mutation neighborhood genetic algorithm achieves superior solution quality, higher computational efficiency, and more stable results for solving multi-depot logistics distribution vehicle routing problems. Additionally, it is verified that the multi-depot semi-open split delivery and pickup distribution mode under joint distribution outperforms the single-depot split delivery and pickup distribution mode under independent distribution. The research findings not only extend the vehicle routing problem but also provide decision-making references for distribution optimization in relevant express logistics enterprises.

Full Text

Preamble

Multi-Depot Half-Open Vehicle Routing Problem with Split Deliveries and Pickups

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Abstract: To address the multi-depot half-open vehicle routing problem with split deliveries and pickups, this paper constructs a mathematical model for the multi-depot half-open split delivery and pickup vehicle routing problem with the objective of minimizing total vehicle delivery distance. A large mutation neighborhood genetic algorithm is designed for solution, employing two-dimensional chromosome coding and sequential crossover strategy, while utilizing large mutation strategy and neighborhood search strategy to enhance both global and local optimization capabilities. Numerical examples verify the effectiveness of the proposed model and algorithm. Experimental results demonstrate that the large mutation neighborhood genetic algorithm achieves superior solution quality, high efficiency, and stable results for multi-depot logistics distribution vehicle routing problems. The study also confirms that the multi-depot half-open split delivery and pickup distribution mode under joint distribution outperforms the single-depot split delivery and pickup mode under independent distribution. The research findings not only extend the vehicle routing problem literature but also provide decision-making references for distribution optimization in relevant logistics enterprises.

Keywords: vehicle routing problem; multi-depot; split deliveries and pickups; genetic algorithm based on big mutation

0 Introduction

In recent years, the rapid development of e-commerce alliances and the implementation of environmental protection policies have accelerated the growth of reverse logistics in China. The Vehicle Routing Problem (VRP) has gradually evolved into a multi-depot transportation problem with return trips under joint distribution, encompassing variants such as the Multi-Depot Open VRP (MDOVRP) [?], Multi-Depot Half-Open VRP (MDHOVRP) [?], and multi-depot closed VRP. As customer demands have evolved from single delivery requirements to simultaneous delivery and pickup needs, additional extensions have emerged, including the VRP with Simultaneous Delivery and Pickup (VRPSDP) [?] and VRP with Split Deliveries and Pickups (SVRPPD) [?]. This paper investigates the Multi-Depot Half-Open VRP with Split Deliveries and Pickups (MDHOSVRPPD) under joint distribution, which integrates SVRPPD and MDHOVRP to more accurately represent complex real-world logistics distribution scenarios. For instance, in a region with multiple distribution centers operating under a joint distribution alliance, delivery vehicles must not only distribute parcels but also collect them. After fulfilling all customer demands, vehicles may return to any distribution center based on proximity, sharing the distribution network. The half-open depot configuration and reasonable splitting of customer delivery/pickup demands help improve vehicle load utilization and reduce distribution costs for logistics and transportation enterprises. Currently, research on this specific problem remains limited, although its constituent problems—SVRPPD and MDHOVRP—continue to be active research areas.

Scholars worldwide have conducted extensive and in-depth research on SVRPPD and MDHOVRP models and solution methods. Mitra [?] first thoroughly analyzed the SVRPPD problem and constructed two heuristic algorithms (NH and SM) for its solution. Archetti et al. [?] provided detailed scientific justification for whether to split customer demands, demonstrating experimentally that splitting yields maximum benefits when the total average demand at customer points is greater than half but less than three-quarters of vehicle capacity, with relatively small demand variance. The primary benefit of splitting lies in reducing vehicle delivery routes, independent of customer point locations. Tang et al. [?] applied a split delivery and pickup vehicle routing model with time window constraints to a real-world third-party logistics case, solving it using a constructive heuristic algorithm—the competitive decision algorithm. Nagy et al. [?] employed tabu search for SVRPPD, comparing it with the algorithm from [?] and demonstrating superior solution accuracy and efficiency. Regarding MDHOVRP research, Cordeau et al. [?] designed a tabu search algorithm incorporating a set of penalty functions for MDHOVRP. Ting et al. [?] combined simulated annealing with ant colony optimization to solve MDHOVRP. Gu et al. [?] studied the multi-depot half-open VRP with time windows, proposing a three-phase solution method: first using K-medoids to transform the multi-depot problem into single-depot routes, then applying a GA-ACO algorithm, and finally using a savings algorithm to split the single depot into multiple depots. Fan et al. [?] improved ant colony algorithm performance by modifying pheromone update mechanisms for MDHOVRP. Wang et al. [?] combined genetic algorithm with particle swarm optimization for MDHOVRP. Fan et al. [?] first investigated the Multi-Depot VRP with Simultaneous Deterministic Delivery and Stochastic Pickup based on Joint Distribution (MDVRPSDDSPJD), designing a variable neighborhood memetic algorithm tailored to the problem characteristics.

