

Determination of neutron-skin thickness using configurational information entropy postprint

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Abstract

Configurational information entropy (CIE) theory was employed to determine the neutron skin thickness of neutron-rich calcium isotopes. The nuclear density distributions and fragment cross-sections in 350 MeV/u $^{40-60}\text{Ca} + ^9\text{Be}$ projectile fragmentation reactions were calculated using a modified statistical abrasion-ablation model. CIE quantities were determined from the nuclear density, isotopic, mass, and charge distributions. The linear correlations between the CIE determined using the isotopic, mass, and charge distributions and the neutron skin thickness of the projectile nucleus show that CIE provides new methods to extract the neutron skin thickness of neutron-rich nuclei.

Full Text

Preamble

Determination of neutron-skin thickness using configurational information entropy

Chun-Wang Ma,^{1,2,†} Yi-Pu Liu,¹ Hui-Ling Wei,¹ Jie Pu,¹ Kai-Xuan Cheng,¹ and Yu-Ting Wang¹

¹School of Physics, Henan Normal University, Xinxiang 453007, China

²Key Laboratory Optoelectronic Sensing Integrated Application of Henan Province, Henan Normal University, Xinxiang 453007, China

Configurational information entropy (CIE) theory was employed to determine the neutron skin thickness of neutron-rich calcium isotopes. Nuclear density distributions and fragment cross-sections in 350 MeV/u $^{40-60}\text{Ca} + ^9\text{Be}$ projectile

fragmentation reactions were calculated using a modified statistical abrasion-ablation model. CIE quantities were determined from the nuclear density, isotopic, mass, and charge distributions. Linear correlations between the CIE values derived from isotopic, mass, and charge distributions and the neutron skin thickness of the projectile nucleus demonstrate that CIE provides novel methods for extracting the neutron skin thickness of neutron-rich nuclei.

Keywords: Neutron-skin thickness, Configurational information entropy, Cross section distribution, Projectile fragmentation

Introduction

Next-generation radioactive nuclear beam facilities will provide unprecedented opportunities to explore exotic nuclei near and beyond the drip lines. Nuclei with large neutron excess can develop exotic neutron skin or halo structures, which have attracted significant experimental and theoretical interest over the past three decades. The neutron skin thickness is defined as $\delta_{np} = \delta_n - \delta_p$, representing the difference between the point neutron and point proton root-mean-square (RMS) radii of a nucleus. Numerous methods have been developed to experimentally determine neutron skin thickness, though most represent indirect measurements that are model-dependent. Typical approaches include measurements of reaction cross sections (σ_R), charge-changing cross sections (σ_{cc}) [?, ?], electric dipole polarizability [?], photon multiplicity [?], π^-/π^+ or Σ^-/Σ^+ ratios [?, ?], $^3\text{H}/^3\text{He}$ ratios [?], and α -decay half-lives [?]. Projectile fragmentation reactions, the primary experimental technique for studying rare isotopes, are particularly suitable for determining neutron skin thickness due to the pronounced experimental signatures induced by neutron skin structure [?, ?]. These signatures include isospin effects in isotopic cross sections [?], neutron-abrasion cross sections (σ_{nabr}) [?], neutron removal cross sections [?], mirror nuclei or isobaric ratios [?], and isoscaling parameters (α) [?]. Parity-violating electron scattering (PVES) remains the only model-independent method for determining neutron skin thickness [?], with a theoretical Bayesian investigation presented in Ref. [?]. Determining the neutron skin thickness of ^{48}Ca and ^{208}Pb is currently of significant interest and has been prioritized in the U.S. 2015 Long Range Plan for Nuclear Science [?]. The Lead Radius Experiment (PREX) has previously measured the neutron skin thickness of ^{208}Pb [?], with recent PREX results indicating a substantially thicker neutron skin than earlier predictions [?]. Determining the neutron skin thickness of nuclei near the neutron drip line remains an important research objective and one of the most compelling topics in the new era of radioactive beam facilities.

