A Method for Determining Sea-Level Exhaust Velocity of Rocket Engines Based on Plume Expansion Angle

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Abstract

In recent years, non-contact measurement research for rocket engines has garnered international attention; however, the primary focus has been on the plume temperature field or vibrational dynamics of rocket engines, rather than on measuring exhaust velocity. Conventional exhaust velocity measurement methods involve complex equipment and have long lacked alternative approaches for reference. Therefore, this paper proposes a method for estimating the sea-level exhaust velocity of rocket engines using expansion angle measurements from plume images, which combines a one-dimensional compressible adiabatic flow model, the Prandtl-Meyer function under atmospheric background conditions, and Brent’ s algorithm. This study employs the proposed method to obtain the relationship between plume expansion angle and sea-level exhaust velocity, and compares it with measurement results from conventional methods to validate the method’ s accuracy. The estimation results demonstrate a linear relationship between sea-level exhaust velocity and plume expansion angle, with a relative error of less than 2% compared to conventional methods; thus, the results indicate that this estimation method possesses high accuracy.

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Full Text

A Method for Determining Rocket Motor Sea-Level Exhaust Velocity Based on Plume Expansion Angle

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Abstract

Non-contact measurement techniques for rocket motors have attracted global research attention in recent years, yet studies have primarily focused on plume temperature fields or vibration mechanics rather than exhaust velocity measurement. Traditional exhaust velocity measurement methods require complex equipment and have lacked alternative references for validation. This paper proposes a method for estimating rocket motor sea-level exhaust velocity using plume expansion angle measurements from images, integrating a one-dimensional compressible adiabatic flow model, the Prandtl-Meyer function in atmospheric conditions, and the Brent algorithm. The relationship between plume expansion angle and sea-level exhaust velocity was derived and validated against traditional measurement results. The estimation reveals a linear relationship between sea-level exhaust velocity and plume expansion angle, with a relative error of less than 2% compared to conventional methods, demonstrating high accuracy.

Keywords: non-contact measurement; rocket motor jet flow; sea-level exhaust velocity; plume expansion angle

1 Introduction

Non-contact measurement offers significant advantages, including minimal disturbance to the measured field, high upper measurement limits, and low latency, attracting considerable research interest. However, rocket motor research has predominantly concentrated on plume temperature field measurements or vibration mechanics studies [?]. The Aerospace Science and Industry Corporation' s Sixth Academy has proposed a condensed-phase particle tracing method to measure particle velocities within the plume [?], though particle velocity differs from the motor’ s exhaust velocity.

Traditional specific impulse or exhaust velocity measurements employ large, complex equipment such as thrust stands, flow meters, and weighing platforms [?]. These methods involve weighing rocket fuel before ignition or prior to entering the thrust chamber, recording thrust throughout the entire test firing, and subsequently calculating average specific impulse or exhaust velocity. Consequently, traditional approaches suffer from complex equipment and procedures, single-principle validation without reference results, and inability to measure instantaneous exhaust velocity [?].

Rocket motor plumes exhibit pressure differences between the exit plane and ambient atmosphere, causing expansion upon leaving the nozzle—a phenomenon of supersonic flow turning. Leveraging the advantages of non-contact measurement and considering the relationship between supersonic jet turning angle and Mach number, this paper proposes an estimation method for sea-level exhaust velocity based on plume images, combined with local speed of sound at the exit plane and a one-dimensional compressible adiabatic flow model. The results are compared with traditional sea-level exhaust velocity measurements to validate the proposed method.

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2 Methodology and Error Sources

2.1 Estimation Theory and Assumptions

The rocket motor plume is simplified as a continuous barrel-shaped wave train [?], as shown in [Figure 1: see original paper]. This jet configuration represents a typical low-degree under-expanded jet [?] where the pressure ratio between the exit plane mainstream pressure p_e and ambient pressure p_a satisfies $1 < n < 1.15$. In this configuration, the plume mainstream forms an initial turning angle δ relative to the motor axis within the first wave train unit.

According to the Prandtl-Meyer relation, this turning angle has a one-to-one correspondence with Mach number. Therefore, the exit plane plume Mach number Ma_e can be determined through inverse solution of the Prandtl-Meyer function [?]. Based on the local speed of sound $v_{a,e}$, the jet velocity v_e can be obtained as a function of the turning angle δ and exit plane temperature T_e. Generally, because rocket motor nozzle flow is extremely rapid and represents a well-adiabatic frozen flow, total enthalpy is conserved [?], enabling establishment of the relationship between v_e, T_e, and chamber temperature T_0. In summary, by combining the Prandtl-Meyer function and total enthalpy conservation, the sea-level exhaust velocity v_e can be determined from the plume initial turning angle δ.

