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Postprint: Accuracy Assessment of GSMAp and IMERG Satellite Precipitation Products in Shaanxi Region

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Abstract

Using ground rainfall observation data provided by the China Meteorological Administration as reference, this study evaluated and compared the accuracy of two high-resolution satellite rainfall products (IMERG and GSMAp) in Shaanxi Province using six different statistical analysis indicators: correlation coefficient, root mean square error, relative bias, rainfall false alarm rate, hit rate, and critical success index, at four different temporal scales (annual, seasonal, monthly, and daily). Additionally, a comparative analysis of their performance in monitoring heavy rainfall processes was conducted. The results show that: (1) At the annual scale, GSMAp demonstrates higher accuracy than IMERG. GSMAp shows high correlation with station observation data, while IMERG shows moderate correlation; GSMAp overestimates annual-scale rainfall, whereas IMERG underestimates it. (2) At the seasonal scale, both datasets achieve highest accuracy in summer; overall, IMERG also demonstrates higher accuracy than GSMAp at the seasonal scale. (3) At the monthly scale, both products show relatively high correlation with ground observation data and both exhibit some degree of overestimation, but IMERG possesses relatively higher accuracy than GSMAp. (4) At the daily scale, GSMAp shows slightly higher accuracy than IMERG. (5) The accuracy of satellite rainfall products is related to rainfall amount, generally manifesting as overestimation for light rainfall and underestimation for heavy rainfall. (6) Satellite rainfall product accuracy exhibits obvious regional differences; GSMAp shows overall underestimation of rainfall in Shaanxi Province, with the most significant underestimation occurring in northern Shaanxi; IMERG shows slight overestimation in Guanzhong but obvious underestimation in both northern and southern Shaanxi. (7) Through analysis of four heavy rainfall events, it was found that GSMAp has slightly stronger monitoring capability for heavy rain and above-intensity rainfall events than IMERG. The research results can provide reference for meteorological and hydrological research in this region when selecting and using rainfall data.

Full Text

Accuracy Assessment of GSMAp and IMERG Satellite Precipitation Products from the Global Precipitation Measurement Mission in Shaanxi Province

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Abstract: Using ground-based precipitation observations from the China Meteorological Administration as reference, this study evaluated and compared the accuracy of two high-resolution satellite precipitation products (IMERG and GSMAp) in Shaanxi Province across four temporal scales (annual, seasonal, monthly, and daily) using six statistical metrics: correlation coefficient, root mean squared error, relative bias, false alarm ratio, probability of detection, and critical success index. The performance of these products in monitoring heavy rainfall events was also analyzed. Results indicate that: (1) At the annual scale, GSMAp exhibits higher accuracy than IMERG, showing high correlation with ground observations ($CC = 0.89$) while IMERG shows moderate correlation ($CC = 0.56$). GSMAp overestimates annual precipitation ($BIAS = 13.60\%$), whereas IMERG underestimates it ($BIAS = -11.41\%$). (2) At the seasonal scale, both products achieve highest accuracy in summer, with IMERG demonstrating higher overall precision than GSMAp. (3) At the monthly scale, both products correlate highly with ground observations but exhibit overestimation, with IMERG showing relatively higher accuracy. (4) At the daily scale, GSMAp demonstrates higher accuracy than IMERG. (5) The accuracy of satellite precipitation products is rainfall-dependent, generally overestimating light rainfall and underestimating heavy rainfall. (6) Significant regional differences exist: GSMAp underestimates precipitation across Shaanxi, most notably in northern Shaanxi, while IMERG slightly overestimates precipitation in the Guanzhong Plain but markedly underestimates it in northern and southern Shaanxi. (7) Analysis of four heavy rainfall events reveals that GSMAp has slightly stronger monitoring capability for heavy precipitation events than IMERG. These findings provide valuable reference for selecting and applying precipitation data in meteorological and hydrological research in the region.

Keywords: satellite precipitation products; Global Precipitation Measurement (GPM); heavy precipitation events; accuracy assessment

1 Introduction

Precipitation represents a crucial exchange of energy and moisture between the Earth's surface and atmosphere and is a key variable in meteorological and cli-

matological research. Although traditional rain gauge observations can provide relatively accurate precipitation data at small scales, they suffer from uneven spatial distribution and sparse station coverage in some regions, making it difficult to accurately characterize precipitation patterns at large scales. Satellite remote sensing technology overcomes these spatiotemporal limitations, enabling comprehensive, multi-temporal, and large-scale continuous precipitation observations.