Information entropy theory, established by C.E. Shannon [?], enables the transformation of system variables into precise information quantities [?] and has found diverse applications [?, ?]. The first application of information entropy theory to heavy-ion reactions traced back to studies of the nuclear liquid-gas transition in nuclear multifragmentation [?]. Recent work has extended this approach to investigate the information entropy carried by individual fragments

produced in projectile fragmentation reactions, revealing scaling phenomena for fragments across a wide range of neutron excess [?, ?, ?]. Configurational information entropy (CIE) was developed to quantify the information entropy of physical distributions [?], connecting the dynamical and informational content of physical systems with localized configurations. CIE methods have been applied to Korteweg-de Vries (KdV) solitons, compact astrophysical systems, scalar glueballs [?], theoretical studies of new Higgs boson decay channels [?], heavier η meson states in AdS/QCD [?], confinement/deconfinement transitions in QCD [?], quarkonium in finite-density plasmas [?], and time evolution in physical systems [?, ?]. In projectile fragmentation reactions, fragment distributions exhibit sensitive dependence on changes in neutron density [?, ?, ?], suggesting that CIE could enable determination of neutron skin thickness in neutron-rich nuclei. In this study, the CIE method was adopted to quantify the information entropy of nuclear density and fragment distributions in projectile fragmentation reactions. The analyzed data were generated using a modified statistical abrasion-ablation (SAA) model, which is known to successfully describe fragment cross sections in projectile fragmentation reactions [?, ?].

II. THEORIES

A. Modified Statistical Abrasion-Ablation Model

The modified statistical abrasion-ablation (SAA) model [?, ?] can be applied to projectile fragmentation reactions at both intermediate and high energies, representing an improvement over the original SAA model by Brohm and Schmidt [?]. In quasi-free nucleon-nucleon collisions, the reaction is described as a two-step process. In the initial stage, nucleons are categorized by a Glauber-type model as “participants” and “spectators,” where participants interact strongly in the overlapping region between projectile and target while spectators remain virtually undisturbed [?]. In the second stage, the excitation energy is compared to the separation energies of protons, neutrons, and α particles to determine the emission type according to $\min(s_p, s_n, s_\alpha)$. After de-excitation calculations, cross sections for final fragments comparable to experimental measurements are obtained. The colliding nuclei are composed of many parallel tubes oriented along the beam direction, with transverse motion neglected and interactions between tube pairs treated as independent. For a specific pair of interacting tubes, absorption of projectile neutrons and protons is assumed to follow a binomial distribution. At a given impact parameter b , the transmission probabilities for neutrons and protons in an infinitesimal projectile tube are calculated using

$$t_i(s-b) = \exp\{-[\mathcal{D}_T^n(s-b)\sigma_{ni} + \mathcal{D}_T^p(s-b)\sigma_{pi}]\},$$

where \mathcal{D}_T represents the normalized integrated nuclear density distribution of the target along the beam direction for protons ($\int d^2s \mathcal{D}_T^p = Z_T$) and neutrons ($\int d^2s \mathcal{D}_T^n = N_T$), with N_T and Z_T being the target neutron and proton numbers, respectively. The variables s and b are defined in a plane perpendicular to the

beam direction, and $\sigma_{i'i}$ denotes the free-space nucleon-nucleon cross sections ($i', i = n$ for neutrons and $i', i = p$ for protons) [?]. The average absorbed mass in the infinitesimal tube limit at a given b is

$$\langle \Delta A(b) \rangle = \int d^2s \mathcal{D}_T^n(s) [1 - t_n(s - b)] + \int d^2s \mathcal{D}_T^p(s) [1 - t_p(s - b)].$$

For a specific fragment, the production cross section can be calculated using

$$\sigma(\Delta N, \Delta Z) = \int d^2b P(\Delta N, b) P(\Delta Z, b),$$

where $P(\Delta N, b)$ and $P(\Delta Z, b)$ are the probability distributions of abraded neutrons and protons at impact parameter b , respectively. $\sigma(\Delta N, \Delta Z)$ represents the residual fragment after the abrasion stage (the prefragment). The prefragment excitation energy is calculated as $E^* = 13.3 \langle \Delta A(b) \rangle$ MeV, where $\langle \Delta A(b) \rangle$ is the number of abraded nucleons and 13.3 MeV is the mean excitation energy per abraded nucleon [?]. After de-excitation calculations, cross sections for final fragments comparable to measured fragments are obtained.

Fermi-type density distributions were adopted for protons and neutrons in the nucleus:

$$\rho_i(r) = \frac{\rho_0^i}{1 + \exp\left(\frac{r - C_i}{t_i/4.4}\right)}, \quad i = n, p$$

where ρ_0^i is the normalization constant for neutrons ($i = n$) or protons ($i = p$), t_i is the diffuseness parameter, and C_i is half the density radius of the neutron or proton distribution.

