New Clues on the Principle of Relativity in Particle Dynamics

Authors: Chen Chiyi, Chen Chiyi

Date: 2023-08-25T19:22:22+00:00

Abstract

This paper focuses on exploring the physical logic of the relativity principle in particle dynamics. Through causal analysis of dynamics, it is found that the reason why Newton' s second law suffers from difficulties regarding inertial frames and inertial forces is fundamentally due to the neglect of causal correspondence in reference frame dynamics. The dynamical properties of a reference frame should be attributed to reference particles, and the artificially distinguished reference particles and investigated particles should be on a completely equal footing in dynamics, thereby constructing a particle dynamics equation that does not rely on inertial frames and maintains form invariance in all translational reference frames. As for rotating reference frames, on the one hand, the nature of inertial forces is revealed by the new form of the particle dynamics equation as the real force on reference particles after equal weighting of masses, making it clear that they are not physically equivalent to gravity. On the other hand, according to the fundamental principle of causal correspondence, the physics incorporated into rotating reference frames must include the dynamical properties of four non-coplanar reference particles, therefore it is impossible to unify them with the new form of the particle dynamics equation in translational reference frames into a concise equation form. Finally, this paper proposes that all claims of relativity principles having been extended to rotating reference frames deserve careful re-examination and verification.

Full Text

New Clue to the Principle of Relativity for Particle Dynamics

ChiYi Chen[1])†
[1]) Department of Physics, Hangzhou Normal University, Hangzhou 311121, China

Abstract. This article investigates the physical logic underlying the principle of relativity for particle dynamics. Through causal analysis of dynamics, we uncover that the fundamental reason for the puzzles of inertial frames and inertial forces in Newton' s second law is the neglect of causal correspondence in reference frame dynamics. The dynamical properties of a reference frame should be attributed to its reference particle, and the artificially distinguished reference particle and the particle under investigation should occupy completely equal status in dynamics. This leads to a new particle dynamics equation that does not depend on inertial frames and maintains form invariance in all translational reference frames. For rotational reference frames, the new form of particle dynamics equation reveals that inertial forces are essentially real forces acting on reference particles after mass-weighting, making it clear that they are not physically equivalent to gravity. Moreover, according to the fundamental rule of causal correspondence, incorporating rotational reference frames requires including the dynamical properties of four non-coplanar reference particles, which cannot be unified into a concise equation form with the new particle dynamics equation in translational frames. Finally, we propose that any claim of extending the principle of relativity to rotational reference frames deserves careful re-examination and verification.

Keywords: particle dynamics, principle of relativity, translational reference frame, rotational reference frame
PACS: 45.20.Dd, 45.50.Pk, 04.20.Àq, 04.20.Cv

Project supported by the Natural Science Foundation of Zhejiang Province (Grant No. Y6110778), the National Natural Science Foundation of China (Grant No. 11105036), and the Chinese Academy of Sciences Knowledge Innovation Program (Grant No. KJCX2-YW-W10).

1. Introduction

The principle of relativity in dynamics represents a fundamental property of physical laws, indicating the scope of applicability within which dynamical equations maintain both their mathematical form and physical definitions. In Newtonian mechanics, the principle of relativity for particle dynamics is the Galilean principle of relativity, which states that classical mechanical laws maintain the same mathematical form in any inertial reference frame (inertial frame), implying that all inertial frames are equivalent. Newton' s second law is thus only applicable to inertial frames (denoted as O), with the object under study being p:

$$\sum \mathbf{F}_{p\to O} = m_p \mathbf{a}_{p-O}$$

However, a significant puzzle arises: the definition of an inertial frame itself depends on Newton' s first law. Specifically, if an object can remain at rest or in uniform linear motion without any interaction, the reference frame is inertial. This definition is substantially contained within Newton' s second law, creating a logical circularity if one cannot *a priori* identify an inertial frame. As demonstrated in Section 2, this logically deficient circular definition is entirely avoidable.