In 1997, the United States launched the Tropical Rainfall Measuring Mission (TRMM) satellite, which carried the first spaceborne precipitation radar (PR). TRMM provided rainfall observations between 35°S and 35°N, offering three-dimensional structures of storms that significantly improved precipitation estimation and created new opportunities for large-scale hydrometeorological research. Building on TRMM's multi-year observations and integrating data from other research and operational satellites, the TRMM Multi-satellite Precipitation Analysis (TMPA) produced precipitation data covering 50°S to 50°N with a spatial resolution of $0.25^\circ \times 0.25^\circ$. In 2014, NASA and JAXA jointly implemented the Global Precipitation Measurement (GPM) mission as a successor to TRMM. The GPM Core Observatory carries the first Ku/Ka-band dual-frequency precipitation radar (DPR), enabling more accurate monitoring of light precipitation ($<0.05 \text{ mm} \cdot \text{h}^{-1}$) and solid precipitation. This advancement marked the transition from TRMM to the GPM era, with satellite precipitation products offering expanded global coverage and higher spatiotemporal resolution ($0.1^\circ \times 0.1^\circ$, 30 minutes).

Numerous evaluation studies of GPM-era products have emerged worldwide. Beuria et al. compared IMERG Final Run and TRMM 3B43 across Indian basins, finding IMERG outperformed TRMM across all rainfall intensities. Studies in China have shown that IMERG V04 significantly overestimates precipitation in most regions, with biases related to rainfall amount. Chen et al. found that satellite precipitation products perform poorly in complex terrain and high-altitude areas, sometimes yielding unreliable results with distinct seasonal variations. Wang et al. and Li et al. both identified underestimation of precipitation in high-altitude regions of the Heihe and Yellow River basins. While satellite precipitation data have been applied in watershed hydrological modeling, their accuracy varies significantly across periods and regions due to multiple influencing factors. Therefore, validating these products before regional application is essential.

In Shaanxi Province, which spans both the Yangtze and Yellow River basins with complex topography and climate, Wan et al. analyzed spatial-temporal precipitation patterns using 50 years of station data, revealing a north-south gradient with more rainfall in the south. Ren et al. evaluated TRMM 3B42 in the Qinling-Daba Mountains, finding high accuracy at annual, seasonal, and monthly scales but relatively poor daily-scale performance. Zeng et al. down-scaled TRMM data using a geographically weighted regression model for the Qinling-Daba region. However, the accuracy of GPM-era products in Shaanxi

remains unclear. Understanding how accuracy changes with improved spatial resolution and coverage is crucial for promoting the application of these new high-resolution products. Therefore, this study evaluates IMERG and GSMAp using ground observations as reference, assessing their accuracy across multiple temporal scales and analyzing their performance in different geomorphic regions of Shaanxi.

2 Study Area and Data

2.1 Study Area

Shaanxi Province is located in the interior of China along the middle reaches of the Yellow River, between $105^{\circ}29' - 111^{\circ}15' E$ and $31^{\circ}42' - 39^{\circ}35' N$. The province spans three climate zones from south to north (north subtropical, warm temperate, and middle temperate), resulting in significant north-south climate differences. Precipitation generally decreases from south to north. Influenced by monsoons and topography, Shaanxi exhibits pronounced seasonal precipitation variation, with rainy summers and dry winters. For analyzing error characteristics across different geomorphic regions, the province is divided into three areas using the North Mountains and Qinling Mountains as boundaries: the Loess Plateau of northern Shaanxi, the Guanzhong Plain, and the Qinling-Daba mountainous region of southern Shaanxi.

2.2 Precipitation Data

Ground reference data were obtained from the China Meteorological Data Network, including monthly and daily datasets from 2014 to 2019. The spatial distribution of 20 meteorological stations is shown in Figure 1. Data from 2014-2018 were used for annual-scale analysis, while all data from 2014-2019 were used for other temporal scales.

