Generalized Short-Circuit Ratio for Operation of Renewable Energy Multi-Infeed Systems Considering Actual Operating Conditions

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Abstract

With the development of renewable energy, the voltage support strength of multi-infeed renewable energy systems is gradually decreasing, necessitating effective strength assessment methods. If all equipment is assumed to operate at rated operating points, existing power grid strength indicators and their critical values can accurately quantify strength; however, the actual operating conditions of equipment are diverse and complex, which violates the definition conditions of existing indicators and renders these methods ineffective. To address this issue, this paper focuses on the quantification of voltage support strength under actual operating conditions. First, the mapping relationship between the dynamic characteristics of grid-following renewable energy equipment and operating condition parameters is identified. Second, based on eigen-subspace perturbation theory, an equivalent single-infeed system that can approximately characterize the stability of the actual system is derived, and upon this basis, the operating generalized short-circuit ratio is defined, thereby enabling rigorous quantification of strength under actual operating conditions using both the operating generalized short-circuit ratio and the equipment critical operating short-circuit ratio. Additionally, this paper reveals the influence mechanism of actual operating conditions on strength. Finally, the effectiveness of the proposed method is verified through case studies.

Full Text

Operational Generalized Short Circuit Ratio of Renewable Energy Multi-infeed Systems Considering Actual Operating Conditions

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Abstract

With the rapid development of renewable energy, the voltage support strength of renewable energy multi-infeed systems is gradually decreasing, necessitating effective strength assessment methods. Existing grid strength indices and their critical values can accurately quantify system strength when all grid-following converters operate at their rated operating points. However, actual operating conditions are diverse, complex, and variable, which disrupts the foundational assumptions of existing indices and renders these methods ineffective. This paper addresses the quantification of voltage support strength under actual operating conditions. First, we discover the mapping relationship between the dynamic characteristics of grid-following renewable energy converters and their operating condition parameters. Second, based on eigen-subspace perturbation theory, we derive an equivalent single-infeed system that can approximate the stability of the actual system. Building upon this foundation, we define the operational generalized short-circuit ratio (OgSCR) and establish a rigorous framework for quantifying strength under actual operating conditions using both OgSCR and the device critical operational short-circuit ratio (OSCR0). Furthermore, this paper reveals the underlying mechanism through which actual operating conditions influence system strength. Finally, case studies validate the effectiveness of the proposed method.

Keywords: voltage support strength; actual operating conditions; operational generalized short-circuit ratio; device critical operational short-circuit ratio

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The construction of new power systems with an increasing proportion of renewable energy is essential for achieving the "dual carbon" goals. Currently, renewable energy typically integrates into the grid through grid-following power electronic converters. However, the phase-locked loop (PLL) vector control employed by these grid-following converters can induce small-signal synchronization instability, particularly in systems with weak voltage support strength (hereinafter referred to as "system strength" considering that system strength issues related to low-inertia characteristics can be considered separately) [1]-[3], thereby threatening the secure and stable operation of power systems [4][5]. This creates an urgent need for scientific assessment methods for system strength.

If all grid-following renewable energy converters operate at their rated operating points, existing grid strength indices and device critical short-circuit ratios can quantify system strength. These indices fall into two categories: those defined based on static power flow equations, such as short-circuit ratio with interaction factors [6], equivalent short-circuit ratio [7], and multi-station short-circuit ratio [8]; and those defined based on sensitivity equations, such as the generalized short-circuit ratio [9]-[11].

However, influenced by environmental factors like weather, renewable energy exhibits randomness, intermittency, and volatility [12][13], causing actual operating conditions to vary significantly and complexly among devices (often operating at non-rated points, primarily manifested as actual power output and terminal voltage frequently deviating from rated values with potentially large differences). These actual operating conditions affect system strength [6][7], yet the aforementioned grid strength indices and critical values are mostly defined based on rated conditions. Direct application of these methods may lead to misjudgment of system stability. Although reference [14] extended the generalized short-circuit ratio to a generalized operational short-circuit ratio to consider the impact of actual power output on system strength, it neither provides a theoretical foundation for considering actual output nor accounts for the influence of device terminal voltage on system strength, thus failing to accurately measure system strength under actual operating conditions.

To address this, this paper proposes the operational generalized short-circuit ratio (OgSCR) index and its critical value, forming a rigorous method for quantifying system strength under actual operating conditions. Specifically, we first discover the mapping relationship between device dynamic characteristics and operating condition parameters. Based on multi-scale methods and the multi-scale control of grid-following converters, we approximately decouple actual operating conditions from device dynamics and equivalently transform them to the network side. Second, using eigen-subspace perturbation theory, we construct an appropriate equivalent homogeneous system. By using the eigen-subspace of this equivalent homogeneous system to approximate that of the actual system, we solve for an equivalent single-infeed system that can characterize the stability of the actual system. Building upon this, we define OgSCR and the device critical operational short-circuit ratio (OSCR0), and present a quantification process for system strength under actual operating conditions. Additionally, we reveal the influence mechanism of actual operating conditions on system strength from two perspectives: index modification and critical value modification. Finally, case studies validate the necessity of considering actual operating conditions and the effectiveness of OgSCR.