B. Configurational Information Entropy Method

To quantify CIE in fragment distributions, we introduce the CIE definitions. For a system with spatially localized clusters, the CIE analysis involves a set of functions $f(x) \in L^2(\mathbb{R})$ and their Fourier transforms $F(k)$ that obey Plancherel's theorem [?]:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(k)|^2 dk,$$

where $f(x)$ is square-integrable and bounded. The model fraction $f(k)$ is defined as

$$f(k) = \frac{|F(k)|^2}{\int |F(k)|^2 dk},$$

with integration over all k , where $F(k)$ is defined and d is the number of spatial dimensions. The model fraction $f(k)$ measures the relative weight of a given mode k . The CIE quantity $S_C[f]$ is defined as the Shannon information entropy of $f(k)$ [?]:

$$S_C[f] = - \sum f_m \ln(f_m).$$

Thus, the CIE quantity contains information about configurations compatible with certain constraints of a given physical system. If all modes k have equal weight, then $f_m = 1/N$ and the discrete configuration entropy reaches its maximum at $S_C = \ln N$. If only one mode exists, $S_C = 0$.

Continuous CIE can also be defined for continuous distributions such as nuclear density distributions. For non-periodic functions on interval (a, b) ,

$$S_C[f] = - \int \tilde{f}(k) \ln[\tilde{f}(k)] d^d k,$$

where $\tilde{f}(k) = f(k)/f(k)_{\max}$ and $f(k)_{\max}$ is the maximum fraction. The normalized function $\tilde{f}(k)$ ensures $\tilde{f}(k) \leq 1$ for all k modes, with $\tilde{f}(k) \ln \tilde{f}(k)$ representing the CIE density.

III. RESULTS AND DISCUSSION

The $^{40-60}\text{Ca} + ^9\text{Be}$ reactions at 350 MeV/u were calculated using the modified SAA model (A_p refers to even mass numbers from 40 to 60). Fragment cross sections with Z ranging from 3 to 20 were obtained. For clarity, only selected results are shown in the figures.

Figure 1 displays the Fermi-type nuclear density distributions and their fast Fourier transform (FFT) spectra. A clear increase in ρ_n is observed from ^{40}Ca to ^{60}Ca , while ρ_p shows the opposite trend. The FFT spectra exhibit a two-peak structure, with the second peak lower than the first. The difference between neutron and proton density distributions $\Delta\rho = \rho_n - \rho_p$ is also shown. For ^{40}Ca , $\Delta\rho$ is very small, but it increases as neutrons are added to the projectile. Based on these FFT spectra $f(k)$, the CIE values for ρ_n , ρ_p , and $\Delta\rho$ can be determined using Eq. (7), denoted as $S_{\rho_n}^C[f]$, $S_{\rho_p}^C[f]$, and $S_{\Delta\rho}^C[f]$, respectively.

The isotopic cross section (σ_Z) distributions from the 350 MeV/u $^{40-60}\text{Ca} + ^9\text{Be}$ reactions are plotted in Figure 2. In panels (ai), from $Z_{fr} = 7$ to 20, the isotopic cross-section distributions show similar patterns for fragments with small Z_{fr} , while a shift toward neutron-rich isotopes is observed for larger Z_{fr} . The symmetric Gaussian-like shape of the isotopic distribution is altered by enhanced cross sections of neutron-rich fragments in neutron-rich reaction systems, demonstrating the isospin effect in fragment production induced by increased neutron density on the surface of neutron-rich nuclei [?]. The FFT spectra of

the isotopic distributions are shown in panels (bj) of Figure 2, each exhibiting a single peak. The amplitudes of these FFT spectra decrease as the projectile becomes more neutron-rich, except for $Z = 20$. Based on these FFT spectra of σ_Z distributions, the CIE quantities are determined according to Eq. (7) and denoted by $S_{\sigma_Z}^C[f]$.

The correlation between the CIE of density distributions and the neutron skin thickness δ_{np} of the projectile nucleus is shown in Figure 3(a). Both $S_{\rho_n}^C[f]$ and $S_{\rho_p}^C[f]$ decrease linearly with increasing δ_{np} from ^{40}Ca to ^{60}Ca . $S_{\Delta\rho}^C[f]$ also decreases linearly with δ_{np} , except for ^{40}Ca . Panel (b) of Figure 3 plots the correlation between $S_{\sigma_Z}^C[f]$ for different Z_{fr} values and δ_{np} of the projectile nuclei. The $S_{\sigma_Z}^C[f]$ values for isotopes from $Z_{fr} = 10$ to 18 decrease with increasing δ_{np} of the projectile nucleus, with fragments near the projectile nucleus showing greater sensitivity to δ_{np} changes.