In summary, existing literature primarily focuses on SVRPPD and MDHOVRP separately, with limited research combining both aspects. In complex real-world logistics distribution networks, addressing only single problems proves insufficient. Comprehensive optimization requires considering vehicle sharing, joint distribution among distribution centers, and reasonable splitting of customer delivery/pickup demands. The MDHOSVRPPD problem, derived from real-world logistics distribution evolution, features multi-depot, half-open, and split delivery/pickup characteristics. Vehicle loads fluctuate when serving customer points, necessitating feasibility checks for each route segment and increasing algorithmic complexity. To effectively solve MDHOSVRPPD and improve solution quality and efficiency, this paper designs a large mutation neighborhood genetic algorithm specifically tailored to MDHOSVRPPD characteristics, employing two-dimensional chromosome coding to enable customer demand splitting while improving solution efficiency. The introduction of sequential crossover operators enhances population information exchange to obtain high-quality satisfactory solutions.

1 Formulation of MDHOSVRPPD

The MDHOSVRPPD problem extends traditional VRP through the following evolutionary path:

1) **VRP with Backhauls (VRPB)** [?]: Emerged when delivery vehicles could only perform rear-loaded operations. Vehicles depart from distribution centers with goods, first serve delivery-demand customers, then pickup-demand customers, and finally return to the distribution center. Figure 1: see original paper illustrates VRPB.

2) **VRP with Backhaul and Mixed Load (VRPBM)** [?]: With advances in logistics technology enabling side-loaded vehicles, restrictions on delivery/pickup sequences were removed, creating VRPBM. Figure 1: see original paper shows VRPBM.

Both VRPB and VRPBM address single-depot scenarios with single-type customer demands, exhibiting limitations for modern logistics.

3) **VRP with Simultaneous Delivery and Pickup (VRPSDP)** [?, ?]: Customers have simultaneous delivery and pickup demands, with vehicles serving each customer only once. VRPSDP extends VRPBM, as shown in Figure 1: see original paper.

4) **VRP with Split Deliveries and Pickups (SVRPPD)**: In practice, some customers have large demands. Restricting each customer to single-vehicle service causes severe empty running, increases vehicle usage, and raises dispatch costs. Dror and Trudeau [?] first proposed the Split Delivery VRP (SDVRP), which eliminates the single-visit restriction through reasonable splitting, improving vehicle utilization while reducing travel distance and fleet size. Subsequent research explored splitting for customers with both delivery and pickup demands (SVRPPD). SDVRP can be viewed as an extension combining SDVRP and VRPSDP. Figure 1: see original paper illustrates SVRPPD.

5) **Multi-Depot VRP with Split Deliveries and Pickups (MDSVRPPD)**: Modern logistics evolution has created scenarios with multiple distribution centers in a region, where vehicles from different depots serve all customers and return to their original depots. MDSVRPPD extends SVRPPD, as shown in Figure 1: see original paper.

6) **Multi-Depot Half-Open VRP with Split Deliveries and Pickups (MDHOSVRPPD)**: MDHOSVRPPD features half-open depots. Open, closed, and half-open configurations differ based on vehicle start/end depot positions. Open depots allow free stops during routes without mandatory return; closed depots require identical start and end depots; half-open depots permit returning to any nearby depot after task completion, enabling shared distribution networks. Therefore, MDHOSVRPPD represents a further cost-reduction evolution and extends MDSVRPPD, as

illustrated in Figure 1: see original paper.

Regarding optimal solution theory for MDHOSVRPPD, literature [?] has proven that multi-depot half-open transportation only affects vehicle start/end positions at depots, while customer service routes remain unaffected. Consequently, the demand-splitting optimal solution theory proposed by Dror and Trudeau also applies to MDHOSVRPPD.

2.1 Problem Description

The MDHOSVRPPD problem can be described as follows: Given multiple distribution centers and customer points where depots can collaborate in joint distribution, each customer simultaneously has delivery and pickup demands, and individual customer total demand may exceed vehicle capacity. Homogeneous vehicles depart from depots in a loaded state carrying required goods, serving customers following a delivery-first-then-pickup sequence without exceeding maximum vehicle capacity, and return to the nearest depot upon completion.