1 Problem Description and System Modeling

1.1 Problem Description

Consider the renewable energy multi-infeed power system shown in Fig. 1, which contains n converter buses, m intermediate passive buses, and an infinite bus. For convenience in subsequent derivation and mechanism analysis, we first consider that the n grid-following converters have similar external characteristics under rated conditions and that the AC network line resistance-to-inductance ratio ($\tau = R/L$) is identical. More complex scenarios will be discussed later. If these devices operate at their rated points, the generalized short-circuit ratio can quantify system strength, and the difference between the grid generalized short-circuit ratio and the device critical short-circuit ratio reflects the small-signal stability margin in the PLL bandwidth and below, primarily concerning small-signal synchronization stability and static voltage stability.

However, considering actual operating conditions—where device terminal voltage ranges between 0.9-1.1 pu and active power output ranges between 0.2-1.0 pu during normal operation [14][15]—the actual operating points of these n converters may differ significantly. This violates the assumptions required for the generalized short-circuit ratio, rendering the method ineffective. This raises the direct question: when these devices deviate from rated operating conditions, how can we rigorously calculate grid strength and its critical value to achieve accurate quantification of system strength? The research problems addressed in this paper can be summarized as:

Problem 1: Similar to the definition of the generalized short-circuit ratio, how should an index be defined to accurately quantify system strength under actual operating conditions?

Problem 2: What is the mechanism through which operating conditions affect system strength?

1.2 Multi-infeed System Modeling

Under the global xy coordinate system, we establish the linearized model of the system shown in Fig. 1 to obtain its closed-loop characteristic equation. First, looking from the converter terminals toward the AC grid, the sensitivity equation relating only the voltage increment $\Delta U_{xy}(s)$ and current increment $\Delta I_{xy}(s)$ at the converter buses in the global synchronous reference frame can be expressed as [10]:

$$\Delta U_{xy}(s) = -\frac{1}{s}\left(I_2 \otimes B\right)\Delta I_{xy}(s) \tag{1}$$

where $\Delta U_{xy} = [\Delta U_{1x}\ \Delta U_{1y}, \dots, \Delta U_{nx}\ \Delta U_{ny}]$, $\Delta I_{xy} = [\Delta I_{1x}\ \Delta I_{1y}, \dots, \Delta I_{nx}\ \Delta I_{ny}]$; for $i = 1, \dots, n$, U_{ix}, U_{iy} are the x and y components of the i-th converter' s terminal voltage in the global coordinate system; I_{ix}, I_{iy} are the x and y components of the i-th converter' s output current in the global coordinate system; B is the network admittance matrix containing only converter nodes, $B \in \mathbb{R}^{n\times n}$; Δ denotes variable increments; s is the Laplace operator; $\otimes$ denotes the Kronecker product; and ω_0 is the synchronous speed of the AC grid.

Looking from the grid toward the converters, the sensitivity equation relating the voltage increment $\Delta U_{xy}(s)$ and current increment $\Delta I_{xy}(s)$ at the converter terminals in the global coordinate system can be expressed as [10]:

$$\Delta I_{xy}(s) = Y_{IBR}(s)\Delta U_{xy}(s) \tag{2}$$

where $Y_{IBR}(s) = \mathrm{diag}\{Y_{IBR1}(s), \dots, Y_{IBRn}(s)\}$ is the admittance transfer function matrix of the converters, and the i-th converter' s admittance transfer function matrix $Y_{IBRi}(s)$ is:

$$Y_{IBRi}(s)=\frac{H_i(s)G_{VF}(s)}{sL_f+H_i(s)}\begin{bmatrix}1 & -H_{pll}(s)\\ H_{pll}(s) & 1\end{bmatrix}+\frac{H_{dc}(s)H_P(s)}{sC_{dc}U_{dc0}}\begin{bmatrix}\cos^2\theta_i & \sin\theta_i\cos\theta_i\\ \sin\theta_i\cos\theta_i & \sin^2\theta_i\end{bmatrix}$$

where $H_i(s)=K_{ip}+K_{ii}/s$ represents the PI control parameters of the inner loop; $H_{pll}(s)=(K_{pllp}+K_{plli}/s)/s$ represents the PI control parameters of the PLL; $H_{dc}(s)=K_{dcp}+K_{dci}/s$ represents the PI control parameters of the DC voltage outer loop; $H_P(s)=K_{Pp}+K_{Pi}/s$ represents the PI control parameters of the active power control loop; C_{dc} is the DC-side capacitance; U_{dc0} is the capacitor voltage; $f_{VF}(s)$ is the voltage feedforward filter function of the current inner loop; L_f is the filter inductance; and U_i, I_i, P_i are the per-unit values of the i-th converter' s terminal voltage, injected current, and output active power, respectively, satisfying $I_i=P_i/U_i$ (using a unified base).