In practice, all real reference frames are non-inertial (denoted as O), and no dynamical formula can be directly applied. Even through mathematical manipulation of Newton' s second law (by transforming the kinematic part of the reference frame), one can force an expression relative to non-inertial frame O:

$$\sum \mathbf{F}_{p\to O} - m_p \mathbf{a}_{O-\text{inertial}} = m_p \mathbf{a}_{p-O}$$

This expression is not a true dynamical law because the second term on the left is not a real force–it cannot be calculated through theoretical formulas like other common forces, nor can it be measured (since this still requires an inertial frame as a prerequisite). This virtual force is defined as the inertial force:

$$\mathbf{F}_{\text{inertial}} = -m_p \mathbf{a}_{O-\text{inertial}}$$

Thus, Newton's second law can be formally simulated for any moving (including rotating) non-inertial frame O as:

$$\sum \mathbf{F}_{p\to O} + \mathbf{F}_{\text{inertial}} = m_p \mathbf{a}_{p-O}$$

Notably, the calculation formula for inertial forces still requires first finding an inertial frame, meaning it does not fundamentally break through the limitation of Newtonian dynamics being restricted to inertial frames. In practical applications, since absolute space cannot be referenced to measure kinematic quantities, one must directly choose approximate inertial frames, and if the approximation is insufficient, must even consider the magnitude and direction of inertial forces.

In special relativity, the mechanical principle of relativity is extended to the special principle of relativity, requiring that all physical laws (mechanical, electromagnetic, and other interaction dynamics) maintain mathematical form in all inertial reference frames. The scope of equivalent reference frames remains limited to inertial frames, not significantly broadening the applicability compared to Newtonian mechanics.

In general relativity, the general principle of relativity is introduced as an *a priori* axiom, stating that all reference frames (including translational and rotational non-inertial frames) are equivalent, and that objective physical laws should maintain mathematical form in all physical reference frames. This represents the maximal extension of the principle of relativity, escaping the inertial frame dilemma that previously seemed unavoidable. For general relativity, general covariance is a crucial theoretical property, and the general principle of relativity serves as its physical foundation, making it one of the cornerstones of Einstein' s theory. Einstein' s "freely falling elevator" thought experiment provided the inspiration for the (non-weak) equivalence principle, which posits the physical equivalence of inertial and gravitational forces. This equivalence principle provides the "conceptual ladder" connecting to the general principle of relativity.

Nevertheless, compared to the Galilean and special principles of relativity, the general principle of relativity still lacks convincing theoretical proof. This paper explores the physical and logical construction of the principle of relativity for particle dynamics, analyzing from the perspective of causal correspondence the possibility of incorporating both translational and rotational reference frames. The paper is organized as follows: Section 1 provides background on the principle of relativity. Section 2 examines the formal logic of Newton' s second law and constructs an explicit particle dynamics equation applicable to general translational reference frames based on causal correspondence. Section 3 reflects on the modification of Newton' s second law, demonstrating the essential and indispensable role of reference objects and reference frames in particle dynamics physics. Section 4 applies the principle of causal correspondence, combined with the physics of reference frames, to analyze the technical requirements for extending the principle to rotational reference frames, identifying fundamental difficulties. Finally, we argue that a moderate principle of relativity can satisfy the basic requirements of physics.

2. Causal Correspondence Formal Logic and Formal Improvement of Newton' s Second Law

The predictive power of physical laws allows us to distinguish cause from effect, making causal correspondence a fundamental principle of physics. Particle dynamics is essentially a causal law relating forces and motion: forces are the cause, and acceleration effects are the effect. To make Newton' s second law applicable to as many real reference frames (denoted as O) as possible, its generalized form would be:

$$\sum \mathbf{F}_{p\to O} = m_p \mathbf{a}_{p-O}$$

The left side depends only on the object under study, while the right side contains the acceleration $\mathbf{a}_{p-O}$ of object p relative to reference frame O (or more precisely, relative to the corresponding reference object o). Thus, the right side actually depends on both the object under study p and the reference object o. Since force is the cause and acceleration is the effect, and considering the choices of p and o are completely independent, Newton' s second law suffers from formal logical causal asymmetry and inconsistency between its two sides. This is precisely why it can only theoretically hold in so-called inertial frames, while in practice no strict inertial frame can be found.

According to the causal correspondence principle, the force term on the left side of the particle dynamics equation should also account for the forces on the reference object. Within the Newtonian framework, there is a conceptual question: are reference objects and reference frames purely mathematical and *a priori*, or are they physical? From a modern perspective, reference frames are not purely mathematical or *a priori*; a physically useful reference frame must have its reference origin established on a real reference object. Based on this materialist view that reference frame physics depends on real reference objects, a new form of particle dynamics equation can be strictly derived from Newton' s second law but with greater generality.