The objective is to find a set of vehicle routes satisfying all customer demands while minimizing total delivery distance. Based on this analysis, this paper constructs a vehicle routing model minimizing transportation distance under the following assumptions: a) Distribution center and customer point locations are known; b) Each customer' s delivery and pickup demands are known; c) All vehicles start and end at distribution centers, have identical capacity, and actual load cannot exceed vehicle capacity during transportation; d) Total delivery/pickup demands at each customer may exceed vehicle capacity, and customers can accept multiple services; e) No sequence requirements for customer service; f) No time requirements for customer service.

Symbol definitions are provided in .

2.2 Mathematical Model

The MDHOSVRPPD mathematical model minimizes total distance as follows:

The objective function (1) minimizes distance. Constraints (2)-(3) ensure all customer demands are satisfied. Constraint (4) ensures vehicles carry only delivery goods when departing depots and only pickup goods when returning. Constraint (5) maintains flow conservation at customer points (vehicles cannot remain at customers). Constraint (6) requires each vehicle to depart from exactly one depot and return to exactly one depot. Constraint (7) ensures actual load does not exceed vehicle capacity. Constraint (8) defines variable domains.

Model symbols are described in .

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3 Large Mutation Neighborhood Genetic Algorithm

MDHOSVRPPD is NP-hard, and the split delivery/pickup characteristic increases solution difficulty. While most scholars use constructive heuristics for split delivery/pickup VRP [?, ?, ?, ?], this paper employs modern heuristic algorithms, which can obtain near-optimal solutions within reasonable time for large-scale VRP problems. Combining the Genetic Algorithm based on big Mutation (GAM) with Variable Neighborhood Search (VNS) [?] addresses MDHOSVRPPD—GAM provides excellent global search capability while VNS offers strong local search ability. Therefore, this paper designs the Genetic Algorithm based on big Mutation and Variable Neighborhood Search (GAMVNS).

3.1 Encoding and Decoding

This paper employs two-stage real-number encoding, splitting each actual customer into two virtual customers—one for delivery tasks and one for pickup tasks. The first stage contains a random permutation of delivery tasks, and the second stage contains a random permutation of pickup tasks. All tasks are stored in [Figure 2: see original paper] shows the service order recording.

During decoding, vehicles follow a delivery-first-then-pickup principle. Decoding occurs in two phases: First, customer tasks are assigned to vehicles from left to right using a greedy approach—vehicles perform as much pickup as possible after satisfying delivery tasks, with vehicle constraint verification until no delivery goods remain and pickup goods reach full capacity, at which point the vehicle exits route assignment and returns to a depot, and a new vehicle serves remaining tasks. Second, each vehicle' s start and end depots are determined based on minimizing distance from its first and last served customers to depots.

For example, solving [Figure 2: see original paper] assumes Vehicle 1 satisfies all tasks of Customer 4, serves delivery tasks and partial pickup at Customer 3, and partial delivery and all pickup at Customer 5, then exits when no delivery goods remain and capacity is full. Vehicle 2 serves remaining tasks—first delivery at Customer 5 and pickup at Customer 3, then Customers 2 and 1. [Figure 3: see original paper] shows the assignment process and final routes for Vehicles 1 and 2, with Depots 6 and 7. Final depots are determined by minimizing distance from each vehicle' s first and last customers to depots.

Subsequent operations only iterate on the first stage of [popPnum]. After operations, customers are reassigned to vehicles based on capacity constraints to obtain routes, then start/end depots are decoded. As long as the first stage encoding is correct, the final routes will be correct—the first stage determines subsequent encoding. This approach enables customer demand splitting while ensuring subsequent perturbation operations always produce feasible solutions, improving algorithmic efficiency.

3.2 Fitness Function

The fitness function is typically determined by objective function (1). Since the model minimizes distance, chromosome [popPnum]' s fitness function [s] is:

[s] = 1 / [G]

where [G] is the chromosome' s objective function value.

3.3 Selection and Crossover Operations

Selection combines roulette wheel selection with elitist preservation. Roulette wheel selection chooses individuals based on probability proportional to fitness value—higher fitness yields higher selection probability. Elitist preservation replaces the worst offspring with the best parent. These strategies enhance gene exchange among all individuals and rapidly form stable next generations.

Crossover employs the order crossover operator. When crossing individuals, four genes are randomly selected from two parent individuals, maintaining the positions of remaining customers while eliminating duplicates to produce offspring. [Figure 4: see original paper] illustrates this process.

3.4 Large Mutation Operation

Traditional genetic algorithms suffer from premature convergence. To overcome this, a large mutation strategy is implemented: First, an appropriate mutation rate is set while tracking each generation' s maximum fitness [maxf] and average fitness [avgf]. When:

[maxf] $\times$ α < [avgf]

where α is the concentration factor ($0.5 < \alpha < 1$), indicating high population "concentration," a mutation operation is performed with probability 5+ times the normal rate.