Combining (1) and (2), the closed-loop characteristic equation of the system shown in Fig. 1 can be expressed using the determinant as:

$$\det\left(I_2\otimes B+sY_{IBR}(s)\right)=0 \tag{3}$$

2 Simplified Description of System Strength Under Actual Operating Conditions

Solving the roots of the closed-loop characteristic equation (3) enables system stability analysis. However, this calculation is difficult to implement due to the large number of converters in the system. Therefore, we consider simplifying the actual system into a simpler one to facilitate streamlined analysis of system strength.

For heterogeneous multi-infeed systems with differing converter dynamic characteristics, converters can be viewed as perturbations to the equipment or network dynamics of a homogeneous system. Based on eigen-subspace perturbation theory, an equivalent single-infeed system that approximately characterizes the stability of the original system can be derived, as detailed in references [9][17]-[20]. We consider adopting this analytical approach. However, based on the above analysis and equation (3), differences in actual operating conditions among converters may be significant and couple complexly with converter dynamics, resulting in large variations in converter external characteristics under actual operating conditions. This prevents direct treatment as perturbations to converter dynamics, making it difficult to solve for an equivalent single-infeed system that approximates the stability of the original system.

To address this, this section first uses multi-scale methods and the multi-scale control of grid-following converters to approximately decouple actual operating conditions from converter dynamics and equivalently transform them to the network side, thereby reducing dynamic differences among converters. Second, based on eigen-subspace perturbation theory, we solve for an equivalent single-infeed system that can approximate the stability of the actual system.

2.1 Approximate Decoupling of Actual Operating Conditions and Converter Dynamics

According to multi-scale methods, when analyzing stability problems in the low-to-medium frequency band (slow time scale), high-frequency (fast time scale) control loop dynamics can be approximately neglected [21]. Therefore, for the small-signal stability problems within the PLL bandwidth and below that are the focus of this paper, the voltage feedforward filter dynamics can be approximately ignored [21][22]. Further considering $I_i = P_i/U_i$, the converter model can be simplified as:

$$Y_{IBRi}(s) \approx \frac{H_i(s)}{sL_f + H_i(s)} \begin{bmatrix} 1 & -H_{pll}(s) \\ H_{pll}(s) & 1 \end{bmatrix} + \frac{P_i}{U_i^2} \begin{bmatrix} \cos^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin\theta_i\cos\theta_i & \sin^2\theta_i \end{bmatrix} \tag{4}$$

To decouple actual operating conditions from converter dynamics and reduce the dynamic differences among converters caused by varying operating conditions, we left-multiply (1) by the matrix $\mathrm{diag}(U_i^2/P_i) \otimes I_2$. The system closed-loop characteristic equation then becomes:

$$\det\left(\mathrm{diag}\left(\frac{U_i^2}{P_i}\right) \otimes I_2 \cdot B + sY'_{IBR}(s)\right) = 0 \tag{5}$$

where $\mathrm{diag}(\cdot)$ denotes a diagonal matrix; $T(s) = Y'_{IBR}(s) + Y'_{net}(s)$ is the inverse of the closed-loop transfer function matrix (referred to as the "actual system matrix").

From (5), after approximate decoupling, both the numerator and denominator of the converter model are linear functions of terminal voltage and do not contain actual power output. Under steady-state conditions, converter terminal voltage and actual power output range from 0.9-1.1 pu and 0.2-1.0 pu, respectively. Consequently, the model differences among converters caused by varying operating conditions are significantly reduced, allowing us to consider that actual operating conditions and converter dynamics are approximately decoupled.

2.2 Approximate Solution for the Equivalent Single-infeed System

Since the converter model after approximate decoupling still contains converter terminal voltage, small differences remain among converter models. We consider using eigen-subspace perturbation theory to construct an appropriate equivalent homogeneous system. By using the eigen-subspace of this equivalent homogeneous system to approximate that of the actual system, we can solve for an equivalent single-infeed system that can characterize the stability of the actual system.

According to eigenvalue definitions, the equivalent network admittance matrix $\mathrm{diag}(U_i^2/P_i)B$ has n double eigenvalues λ_i $(i = 1, \dots, n)$, with a corresponding set of eigen-subspaces (X_i, Y_i), where $X_i = x_i \otimes I_2$ and $Y_i = y_i \otimes I_2$. The column vectors of X_i and Y_i are the right and left eigenvectors of the equivalent admittance matrix corresponding to λ_i, respectively, and satisfy [23]:

$$\mathrm{diag}\left(\frac{U_i^2}{P_i}\right) B x_i = \lambda_i x_i, \quad y_i^T \mathrm{diag}\left(\frac{U_i^2}{P_i}\right) B = \lambda_i y_i^T$$