The satellite precipitation products used were IMERG and GSMAp. IMERG provides three processing levels: Early, Late, and Final. This study used the research-grade Final product, which incorporates monthly gauge data and offers higher accuracy than Early and Late products. GSMAp offers three products: GSMAp_{NRT} (Near Real Time), GSMAp_{MVK} (Moving Vector with Kalman filter), and GSMAp_{Gauge} (gauge-calibrated). This study used the GSMAp_{Gauge} product, which is calibrated with global rain gauge data and provides $0.1^{\circ} \times 0.1^{\circ}$ high-precision data. Table 1 summarizes the key characteristics of both products.

Satellite precipitation products represent average precipitation rates per unit time within a grid cell ($mm \cdot h^{-1}$ for hourly products or $mm \cdot d^{-1}$ for daily products), while ground stations record point measurements with the same units, enabling direct comparison. Spatially, each station was matched to its corresponding grid cell based on latitude and longitude. Given the sparse station network, no grid cell contained multiple stations, resulting in a one-to-one correspondence. All satellite and ground observation data underwent quality control

by the data providers, with additional checks performed on ground data before use.

2.3 Evaluation Methods

Using ground station data as reference, this study conducted point-scale validation of both satellite products. Six evaluation metrics were selected: correlation coefficient (CC), root mean squared error (RMSE), relative bias (BIAS), false alarm ratio (FAR), probability of detection (POD), and critical success index (CSI). CC measures the linear relationship between satellite estimates and ground observations. RMSE reflects overall error magnitude and precision, with values closer to 0 indicating smaller errors. BIAS indicates systematic overestimation (positive values) or underestimation (negative values). FAR, POD, and CSI assess the detection capability for precipitation events.

The formulas for the primary metrics are:

$$CC = \frac{\sum_{i=1}^n (D_i - \bar{D})(W_i - \bar{W})}{\sqrt{\sum_{i=1}^n (D_i - \bar{D})^2 \sum_{i=1}^n (W_i - \bar{W})^2}}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (D_i - W_i)^2}$$

$$BIAS = \frac{\sum_{i=1}^n (W_i - D_i)}{\sum_{i=1}^n D_i} \times 100\%$$

where n is the sample size, D is ground observation, W is satellite estimate, and overbars denote means.

For heavy rainfall event analysis, a threshold of $25 \text{ mm} \cdot \text{d}^{-1}$ was used to define heavy rain events. POD, FAR, and CSI were calculated as:

$$POD = \frac{H}{H + M}$$

$$FAR = \frac{F}{H + F}$$

$$CSI = \frac{H}{H + F + M}$$

where H represents correct heavy rain detection, F represents false alarms, and M represents missed events.

3 Results

3.1 Annual-Scale Accuracy Assessment

Based on ground station data, Shaanxi's annual precipitation shows a clear spatial pattern of decreasing rainfall from south to north (Figure 2). Kriging interpolation of station observations reveals this gradient distribution. At the annual scale, GSMAp correlates highly with station data ($CC = 0.89$) while IMERG shows moderate correlation ($CC = 0.56$). GSMAp overestimates annual precipitation ($BIAS = 13.60\%$), whereas IMERG underestimates it ($BIAS = -11.41\%$). Both products can reflect Shaanxi's annual precipitation patterns, but GSMAp demonstrates higher accuracy (Table 2).

3.2 Seasonal-Scale Accuracy Assessment

Shaanxi exhibits pronounced seasonal precipitation variation. To determine whether IMERG and GSMAp accuracy varies seasonally, precipitation was aggregated by season: December–February (winter), March–May (spring), June–August (summer), and September–November (autumn). Both products show highest accuracy in summer and lowest in winter. IMERG demonstrates higher seasonal-scale accuracy than GSMAp overall.

Both products correlate highly with ground observations across all seasons, with the strongest correlation in autumn ($CC > 0.9$). RMSE is smallest in summer and largest in winter, indicating more stable performance during rainy seasons. BIAS patterns reveal that GSMAp overestimates precipitation in spring and autumn but shows mixed performance in winter and summer. IMERG underestimates precipitation in winter and summer while overestimating in spring and autumn. The relationship between satellite estimates and ground observations (Figure 3) shows that accuracy is rainfall-dependent: both products tend to overestimate light rainfall and underestimate heavy rainfall across seasons.