Mutation employs three neighborhood structures—insert, swap, and 2-opt—to enhance local search. Parameters [Se_{Max}] and [Se] control neighborhood search termination. When an individual' s first neighborhood operation finds no improvement after [Se] iterations or reaches maximum cycles [Se_{Max}], it proceeds to the next neighborhood until all structures are exhausted. These strategies significantly improve individual fitness, enhancing GAMVNS' s local search capability and computational efficiency. The algorithm flowchart appears in [Figure 6: see original paper].

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4 Experiments and Results Analysis

This paper validates algorithm effectiveness using MDVRP, MDHOSVRPPD, and SVRPPD benchmark instances. SVRPPD instances demonstrate both algorithm effectiveness and the superiority of multi-depot half-open split delivery/pickup under joint distribution versus single-depot split delivery/pickup under independent distribution. GAMVNS performance is tested through extensive experiments programmed in MATLAB2016B on an Intel® Core™ i5-1035G1 CPU 2.19GHz Windows 10 system with 16GB RAM. Basic parameters appear in .

Experiment 1

To verify algorithm effectiveness, MDVRP instances from literature [?] are tested. These include 30 customers (IDs 1-30), 3 depots (IDs 31-33), vehicle capacity 10t, maximum travel distance 50km, and customer demands $ $2t. compares results from Wolf Pack Algorithm (WPA) [?], Competitive Decision Algorithm (CDA) [?], Tabu Search (TS) [?], Quantum Genetic Algorithm (QGA) [?], and GAMVNS across 10 runs. KOS denotes known optimal solution. Best, Worst, and Avg represent optimal, worst, and average solutions over 10 runs. %Dev shows deviation from optimal, and CPU/s indicates average runtime. Vehicle routes for the optimal solution appear in .

Results show GAMVNS achieves the known optimal solution (116.01) with better worst and average solutions than other algorithms. Runtime averages 7.84s, significantly faster than competitors.

Experiment 2

MDHOSVRPPD involves multiple constraints: multiple depots, multiple customers with delivery/pickup demands, and no standard benchmark set. This paper adapts MDVRP instance P02 from the standard library (https://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances/) to MDHOSVRPPD characteristics. P02 includes 50 customers (IDs 1-50), 4 depots (IDs 51-54), and vehicle capacity 160. Customer delivery/pickup demands are generated using Nagy' s [?] separation rules based on coordinates and original demand. Instance details appear in .

GAMVNS is compared with traditional Genetic Algorithm (GA), Ant Colony Optimization (ACO), Genetic Algorithm based on big Mutation (GAM), and Variable Neighborhood Search (VNS). Results over 20 runs appear in and , showing Best, Num (vehicle count), %Dev, and CPU/s.

Traditional GA, GAM, ACO, and VNS employ single search strategies prone to premature convergence, yielding poor solution quality. GAMVNS outperforms these by 5.66%, 6.88%, 12.83%, and 16.46% in total distance, respectively, with ideal average runtime of 7.2s and optimal deviation of 0.8%. [Figure 7: see original paper] shows rapid convergence with few iterations, demonstrating strong optimization capability.

Experiment 3

To validate that multi-depot half-open split delivery/pickup under joint distribution outperforms single-depot split delivery/pickup (SVRPPD) under independent distribution, MDVRP instances P01-P06 are tested in both modes. Multi-depot coordinates use original data; single-depot coordinates appear in . Customer demands follow Experiment 2 standards. presents Best, Worst, Avg, %Dev, Num, and CPU/s over 20 runs.

Results show maximum optimal deviation of 8.9% and minimum of 0.5%, with all solution times under 8s, demonstrating fast, stable convergence. Comparing the two modes, distance reductions of 31.86%, 13.0%, 25.57%, 9.89%, 27.95%, and 6.55% are achieved. [Figure 8: see original paper] illustrates that multi-depot half-open split delivery/pickup significantly reduces frequent depot-return trips, lowering distribution costs.

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5 Conclusion

This paper addresses the multi-depot half-open vehicle routing problem with split deliveries and pickups under joint distribution through: (1) constructing a mathematical model minimizing total delivery distance; (2) designing a large mutation neighborhood genetic algorithm where large mutation and neighborhood search strategies overcome premature convergence and improve search efficiency; and (3) validating the model and algorithm through three benchmark sets. Results demonstrate that the proposed model and algorithm effectively solve MDHOSVRPPD with stable search and clear advantages.

Furthermore, MDHOSVRPPD suits e-commerce alliances. To provide more realistic distribution routes for e-commerce enterprises, future research should incorporate customer satisfaction and time window constraints into the multi-depot half-open split delivery/pickup vehicle routing problem.

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Note: Figure translations are in progress. See original paper for figures.