Let $X = (X_1 \dots X_n)$ and $Y = (Y_1 \dots Y_n)$. Using X and Y, the equivalent network admittance matrix can be transformed as:

$$\mathrm{diag}\left(\frac{U_i^2}{P_i}\right) B = X \cdot \mathrm{diag}(\lambda_i I_2) \cdot Y^T \tag{6}$$

Using the eigen-subspace (X_1, Y_1) corresponding to the minimum eigenvalue λ_1, we construct an equivalent homogeneous system. Its inverse closed-loop transfer function matrix (referred to as the "equivalent homogeneous system matrix") is:

$$T_{hom}(s) = Y_{IBR\_hom}(s) + \lambda_1 I_2 \tag{7}$$

where x_{1i}, y_{1i} are the i-th elements of x_1, y_1, respectively, and p_{1i} is the participation factor of the i-th converter; $Y_{IBR\_hom}(s)$ is the equivalent converter model.

Combining (6) and (7) and using X and Y, the closed-loop characteristic equation of the equivalent homogeneous system can be transformed as:

$$\det\left(T_{hom}(s)\right) = \det\left(Y_{IBR\_hom}(s) + \lambda_1 I_2\right) = 0 \tag{8}$$

From (8), using (X_i, Y_i), the closed-loop characteristic equation of the equivalent homogeneous system can be decoupled into n single-infeed system closed-loop characteristic equations with different short-circuit ratios. The equivalent single-infeed system corresponding to (X_1, Y_1) can characterize the stability of the equivalent homogeneous system. Furthermore, it can be proven that this equivalent homogeneous system approximates the stability of the actual system. Meanwhile, (X_i, Y_i) are the eigen-subspaces of the equivalent homogeneous system matrix. According to eigen-subspace perturbation theory [23], the actual system matrix can be viewed as a perturbation of the equivalent homogeneous system matrix. Using (X_1, Y_1), we can solve for an equivalent single-infeed system that approximates the stability of the actual system, with its inverse closed-loop transfer function matrix given below. The detailed proof is provided in Appendix A.

$$T_{eq}(s) = \sum_{i=1}^{n} p_{1i} Y'_{IBRi}(s) + \lambda_1 I_2 \tag{9}$$

From (9), this equivalent single-infeed system consists of an equivalent converter and equivalent line, where the equivalent converter model is the weighted sum of all approximately decoupled converter models with respect to the equivalent network, and the admittance of the equivalent line is the minimum eigenvalue of the equivalent admittance matrix. This equivalent single-infeed system can approximate the stability of the original system under actual operating conditions, enabling quantification of system strength under such conditions.

3 Operational Generalized Short-Circuit Ratio

This section defines the operational generalized short-circuit ratio and its critical value, analyzes the influence mechanism of actual operating conditions on system strength, and proposes a system strength and stability margin quantification method based on OgSCR.

3.1 Definition of Operational Generalized Short-Circuit Ratio

The equivalent single-infeed system shown in (9) can approximate the stability of the original system under actual operating conditions. Therefore, when converter dynamics are fixed, the actual system strength depends on the minimum eigenvalue of the equivalent network admittance matrix. We define this as the operational generalized short-circuit ratio (OgSCR), expressed as:

$$\text{OgSCR} = \lambda_{\min}\left(\text{diag}\left(\frac{U_i^2}{P_i}\right) B\right) \tag{10}$$

The proposed index can also be used to measure system strength from the perspective of static voltage stability.

3.2 Influence Mechanism of Actual Operating Conditions on System Strength

Based on the above analysis, actual operating conditions modify the equivalent network dynamics, thereby modifying the OgSCR. Since the elements of the equivalent admittance matrix are scalars, the influence mechanism of operating conditions on OgSCR is relatively straightforward to analyze. However, from (9), the equivalent converter model is the weighted sum of all approximately decoupled converter models with respect to the equivalent network. While operating conditions indirectly modify the equivalent converter model through the equivalent network dynamics, they also directly modify the equivalent converter dynamics because the decoupled converter model still retains partial terminal voltage information. This makes analyzing the influence mechanism of operating conditions on the equivalent device critical operational short-circuit ratio (OSCR0) more complex.

When converter dynamics are fixed, OgSCR has a monotonic relationship with system strength. The minimum short-circuit ratio required for the equivalent converter to reach critical stability or meet specified performance requirements under actual operating conditions (referred to as "equivalent device critical operational short-circuit ratio OSCR0") is precisely the critical value of OgSCR (COgSCR), expressed as:

$$\text{OSCR0} = \arg\{\det(T_{eq}(s_d)) = 0\} \tag{11}$$

where arg{} denotes solving for the root of the equation, and s_d is the dominant system eigenvalue obtained when the equivalent converter is in a critical stable state or meets specified performance requirements.