3.3 Monthly-Scale Accuracy Assessment

At the monthly scale, both products correlate highly with ground observations (Figure 5). IMERG shows stronger correlation and smaller RMSE than GSMAp (Table 3). However, both products exhibit overestimation, with BIAS values of 15.57% for IMERG and 13.60% for GSMAp. The scatter plots demonstrate strong consistency between satellite estimates and ground observations, with IMERG showing slightly better performance. This overestimation tendency at monthly scales contrasts with daily-scale patterns, suggesting scale-dependent error characteristics.

3.4 Daily-Scale Accuracy Assessment

Using data from the rainy season (May–September) with effective precipitation events (daily rainfall > 1 mm), daily-scale accuracy was assessed. Both products show moderate correlation with ground observations, with GSMAp performing

slightly better than IMERG (Figure 6). RMSE values are 9.42 mm for IMERG and 8.81 mm for GSMAp. Both products exhibit underestimation at the daily scale, with BIAS values of -11.41% for IMERG and -8.51% for GSMAp (Table 4).

To analyze error characteristics across rainfall intensities, precipitation events were classified as light rain ($0.1\text{--}10 \text{ mm} \cdot \text{d}^{-1}$), moderate rain ($10\text{--}25 \text{ mm} \cdot \text{d}^{-1}$), heavy rain ($25\text{--}50 \text{ mm} \cdot \text{d}^{-1}$), and extreme rain ($\geq 50 \text{ mm} \cdot \text{d}^{-1}$). Correlation is lowest for moderate rain and highest for extreme rain. Both products severely overestimate light rainfall (largest BIAS) and underestimate heavy rainfall. RMSE increases with rainfall intensity, indicating larger absolute errors for heavy precipitation. GSMAp shows smaller RMSE than IMERG across all intensity classes, confirming its superior daily-scale performance.

3.5 Regional Accuracy Comparison

To quantify regional differences in daily-scale accuracy, evaluation metrics were calculated for the three geomorphic regions (Figure 8). Both products show lowest correlation in the Guanzhong Plain but also smallest RMSE there. In northern Shaanxi, both products exhibit marked underestimation, with BIAS values of -27.16% for IMERG and -13.99% for GSMAp. In southern Shaanxi, underestimation is also significant (-20.44% for IMERG, -15.23% for GSMAp). In Guanzhong, IMERG slightly overestimates precipitation (BIAS = 1.48%) while GSMAp underestimates it (BIAS = -8.51%). Overall, GSMAp demonstrates higher accuracy than IMERG across all regions, though both products show pronounced regional variability likely related to topography, climate, and station density.

3.6 Heavy Rainfall Event Monitoring Capability

To compare monitoring capabilities for heavy rainfall events, four storm events (July 2018, July 2019, August 2019, and August 2020) were analyzed. These events primarily occurred in northern and southern Shaanxi. Both satellite products captured the temporal evolution of rainfall during storm periods, showing more intra-day variability than daily gauge observations (Figure 9). However, both underestimated peak rainfall, with IMERG showing greater underestimation than GSMAp.

Using a $25 \text{ mm} \cdot \text{d}^{-1}$ threshold, detection capability metrics were calculated for heavy and extreme rainfall events across regions (Table 5). In northern Shaanxi, both products show low POD and CSI, with high FAR, indicating poor detection capability. In Guanzhong, IMERG achieves POD of 0.85 with the lowest FAR (0.22) and highest CSI (0.69), while GSMAp shows POD of 0.78, FAR of 0.31, and CSI of 0.58. In southern Shaanxi, both products perform moderately. Overall, GSMAp demonstrates slightly stronger monitoring capability for heavy rainfall events than IMERG, particularly in Guanzhong where it shows the highest POD and lowest FAR.

4 Conclusions

This study evaluated the accuracy of IMERG and GSMAp satellite precipitation products across multiple temporal scales in Shaanxi Province using ground observations from 2014-2019. The main conclusions are:

- (1) At annual and monthly scales, both products correlate highly with ground observations but show systematic biases: GSMAp tends to overestimate while IMERG tends to underestimate precipitation. GSMAp demonstrates higher accuracy at annual and daily scales, while IMERG performs better at seasonal and monthly scales.
- (2) Seasonal accuracy varies significantly, with both products performing best in summer and worst in winter. Accuracy is rainfall-dependent, with overestimation of light rain and underestimation of heavy rain across all temporal scales.
- (3) Regional differences are pronounced: both products markedly underestimate precipitation in northern and southern Shaanxi, while IMERG slightly overestimates precipitation in the Guanzhong Plain. GSMAp shows overall underestimation across the province, most severely in northern Shaanxi.
- (4) For heavy rainfall events ($25\text{mm} \cdot d^{-1}$), both products can capture the temporal evolution but underestimate peak intensities. GSMAp shows slightly stronger detection capability than IMERG, particularly in the Guanzhong region.