The mass yield (σ_A) distributions in the 350A MeV $^{40-60}\text{Ca} + ^9\text{Be}$ reactions are shown in Figure 4(a). In each reaction, the mass yield increases with fragment mass number A_{fr} until approaching the projectile nucleus. Different reactions exhibit similar mass distribution trends that decrease with increasing projectile mass number. The corresponding CIE quantities determined from σ_A distributions are labeled $S_{\sigma_A}^C[f]$. The correlation between $S_{\sigma_A}^C[f]$ and δ_{np} for projectile nuclei is shown in Figure 4(b). Except for a bend point at δ_{np} for ^{42}Ca due to the transition from proton skin to neutron skin, the $S_{\sigma_A}^C[f] \sim \delta_{np}$ correlation is linear for $A_p \geq 44$.

The charge cross section is defined as the summation of isotopic cross sections $\sigma_C = \sum_o \sigma(A_o, Z)$. The charge cross section distributions in the 350A MeV $^{40-60}\text{Ca} + ^9\text{Be}$ reactions are shown in Figure 5. Similar trends to those of σ_A distributions are observed. The CIE determined from σ_C distributions, labeled $S_{\sigma_C}^C[f]$, shows linear correlation with the neutron skin thickness of the projectile nuclei.

The CIE approach transforms experimental distributions into quantified parameters, providing information probes for determining system properties. From the $S_{\sigma_Z}^C[f] \sim \delta_{np}$, $S_{\sigma_A}^C[f] \sim \delta_{np}$, and $S_{\sigma_C}^C[f] \sim \delta_{np}$ correlations, we observe that CIE values determined from isotopic, mass, and charge distributions decrease with increasing neutron skin thickness, exhibiting good linear correlations. Determining neutron skin thickness, particularly for nuclei near the neutron drip line, is limited by the lack of effective probes. The linear correlation between CIE and neutron-skin thickness of neutron-rich nuclei provides new approaches for determining projectile neutron skin thickness through measurement of fragment distributions in projectile fragmentation reactions.

IV. SUMMARY

With the vast opportunities for studying highly asymmetric nuclei at new radioactive ion beam facilities, neutron skin thickness represents one of the most

important questions in nuclear physics. In this study, CIE theory was adopted to quantify the information entropy contained in nuclear density distributions and fragment cross-section distributions from 350 MeV/u $^{40-60}\text{Ca} + ^9\text{Be}$ projectile fragmentation reactions calculated using the modified SAA model. CIE quantities were determined for nuclear density distributions ($S_{\rho_{n,p}}^C[f]$ and $S_{\Delta\rho}^C[f]$), isotopic cross-section distributions ($S_{\sigma_z}^C[f]$), mass cross-section distributions ($S_{\sigma_A}^C[f]$), and charge cross-section distributions ($S_{\sigma_C}^C[f]$). Correlations between $S_{\rho_p}^C[f] \sim \delta_{np}$, $S_{\Delta\rho}^C[f] \sim \delta_{np}$, $S_{\sigma_z}^C[f] \sim \delta_{np}$, $S_{\sigma_A}^C[f] \sim \delta_{np}$, and $S_{\sigma_C}^C[f] \sim \delta_{np}$ were investigated. For neutron-rich calcium projectiles, clear linear dependences of $S_{\rho_n}^C[f]$, $S_{\rho_p}^C[f]$, and $S_{\Delta\rho}^C[f]$ on δ_{np} were observed. The $S_{\sigma_z}^C[f]$ values for fragments with different Z_{fr} were shown to depend linearly on the projectile δ_{np} . It was found that when isotopic distributions are sensitive to isospin effects in projectiles, the extracted $S_{\sigma_z}^C[f]$ is also sensitive to their δ_{np} . Good linear correlations between $S_{\sigma_A}^C[f]$, $S_{\sigma_C}^C[f]$, and δ_{np} of the projectile nucleus were also observed. This suggests that from the CIE perspective, isotopic, mass, and charge distributions in projectile fragmentation reactions may serve as effective probes for determining neutron-skin thickness of neutron-rich nuclei.

In this work, the simple description of nuclear density for projectile nuclei has difficulty treating magic number nuclei and large shape distortions. Future improvements should focus on more sophisticated nuclear density inputs, such as those obtained from density functional theory and relativistic mean-field theory, to better investigate nuclear density effects on fragment cross-section distributions and related CIE quantities.

AUTHOR CONTRIBUTIONS

All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Yi-Pu Liu, Hui-Ling Wei, and Chun-Wang Ma. The first draft was written by Yi-Pu Liu and Chun-Wang Ma, and all authors commented on previous versions. All authors read and approved the final manuscript.

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