Based on the above analysis and equations (9)-(10), OgSCR depends on network parameters, converter terminal voltage, and actual power output, reflecting the maximum sensitivity of AC network voltage to current at converter buses after modification by actual operating conditions. Therefore, it can measure grid strength under actual operating conditions. OSCR0 is related to equivalent converter dynamics, reflecting the equivalent converter' s tolerance to grid strength under actual operating conditions. Consequently, combining OgSCR and OSCR0 enables measurement of system strength under actual operating conditions, with the stability criterion:

$$\text{OgSCR} > \text{OSCR0} \tag{12}$$

Equation (12) can determine whether the system meets minimum stability requirements. In engineering practice, the OgSCR should not only exceed OSCR0 but also include additional margin based on field experience. Therefore, we calculate the relative value $\beta\%$ between OgSCR and OSCR0 to assess whether stability margin requirements are met. For instance, requiring at least $\beta_0\% = 20\%$ margin should satisfy:

$$\frac{\text{OgSCR} - \text{OSCR0}}{\text{OSCR0}} \times 100\% \geq 20\% \tag{13}$$

Notably, from (10) and (11), when all converters operate at rated conditions, $\text{diag}(U_i^2/P_i) = \text{diag}(S_{Bi}^{-1})$ (where S_{Bi} is the rated capacity of the i-th converter), and the definitions of OgSCR and its critical value become identical to those of the generalized short-circuit ratio and its critical value. Thus, the generalized short-circuit ratio and its critical value are special cases of OgSCR and its critical value under rated conditions.

To further analyze the influence mechanism of operating conditions on OSCR0, we consider simplifying the equivalent converter model using multi-scale methods and converter control. According to the above analysis, for small-signal stability problems within the PLL bandwidth and below, high-frequency current inner loop control dynamics can be further neglected [21][22]. The equivalent converter dynamics can then be simplified as (detailed derivation in Appendix B):

$$Y_{IBR\_hom}(s) \approx \frac{1}{sL_{eq}} \begin{bmatrix} 1 & -H_{pll}(s) \\ H_{pll}(s) & 1 \end{bmatrix} + \frac{1}{U_{eq}^2} \begin{bmatrix} \cos^2\theta_{eq} & \sin\theta_{eq}\cos\theta_{eq} \\ \sin\theta_{eq}\cos\theta_{eq} & \sin^2\theta_{eq} \end{bmatrix} \tag{14}$$

where U_{eq} is the equivalent terminal voltage.

From (9) and (14), the equivalent converter model can be simplified to a single converter model with terminal voltage U_{eq}. Correspondingly, OSCR0 can be understood as the tolerance of a single converter to grid strength when its terminal voltage is U_{eq}. The equivalent terminal voltage comprehensively considers all converters' terminal voltages and weighting factors related to the equivalent network, and is simple to calculate. Therefore, analyzing the influence mechanism of equivalent terminal voltage on OSCR0 enables analysis of how operating conditions affect OSCR0. Since both the numerator and denominator of the equivalent converter model are linear functions of equivalent terminal voltage, the impact of equivalent terminal voltage on the equivalent converter model and OSCR0 is relatively small. Detailed analysis results are omitted here and will be addressed in future research.

It is worth noting that since inner loop control typically participates in high-frequency resonance and has minimal impact on the stability issues of interest in this paper, equation (14) and the corresponding mechanism analysis have certain universality for the problems studied, which will be further validated through case studies in Section 4.

Furthermore, considering small-signal synchronization stability dominated solely by the PLL, control loops with bandwidth lower than the PLL can be further neglected [21][22], and the equivalent converter dynamics simplify to:

$$Y_{IBR\_hom}(s) \approx \frac{1}{sL_{eq}} \begin{bmatrix} 1 & -H_{pll}(s) \\ H_{pll}(s) & 1 \end{bmatrix} \tag{15}$$

Substituting (15) into (9) allows analytical derivation of the dominant system eigenvalue expression, from which OSCR0 can be solved as the OgSCR value when the real part of the dominant eigenvalue is zero:

$$\text{OSCR0} = \frac{K_{pllp}}{2\omega_0 L_{eq}} \tag{16}$$

From (16), OSCR0 is independent of operating conditions and determined only by converter control parameters.

In summary, actual operating conditions primarily modify AC network dynamics, thereby modifying OgSCR. The impact on equivalent converter dynamics and OSCR0 is relatively small and depends on the stability issue of interest: when system stability is affected by both outer loops and PLL, equivalent converter dynamics are influenced by equivalent terminal voltage, and OSCR0 varies with equivalent terminal voltage; when system stability is dominated solely by PLL, OSCR0 is independent of operating conditions and determined only by converter control parameters. Moreover, the simplified equivalent converter model can streamline the OSCR0 solution process, whether through analytical calculation or simulation/hardware-in-the-loop testing.

4 Case Studies and Verification

This section builds electromagnetic transient simulation models of a 33-converter chain system in MATLAB/Simulink and a renewable energy base in northwest China on the CloudPSS platform to verify the effectiveness and necessity of OgSCR for quantifying system strength under actual operating conditions.