From the total enthalpy expression, any nozzle cross-section satisfies the following relation [?]:

$$h^* = c_p T + \frac{v^2}{2} \tag{1}$$

where h^* is the specific total enthalpy (J/kg) and c_p is the specific heat at constant pressure (J/(kg • K)), with rocket motor gas treated as a calorically perfect gas. For a given engine, the energy relationship between the exit plane and thrust chamber can be expressed as:

$$c_p T_0 + \frac{v_0^2}{2} = c_p T_e + \frac{v_e^2}{2} \tag{2}$$

where subscript 0 denotes thrust chamber parameters and subscript e denotes exit plane parameters. Since the gas velocity in the thrust chamber is negligible compared to the exit jet, v_0 can be omitted from Equation (2), which simplifies to:

$$c_p T_0 = c_p T_e + \frac{v_e^2}{2} \tag{3}$$

For an ideal gas, the relationship between specific heat at constant pressure c_p, specific heat at constant volume c_v, adiabatic index k, and gas constant R is given by [?][?]:

$$c_p = \frac{k}{k-1} R = \frac{k}{k-1} \frac{R_g}{M} \tag{4}$$

where $R_g = 8.314$ J/(mol • K) is the universal gas constant and M is the molar mass (kg/mol).

Combining the Mach number definition $Ma = v/v_a$ and the local speed of sound expression $v_a = \sqrt{kRT}$, Equations (3) and (4) yield:

$$Ma_e = \frac{v_e}{\sqrt{\frac{kR_g}{M} T_e}} \tag{5}$$

where Ma_e is the exit plane Mach number and $v_{a,0}$ is the local speed of sound in the thrust chamber. According to the Prandtl-Meyer function, the relationship between Ma_e and the exit plane plume expansion angle δ in the right-running wave system shown in [Figure 1: see original paper] is described by:

$$\delta = \sqrt{\frac{k+1}{k-1}} \arctan\left(\sqrt{\frac{k-1}{k+1}(Ma_e^2 - 1)}\right) - \arctan\left(\sqrt{Ma_e^2 - 1}\right) \tag{6}$$

It is generally believed that Equation (6) lacks a universal continuously differentiable inverse function [?], meaning the existence of $Ma_e = Ma_e(\delta)$ has not been proven analytically. Therefore, this paper employs the Brent method to obtain a numerical solution for $Ma_e = Ma_e(\delta)$ [?].

2.2 Error Source Analysis

Measurement Errors

1. **Viewing Angle and Measurement Angle**: Due to the relative positioning between the camera and rocket launch pad, the tangential angle of the plume cannot be measured accurately in images (see [Figure 3: see original paper]).
2. **Plume Luminous Components**: While liquid rocket motor plumes have relatively transparent compositions and reaction products, solid rocket motor plumes contain aluminum, magnesium, and other materials that produce intense light during combustion, obscuring the plume boundary.
3. **Flow Interference Near Nozzle**: Some motor designs exhibit plume turbulence due to internal choking [?], causing non-axisymmetric plume structures, as shown in the A-4 rocket motor plume in [Figure 3: see original paper]. Additionally, plume shock bows [?] may alter air refraction near the nozzle, affecting measurements.

Gas Parameters and Engine Design

The gas parameters used in this study—combustion temperature T_0, adiabatic index k, and molar mass M—are referenced from [?] and [?]. The reference T_0 values are calculated using the minimum free energy method under theoretical conditions of specified pressure and stoichiometric mixture ratio [?]. Since actual chamber operating pressure and initial ignition mixture ratio deviate from theoretical assumptions, the actual T_0 differs from that of operational rocket motors. Furthermore, actual rocket motor nozzles are not perfectly adiabatic, resulting in reduced total enthalpy at the exit plane compared to the thrust chamber. Liquid rocket motors feature adjustable thrust, while solid rocket motors have diverse propellant formulations. Consequently, the adopted k and M values differ somewhat from actual operating conditions. Additionally, complex design factors such as grain geometry, thermodynamic cycle, and internal flow field distribution may affect exit plane flow properties.