The derivation proceeds as follows: assume an inertial frame exists, choose an arbitrary reference object o, and let the object under study be p. The distinction between "object under study" and "reference object" is a human choice, and at the fundamental physical level, they occupy completely equal status. Therefore, under Newtonian dynamics, both obey the same laws:

$$\sum \mathbf{F}_{p\to\text{inertial}} = m_p \mathbf{a}_{p-\text{inertial}}, \quad \sum \mathbf{F}_{o\to\text{inertial}} = m_o \mathbf{a}_{o-\text{inertial}}$$

Dividing both equations by their respective masses and subtracting yields:

$$\sum \frac{\mathbf{F}_{p\to\text{inertial}}}{m_p} - \sum \frac{\mathbf{F}_{o\to\text{inertial}}}{m_o} = \mathbf{a}_{p-\text{inertial}} - \mathbf{a}_{o-\text{inertial}} = \mathbf{a}_{p-o}$$

The final step uses the natural definition that o is the reference origin of frame O, and also implies that O must be a translational reference frame. This equation (7) applies to any reference object o and any non-rotating reference frame O with o as its origin (similar to an inertial frame). Thus, the new particle dynamics equation already demonstrates a principle of relativity that maintains form invariance in any translational reference frame.

Some might argue that a better form of Newtonian particle dynamics is $\frac{d(m\mathbf{v})}{dt} = \mathbf{F}$. However, this only adds applicability to variable-mass problems. Variable-mass issues in classical low-speed scenarios (such as rocket problems) essentially reduce to separation and relative motion between particles in a system, not the dynamics of a single particle. Therefore, the fundamental particle dynamics equation in Newtonian mechanics remains $\mathbf{F} = m\mathbf{a}$, with the alternative form being an effective version for particle systems. In special relativity, the dynamics tends to support $\mathbf{F} = \frac{d(m\mathbf{v})}{dt}$ because particle mass can change, with its physical origin traceable to the constancy of the speed of light. But even in relativistic mechanics, the true fundamental starting point remains $\mathbf{F} = m\mathbf{a}$, from which relativistic momentum in specific frames is derived.

From a comprehensibility perspective, traditional Newton' s second law should be reducible to an empirical law derived from numerous classical mechanics experiments: under a fixed reference frame, describing a moving object, the causal relationship between newly applied forces and the resulting new relative acceleration compared to its previous mechanical state. Generally, this empirical law can be expressed as:

$$\Delta \mathbf{F} = m\Delta \mathbf{a}$$

where $\Delta \mathbf{F}$ represents the increment in force compared to the previous state, and $\Delta \mathbf{a}$ represents the resulting increment in acceleration. Historically, this differential relationship formed the basis for determining force formulas, including gravitation, friction, and elasticity. Once force calculation formulas were established in specific cases, the causal dynamics could be tested in general situations. If we view force source statistics and summation as a kind of "integration," then the definite integral form of this differential causal relationship should be:

$$\sum \mathbf{F}_{p\to O} - \sum \mathbf{F}_{o\to O} = m_p \mathbf{a}_{p-o}$$

rather than the theoretical form given by Newton' s second law:

$$\sum \mathbf{F}_{p\to O} = m_p \mathbf{a}_{p-O}$$

where o represents the reference object that every reference frame must have. Strictly speaking, the new particle dynamics equation and the traditional Newton' s second law are not simply equivalent; rather, it corrects an independent term omitted during the "definite integration" process of the original causal integral form.

3. The Indispensable Status of Reference Objects in Particle Dynamics

Newton' s second law considers only the motion state of reference objects while ignoring their forces, which is why it can only theoretically hold in inertial frames while no strict inertial frame can be found in practice. Some might recall that traditional applications of Newton' s second law seem not to require reference objects, but careful examination reveals otherwise. Any real physical application requires a reference object; otherwise, the reference frame origin cannot be defined. This is particularly obvious when selecting non-inertial frames, where one must specify the reference object to understand what the non-inertial frame is. However, when selecting ground or laboratory frames, this requirement is very subtle, as if reference objects were irrelevant. The fundamental reason is that these frames are approximated as inertial, so the reference object force term corresponding to inertial forces is ignored, making reference objects seem optional.

But even if reference object forces are ignored in the force term, acceleration must still be measured relative to the actual reference object; otherwise, there would be no issue of finding inertial frames. A physical reference frame needs to be found in the real world, and its existence is questionable, whereas a mathematical reference frame is a definitional concept that always exists. When using ground or laboratory frames, one actually selects any stationary object in that environment as the reference object. Acceleration measurement is performed relative to any point stationary on the ground (or in the laboratory), which means relative to any stationary object at that point. In this case, the reference object is that stationary object, whose size can be arbitrarily determined as long as it can be treated as a particle. When the ground or laboratory frame is approximated as inertial, the reference object' s mass and other properties become irrelevant to calculations. However, a real object must exist at that point; otherwise, the validity and operability of physical measurements (such as acceleration) cannot be guaranteed.