4.1 Case 1: 33-Converter Chain System

This subsection verifies the necessity and effectiveness of OgSCR for quantifying system strength under actual operating conditions using a 33-converter chain system. The system topology is shown in Appendix D Fig. D1(a), with converter control parameters and network parameters provided in Appendix D Tables D1-D2.

Table 1 Equivalent terminal voltage, OgSCR, OSCR0, gSCR, SCR0, and stability assessment results for the 33-converter chain system

Case	Equivalent U_{eq} (pu)	OgSCR	OSCR0	η%	gSCR	SCR0	Assessment based on OgSCR	Assessment based on gSCR
1	0.91	1.84	2.07	-11.1%	2.21	2.10	Unstable	Stable
2	0.95	2.21	2.10	5.2%	2.21	2.10	Stable	Stable
3	0.99	2.61	2.14	22.0%	2.21	2.10	Stable	Stable
4	1.03	3.99	2.17	83.9%	2.21	2.10	Stable	Stable

Based on the above analysis, the quantification of system strength and stability margin for multi-infeed systems under actual operating conditions can be transformed into two sub-problems: OgSCR calculation and OSCR0 calculation. The calculation flowchart is shown in Fig. 2, with specific steps as follows:

1. Solve the system power flow distribution.
2. Form the equivalent admittance matrix $\mathrm{diag}(U_i^2/P_i)B$.
3. Calculate OgSCR using equation (10).
4. Based on required analysis accuracy and actual stability modes, simplify the equivalent converter dynamics using equations (14), (15), or other appropriate forms, and obtain OSCR0 through analytical calculation, simulation, or hardware-in-the-loop testing using equation (11).
5. Determine whether the system meets minimum stability requirements using equation (12), and calculate the relative index $\beta\%$ using equation (13) to analyze stability margins.

Specifically, we consider four cases with different converter operating conditions but identical network parameters. The converters' terminal voltages and actual power outputs are provided in Appendix D Table D3. First, we solve for the simplified equivalent single-infeed system considering both outer loop and PLL dynamics. Table 1 presents the equivalent terminal voltages for the four cases, and Fig. 3 compares the dominant eigenvalues of the original system and the equivalent single-infeed system. The results show that the eigenvalues of the two systems are essentially consistent, verifying that the equivalent single-infeed system can approximate the stability of the actual system. Additionally, modal analysis results indicate that when network parameters are identical but operating conditions differ, the system stability status varies, demonstrating the necessity of measuring system strength under actual operating conditions.

Second, based on the equivalent single-infeed system, we obtain OSCR0 for each case through simulation or analytical methods. Combined with OgSCR, we assess system stability and evaluate system strength. Table 1 presents the index values and assessment results, showing that the stability judgments based on OgSCR are consistent with the modal analysis results shown in Fig. 3, thereby validating the effectiveness of OgSCR for measuring system strength under actual operating conditions.

Furthermore, Table 1 presents stability assessment results based on the generalized short-circuit ratio (gSCR). Since the network parameters are identical across the four cases, their gSCR and critical values (SCR0) are the same, leading to unchanged stability judgments and stability modes based on gSCR, which contradicts the modal analysis results. Specifically, based on gSCR criteria, Case 1 should be small-signal stable, but this contradicts the modal analysis results. This demonstrates that grid strength indices defined under rated conditions cannot accurately quantify system strength under actual operating conditions and may lead to misjudgment of system stability.

Finally, at simulation time $T = 0.1$ s, a 5% terminal voltage step is applied at the infinite bus and quickly cleared. The converter terminal voltage waveforms for Cases 1, 2, and 4 are shown in Figs. 4(a), (b), and (c), respectively. The time-domain simulation trends are consistent with the system strength assessment results based on modal analysis and OgSCR, further validating the effectiveness of OgSCR.

4.2 Case 2: Actual Renewable Energy Base

To verify the effectiveness of OgSCR in actual large-scale systems, this section builds an electromagnetic transient simulation model of a renewable energy station in northwest China on the CloudPSS platform. The system topology is shown in Appendix D Fig. D1(b), where nodes 1-54 are grid-following converter injection nodes, nodes 55-92 are passive intermediate nodes, and node 93 is the infinite source node. Converter control parameters and network parameters are provided in Appendix D Tables D1 and D4.

Table 2 Equivalent terminal voltage, OgSCR, OSCR0, and stability assessment results for the actual renewable energy base

Case	Equivalent U_{eq} (pu)	OgSCR	OSCR0	β%	Assessment
1	1.01	3.47	2.31	50.2%	Stable
2	1.00	2.36	2.35	0.4%	Critical
3	0.99	2.21	2.35	-6.0%	Unstable

Specifically, we consider three cases with different converter operating conditions but identical network parameters. The actual operating conditions are provided in Appendix D Table D5, and Table 2 presents the index values and assessment results. The results show that in Case 1, OgSCR=3.47 > OSCR0=2.31, indicating small-signal stability; in Case 2, OgSCR=2.36   OSCR0=2.35, indicating critical stability; and in Case 3, OgSCR=2.21 < OSCR0=2.35, indicating failure to meet small-signal stability requirements.