Numerical Solution Errors

Since the Brent method is employed to solve the inverse Prandtl-Meyer function, a backward error in Ma_e is introduced. To achieve sufficient accuracy, this paper sets the backward error tolerance as $E_b(Ma_e) \leq 1.0 \times 10^{-5}$.

2.3 Solution Algorithm

The sea-level exhaust velocity estimation process consists of two steps: (1) using the inverse Prandtl-Meyer function with the Brent method to determine the exit plane Mach number, and (2) applying one-dimensional adiabatic gas dynamics relations to calculate the sea-level exhaust velocity. Input parameters are categorized into three types: (1) aerodynamic/geometric parameters such as plume expansion angle δ; (2) thrust chamber gas thermodynamic parameters including thrust chamber gas temperature T_0, adiabatic index k, and molar mass M; and (3) numerical computation parameters such as backward error E_b. This process is illustrated in [Figure 4: see original paper].

3 Comparison Between Estimated and Traditional Measurement Results

Three types of rocket motors were selected for analysis: composite solid propellant, hydrazine liquid propellant, and ethanol liquid propellant motors, with four models for each type. Their plume images, expansion angles, and designations are shown in [Figure 5: see original paper], [Figure 6: see original paper], and [Figure 7: see original paper].

Estimating sea-level exhaust velocity requires thrust chamber gas thermodynamic properties, listed in .

Table 1 Gas Characteristics Inside Thrust Chamber [**?**, **?**]

Propellant Type	T_0 (K)	Adiabatic Index k	M (g/mol)
Composite Propellant	3000	1.25	25
High-Concentration Ethanol	3100	1.20	23

The sea-level exhaust velocities obtained through traditional measurement methods for the rocket motor models shown in [Figure 5: see original paper], [Figure 6: see original paper], and [Figure 7: see original paper] are presented in .

Table 2 Sea-Level Exhaust Velocities of Various Rocket Motors Measured by Traditional Method [**?**]

Motor Model	Sea-Level Exhaust Velocity (m/s)	Motor Model	Sea-Level Exhaust Velocity (m/s)
UA1205	2375	YF-21	2550
CZ-11 First Stage	2450	DaFY6-2	2650
MGM-31B	2400	RD-253	2750
M55A1	2425	LR-87-AJ-5	2600
RD-100	2200		

Motor Model	Sea-Level Exhaust Velocity (m/s)	Motor Model	Sea-Level Exhaust Velocity (m/s)
RD-103	2300		

Applying the estimation algorithm yields the sea-level exhaust velocity versus expansion angle relationship curves for the three propellant types, shown in [Figure 8: see original paper]. The figure demonstrates a linear relationship between sea-level exhaust velocity and plume expansion angle in both experimental results and algorithm predictions. Based on [Figure 8: see original paper], the relative errors between the estimated and traditional method results are calculated and presented in .

Table 3 Fractional Error Between Plume-Expansion Method and Traditional Method

Motor Model	Relative Error	Motor Model	Relative Error
UA1205	0.99%	YF-21	0.46%
CZ-11 First Stage	1.86%	DaFY6-2	1.86%
MGM-31B	1.46%	RD-253	1.22%
M55A1	0.63%	LR-87-AJ-5	0.08%
RD-100	0.46%		
RD-103	1.24%		

As shown in , the results from the plume expansion angle method differ only slightly from traditional measurements, with relative errors ranging from 0.08% to 1.86%—all less than 2%. This indicates that the plume expansion angle estimation method achieves high accuracy.

Conclusion

This paper integrates thrust chamber gas characteristics with rocket motor jet flow characteristics to propose an estimation method for determining rocket motor sea-level exhaust velocity based on plume expansion angle. The method incorporates a one-dimensional compressible adiabatic flow model, the Prandtl-Meyer function for jet turning in atmospheric conditions, and the Brent algorithm.

Using composite propellant, hydrazine, and high-concentration ethanol rocket motors as examples, the study derived sea-level exhaust velocity versus plume expansion angle curves and compared the calculated results with thrust stand measurements. The research demonstrates that:

1. For the three examined rocket motor types, sea-level exhaust velocity exhibits a linear relationship with plume turning angle, with a proportional coefficient of approximately 66.6–87.03 m/(s · °).
2. The relative error between the estimation method and traditional methods does not exceed 2%, indicating that this approach provides a theoretical foundation for highly accurate non-contact measurement of rocket motor sea-level exhaust velocity.

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