In summary, regardless of the specific form of particle dynamics, whether relativistic or non-relativistic, if the reference object' s forces do not appear in the dynamical equation, the equation cannot be extended to real reference frames.

4. Technical Requirements for Incorporating Rotational Reference Frames into the Principle of Relativity

First, we examine the physical foundation of the general principle of relativity. Compared to Newton' s second law (equation 1), equation (7) contains an additional term that explains the extra term appearing during reference frame transformation—the inertial force. Comparing the last line of equation (2) with equation (7) reveals that inertial forces, originating from Newton' s second law' s formal system, have the following physical essence in translational reference frame transformations:

$$\mathbf{F}_{\text{inertial}} = -\frac{m_p}{m_o} \sum \mathbf{F}_{o \to \text{inertial}}$$

Thus, the essence of inertial force is the real force acting on reference object o, weighted by mass ratio. This force can be gravitational or any other common non-gravitational interaction. Importantly, this force acts not on the object under study but on the reference object. Since the concept and puzzles of inertial forces originate from Newton' s second law' s formal system, the best and most thorough solution is to eliminate the inertial force problem within the original framework. The above logical deduction achieves this without any additional assumptions or ambiguities. Given this clear explanation, inertial forces are not physically equivalent to gravitational forces, providing counter-evidence to the equivalence principle that Einstein used to build the general principle of relativity.

Second, from a mathematical perspective, analyzing the invariance of particle dynamics under rotational reference frame transformations while adhering to causal correspondence allows qualitative analysis from physical logic. As is well known, determining the complete kinematic properties of a physically rigid rotating reference frame requires information from at least four non-coplanar particles. Therefore, when constructing dynamical equations for physical rotating reference frames, the theory must simultaneously incorporate the causal correspondence of at least four reference particles while maintaining a concise form. This has never been considered or achieved, and at least under current mathematical language frameworks, cannot be done naturally. Moreover, examining a moving particle in a rotating reference frame would require simultaneously introducing four non-coplanar reference particles, which is extremely uneconomical formally. Therefore, Einstein' s general principle of relativity in its current form is unreliable.

We therefore recommend that the physical relativity concept of particle dynamics equations be extended only to all translational reference frames. For rotational parts, rotational reference frames can be used in principle, but should be understood as first transforming mathematically to a translational reference frame, then applying the new particle dynamics equation.

In summary, the only correct approach to extending the principle of relativity for particle dynamics is:

$$\begin{cases} \text{Translational frames:} & \sum \mathbf{F}_{p\to O} - \sum \mathbf{F}_{o\to O} = m_p \mathbf{a}_{p-o} \\ \text{Rotational frames:} & \text{Physically depends on 4 non-coplanar reference particles} \\ & \text{Currently impossible to implement mathematically} \end{cases}$$

Based on these physical and mathematical analyses, we propose that any claim of extending the principle of relativity to rotational reference frames deserves careful re-examination and verification.

5. Conclusion

This paper addresses the practical applicability of dynamics, focusing on real reference frames defined through definite mathematical relationships. Hypothetical reference frames derived through mathematical transformation from real ones are not discussed, as their dynamics can be obtained through mathematical transformation of dynamics in real frames. What physics most urgently needs to solve is invariance in physical reference frames, not formal invariance in mathematical reference frames. Regarding the relationship between reference objects and reference frames: if the reference object is specified first, a reference frame can be naturally established with it as the origin; if the reference frame is specified first, any real object fixed in the frame can serve as the reference object. For example, the commonly used ground frame actually selects any object fixed on the ground as the reference object.

From a practical standpoint, any reference object we can find is in perpetual motion in the universe, yet we can never determine its exact position, velocity, or acceleration in the cosmos. Although distant galaxies can help determine any object' s rotation relative to the universe, the ideal solution for dynamics is that once a reference object is chosen, one can completely determine any object' s motion relative to this reference object based solely on force analysis, without changing the mathematical form of dynamical laws. This is the basic spirit of Einstein' s principle of relativity.

As discussed, the new particle dynamics equation (9) demonstrates a relativity principle with form invariance under arbitrary translational reference frame transformations. Compared to the Galilean principle based on inertial frames and the general principle based on arbitrary frames, the translational invariance principle occupies a middle ground, supported by the logical derivation of the new equation. We can therefore call the form invariance under translational reference frame transformations demonstrated by equation (9) the **moderate principle of relativity**.