Furthermore, at simulation time $T = 0.1$ s, a 5% terminal voltage step is applied at the infinite bus and quickly cleared. The converter terminal voltage waveforms for Case 2 are shown in Fig. 5, revealing sustained oscillations and critical stability. The time-domain simulation assessment results are consistent with those based on OgSCR, thereby validating the effectiveness of OgSCR and its critical value in actual large-scale systems.

5 Conclusions and Outlook

This paper defines the operational generalized short-circuit ratio and its calculation method to address the quantification of small-signal strength and stability margins for multi-infeed grid-following renewable energy systems under actual operating conditions (including both rated and non-rated conditions). The main conclusions are:

1) We discover the mapping relationship between converter dynamic characteristics and operating condition parameters, provide the physical meaning of OgSCR and its critical value, and propose a system strength quantification method for renewable energy equipment under actual operating conditions.

2) We reveal the influence mechanism of actual operating conditions on system strength: operating conditions primarily modify the AC network, thereby modifying OgSCR; their impact on OSCR0 is relatively small and depends on the stability issue of interest. Generally, OSCR0 varies with equivalent terminal voltage, but when stability is dominated solely by PLL, OSCR0 is independent of operating conditions and determined only by converter control parameters.

When the system includes diverse renewable energy equipment such as grid-following converters with different external characteristics under rated conditions, static var generators (SVGs), synchronous condensers, and grid-forming converters, further analysis can be performed using OgSCR based on eigen-subspace perturbation theory. Detailed discussions are beyond the scope of this paper.

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Appendix A: Proof of Stability Approximation

First, we prove that the equivalent homogeneous system approximates the stability of the actual system.

Proof: From the analysis in Section 2.2, the equivalent single-infeed system corresponding to the minimum eigenvalue shown in (8) can approximate the stability of the equivalent homogeneous system, while the equivalent single-infeed system shown in (9) can approximate the stability of the actual system. Combining (8) and (9), the two equivalent single-infeed systems are identical. Therefore, the equivalent homogeneous system and the actual system have approximate stability.

Second, we prove that (X_i, Y_i) are the eigen-subspaces of the equivalent homogeneous system matrix.

Proof: $T_{hom}(s)$, X_i, and Y_i satisfy the following equation:

$$T_{hom}(s)X_i = \lambda_i X_i, \quad Y_i^T T_{hom}(s) = \lambda_i Y_i^T$$

Combining this with the definition of eigen-subspaces [23], (X_i, Y_i) are the eigen-subspaces of the equivalent homogeneous system matrix.

Appendix B: Simplified Equivalent Converter Model

After approximately neglecting high-frequency current inner loop control dynamics, the converter' s admittance transfer function can be simplified as:

$$Y_{IBRi}(s) \approx \frac{1}{sL_f}\begin{bmatrix} 1 & -H_{pll}(s) \\ H_{pll}(s) & 1 \end{bmatrix} + \frac{P_i}{U_i^2}\begin{bmatrix} \cos^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin\theta_i\cos\theta_i & \sin^2\theta_i \end{bmatrix} \quad \text{(B1)}$$

Combining (B1) and assuming the converter itself has no unstable modes, the converter' s impedance transfer function matrix is:

$$Z_{IBRi}(s) = Y_{IBRi}^{-1}(s) \quad \text{(B2)}$$

Combining with the equivalent network impedance matrix and using the determinant, the system closed-loop characteristic equation composed of converter and network impedance models is:

$$\det\left(Z_{net}(s)+Z_{IBR}(s)\right)=0 \tag{B3}$$

Similar to the analysis in Section 2.2, using the eigen-subspace of the equivalent impedance matrix $Z_{net}(s)=\left(\mathrm{diag}(U_i^2/P_i)B\right)^{-1}$ corresponding to its maximum eigenvalue allows construction of an equivalent converter, whose impedance transfer function matrix is expressed as:

$$Z_{IBR\_hom}(s)=\sum_{i=1}^{n}p_{1i}Z_{IBRi}(s) \tag{B4}$$

Taking the inverse of the equivalent converter' s impedance transfer function matrix (B4) yields the equivalent converter' s admittance transfer function matrix:

$$Y_{IBR\_hom}(s)=Z_{IBR\_hom}^{-1}(s) \tag{B5}$$

Appendix C: Derivation of OSCR0 Under PLL Dominance

Substituting (15) into (9) allows analytical derivation of the dominant system eigenvalue expression:

$$s^2+\frac{K_{pllp}}{\omega_0 L_{eq}}s+\frac{K_{plli}}{\omega_0 L_{eq}}=0 \tag{C1}$$

When the real part of the dominant eigenvalue is zero, the system reaches critical stability, and the corresponding OgSCR is OSCR0. Combining (C1) yields:

$$\mathrm{OSCR0}=\frac{K_{pllp}}{2\omega_0 L_{eq}} \tag{C2}$$

From (C2), OSCR0 is independent of operating conditions and determined only by converter control parameters.