These findings provide important reference for selecting appropriate precipitation data in meteorological and hydrological applications in Shaanxi Province. Future work should incorporate topographic and climatic factors to improve algorithm performance and error correction.

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Figures

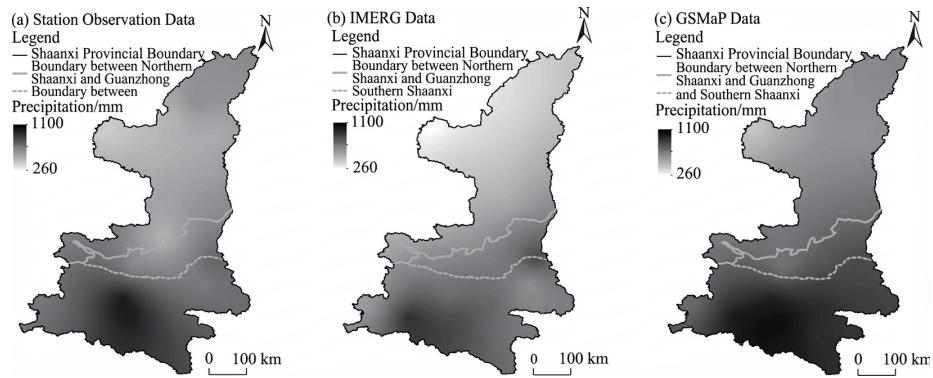


Figure 1: Figure 2

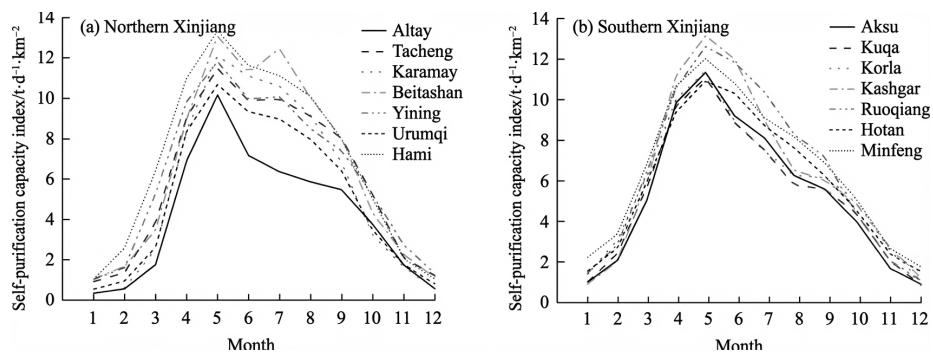


Figure 2: Figure 6

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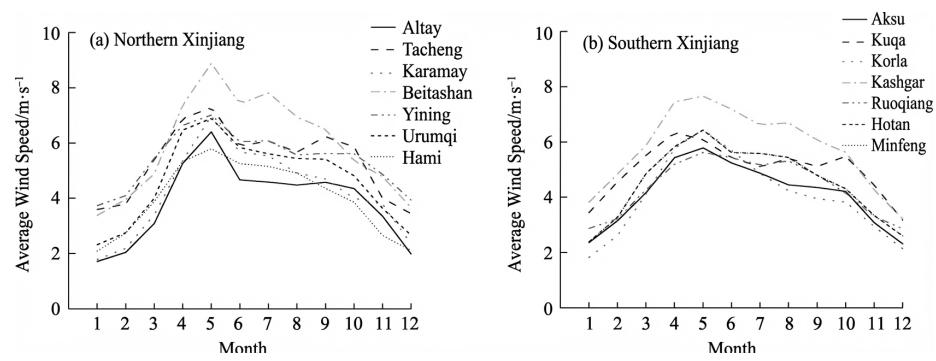


Figure 3: Figure 8