As the first revelation of the essence of inertial forces by the new particle dynamics equation, equation (10) clearly does not support the general principle of relativity as an axiom in Einstein' s general relativity. According to the fundamental spirit of causal correspondence in particle dynamics, regardless of the specific form—relativistic or non-relativistic—if reference object forces do not appear in the dynamical equation, the equation cannot be extended to real reference frames. Further extension to rotational reference frames would require simultaneously incorporating at least four reference particles' kinematics and forces while maintaining a concise form, which has never been achieved. Therefore, any extension of the particle dynamics principle of relativity to rotational reference frames requires cautious treatment and careful verification.

In practice, the moderate principle of relativity already meets basic observational and application needs. On one hand, we can never determine Earth' s exact position, velocity, or acceleration in cosmic space, making absolute division of translational motion impossible. On the other hand, due to the constant and unchanging cosmic background, we can always determine any reference frame' s rotation relative to it using sufficiently distant galaxies. In practice, since cosmic space background is objective and constant, its directions are also objective, allowing us to define coordinate axes using distant galaxies and establish non-rotating reference frames. Thus, rotation of reference frames can be distinguished in principle. For particle dynamics, any translational phenomenon can always be reduced to relative motion between particles, while a single particle has no concept of self-rotation. After treating any reference object as a particle, the rotation problem disappears. Therefore, the rotation issue of reference frames is essentially a mathematical problem that can be separated from the physics of dynamical relativity. The most concise yet sufficient principle of relativity should be the invariance of physical laws under any translational reference object (i.e., any non-rotating reference frame).

Acknowledgments

We thank Professors Zhao Zheng and Liang Canbin of the Physics Department at Beijing Normal University for explaining and emphasizing the background of this problem in their lectures. We thank Professor Zhang Hongsheng of Jinan University for persistent long-term discussions. We thank Researcher Zhang Xinmin for guidance and support during my year-long study at the Institute of High Energy Physics—my mentor in cosmology, without whose thoughts on cosmological metrics this work would not have emerged. We thank Researcher Chen Daming, Researcher Cao Zhoujian, and senior colleagues Li Mingzhe and Gao Changjun for in-depth discussions. Finally, we thank my supervisors Researcher Shen Yougen, Professor Li Kang, and Professor Zhu Chuanjie for their consistent support over more than a decade.

References

[1] Cheng C Z, Jiang Z Y 2006 *General Physics* (Beijing: Higher Education Press) 6th Edition p1-p90 (in Chinese)
[2] Zhou Y B 2009 *A Course in Theoretical Mechanics* (Beijing: Higher Education Press) 3rd Edition p8-p20; p31-p33 (in Chinese)
[3] Liu L, Zhao Z 2004 *General Relativity* (Beijing: Higher Education Press) 2nd Edition p9-12; p26-p30; p188-p190 (in Chinese)
[4] Douglas M J 1998 Leibniz on the Foundations of the Calculus: The Question of the Reality of Infinitesimal Magnitudes *Perspectives on Science* Vol. 6 p6-p40
[5] Hanoch G, Jürgen R 2017 *The Road to Relativity: The History and Meaning of Einstein' s "The Foundation of General Relativity"* Princeton University Press, Annotated, p51
[6] Kinsler, Paul 2011 How to be causal: time, spacetime and spectra *European Journal of Physics* Vol. 32 Issue 6 p1687-p1700
[7] Chen C Y 2014 Investigation of the Formalism of Particle Dynamics under the Framework of Classical Mechanics *Journal of Zhejiang University (Science Edition)* Vol.41 Issue 5 p531-p541 (in Chinese)
[8] Li K, Yang J S 2011 *Introduction to Modern Physics* (Beijing: Tsinghua University Press) p8 (in Chinese)
[9] Isaac Newton 1729 *Mathematical Principles of Natural Philosophy and His System of the World*, translated into English by Andrew Motte in 1729. The translations revised, and supplied with a historical and explanatory appendix, by Florian Cajori. 1934 Cambridge: At the University Press p1-p50
[10] Essen H 2013 Mechanics, cosmology and Mach' s principle *Eur. J. Phys.* 34 p139-p145
[11] Gao B K, Xie T Z 1991 An easily overlooked inertial force acting on the Earth *College Physics* Vol 10 Issue 11 p46-47 (in Chinese)
[12] Gao B K 1995 A puzzling problem in Mechanics *College Physics* Vol.14 Issue 5 p20-24 (in Chinese)
[13] Li F, Gao B K 2005 No circular argument in the law of inertia *College Physics* Vol.24 Issue 4 p14-17 (in Chinese)

[14] Bernard F S 1985 *The First Course in General Relativity* Cambridge University Press p30-p50
[15] Weinberg S. 1972 *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* Part 3 (New York: John Wiley & Sons, Inc.)

Note: Figure translations are in progress. See original paper for figures.