Appendix D: Simulation Parameters

Table D1 Converter control parameters

Parameter	MATLAB	CloudPSS
AC system rated capacity	1500 kVA	1500 kVA
AC system rated voltage	1100 V	1100 V
Converter rated capacity	1500 kVA	1500 kVA
DC-side capacitance C_{dc}	0.05 pu	0.05 pu
DC-side rated voltage	1500 V	1500 V
DC voltage outer loop $H_{dc}(s)$	0.4+20/s	0.5+40/s
Filter capacitance and resistance C_f, R_d	0.05 pu, 0 pu	0.05 pu, 0 pu
Current inner loop transfer function $H_i(s)$	0.2+10/s	0.2+60/s
PLL transfer function $H_{pll}(s)$	12+11100/s	14+10600/s
Voltage feedforward transfer function $G_{FF}(s)$	1/(1+0.0001s)	1/(1+0.0001s)

Table D2 Network parameters for the 33-converter chain system

Parameter	Value (pu)
Line reactance (1-2, 2-3, ···, 32-33)	0.01
Transformer reactance	0.05
Other line reactances	0.005

Table D3 Actual operating conditions for the 33-converter chain system

Case	Terminal Voltage (pu)	Active Power (pu)
1	1-3,12-14,23-25: 0.92; Others: 0.91	1,12,23,10,21,32: 1.00; 2,13,24,11,22,33: 0.90; 3,14,25: 0.8; 4,15,26: 0.7; 5,16,27: 0.6; 6,17,28: 0.5; 7,18,29: 0.4; 8,19,20: 0.3; 9,20,31: 0.2
2	1-2,11,12-13,22,23-24,33: 0.98; 10,21,32: 0.97	1-5,12-26,23-37: 0.6; 6-8,17-19,28-30: 0.4; 9-11,20-22,31-33: 0.2
3	1-5,12-26,23-37: 1.02; 6-11,17-22,28-33: 1.03	3-4,14-15,25-26: 0.99; 5-8,16-19,27-30: 1.00; 9,20,31: 1.01
4	1-5,12-26,23-37: 1.02; 6-11,17-22,28-33: 1.03	1-5,12-26,23-37: 1.00; 6-11,17-22,28-33: 1.00

Table D4 Network parameters for the actual renewable energy base simulation model ($\times10^{-3}$ pu)

Line	Reactance	Line	Reactance	Line	Reactance
$X_{55,86}$	0.24	$X_{68,87}$	0.35	$X_{81,89}$	2.20
$X_{56,86}$	1.00	$X_{69,87}$	0.16	$X_{82,91}$	1.50
$X_{57,86}$	2.40	$X_{70,87}$	0.02	$X_{83,91}$	1.30
$X_{58,86}$	3.30	$X_{71,88}$	2.50	$X_{84,91}$	1.50
$X_{59,86}$	2.90	$X_{72,88}$	2.20	$X_{85,91}$	1.30
$X_{60,86}$	2.00	$X_{73,88}$	1.50	$X_{86,87}$	1.00
$X_{61,86}$	1.70	$X_{74,88}$	1.30	$X_{87,92}$	3.10
$X_{62,86}$	0.03	$X_{75,88}$	0.81	$X_{88,92}$	4.40
$X_{63,87}$	1.10	$X_{76,88}$	1.30	$X_{89,90}$	7.00
$X_{64,87}$	3.10	$X_{77,88}$	0.81	$X_{90,92}$	7.80
$X_{65,87}$	2.30	$X_{78,90}$	0.19	$X_{91,92}$	0.87
$X_{66,87}$	1.90	$X_{79,90}$	2.20	$X_{92,93}$	6.90
$X_{67,87}$	1.40	$X_{80,89}$	0.19	Transformer reactance	64.0

Table D5 Actual operating conditions for the actual renewable energy base

Case	Active Power (pu)	Terminal Voltage (pu)
1	1-15: 0.2; 16-29: 0.4; 30-38: 0.6; 39-54: 0.8	1-29: 1.00; 30-38: 1.01; 39-54: 1.02
2	1-29: 1.00; 30-38: 0.80; 39-48: 0.5; 49-54: 0.4	1-12: 1.05; 13-29: 1.03; 30-38: 1.00; 39-47: 0.99; 48-54: 0.98
3	1-12: 1.04; 13-29: 1.03; 30-54: 1.02	1-12: 1.04; 13-29: 1.03; 30-54: 1.02

Fig. D1 The 33-converter chain system and a practical renewable energy plant simulation models

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