# Small-Signal Stability Analysis of Grid-Connected Equipment in Renewable Energy Power Systems (Part I): Applicability of Physics-Based Models and Stability Criteria

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## Abstract

The dynamic characteristics of new energy power systems are complex, featuring numerous stability analysis methods and stability criteria. Different stability methods/criteria correspond to different physical interpretations and applicable scopes, making it currently difficult to theoretically address what problems each criterion is suitable for solving and whether their corresponding physical interpretations are reasonable. This paper is divided into two parts: the first part proposes an applicability assessment method for stability criteria and attempts to answer whether the physical interpretations corresponding to stability criteria are reasonable; the second part applies this method to analyze the applicable occasions of some typical criteria and attempts to provide their physical interpretations. The first part begins by reviewing and summarizing the deduction mechanisms of existing stability criteria, as well as the physical meanings underlying each criterion. Secondly, from three perspectives—stability equivalence, nominal property, and robustness—three qualitative principles for the applicability of stability criteria are proposed, along with a quantitative indicator based on loop gain sensitivity. Finally, using grid-connected converters as an example, the properties of several impedance-based analysis methods in frequency-domain analysis are illustrated, and their applicable scopes are discussed. The applicability and physical interpretations of other typical criteria in power systems will be explored in the second part.

# Full Text

## Preamble

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## Small Signal Stability Analysis of Equipment in Renewable Energy Power System (Part I): Mechanism Model and Adaptation of Stability Criterion

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### ABSTRACT

Many methods and criteria for stability assessment have been proposed due to the complex dynamics of renewable power systems. The physical implications of these methods/criteria and their scopes of adaptation differ. It is difficult to theoretically answer, for a given stability issue, which stability criterion is suitable and whether the corresponding physical interpretation is reasonable. This paper is divided into two parts. Part I proposes methods for assessing the adaptation of stability criteria and attempts to answer whether the corresponding physical interpretations are reasonable. Part II applies these methods to analyze the adaptation of typical criteria and derives the physical interpretations. In Part I, we first review the derivation processes of existing stability criteria and their underlying physical significance. Second, we propose selection principles for stability criteria from three perspectives: stability equivalence, nominal performance, and robust stability, and introduce a quantitative index called loop gain sensitivity. Finally, we analyze the adaptation of several impedance criteria for grid-connected converters.

**KEYWORDS:** stability criterion; stability mechanism; loop gain sensitivity; robustness

## 1 Introduction

The development of renewable energy power systems represents a crucial component of China' s energy transition and "dual carbon" strategy, with rapid growth anticipated for wind power, photovoltaic, and other renewable sources. However, the complex dynamic characteristics of renewable energy equipment present numerous challenges for power system stability analysis and control [1-3].

The single-machine infinite-bus system of grid-connected equipment serves as one of the simplest systems for understanding complex power system dynamics [4-6]. Nevertheless, such systems remain multiple-input multiple-output (MIMO) systems with intricate dynamic characteristics, making their stability (including stability margins, hereinafter referring specifically to small-signal stability unless otherwise stated) difficult to analyze and quantify [7]. Numerous time-domain and frequency-domain analysis methods and stability criteria have been developed. Frequency-domain methods are particularly favored due to their suitability for black-box analysis and control strategy design. A mainstream approach retains key variables or models that determine stability while equivalently transforming the remaining components, aiming to obtain a single-input single-output (SISO) system equivalent to the original MIMO system in terms of stability, accompanied by a stability criterion for this equivalent SISO system.

For example, for synchronous machines, the damping torque method—suitable for analyzing low-frequency oscillations—is derived by focusing on rotor angles and eliminating intermediate variables based on physical understanding of low-frequency oscillations [4]. For subsynchronous oscillation (SSO), the complex torque coefficient method and equivalent impedance method are derived by focusing on shaft rotor dynamics and circuit characteristics, respectively [5]. Similarly, for multi-timescale oscillation issues in power electronic devices, various stability criteria have emerged based on different physical insights: generalized/polar impedance criteria focusing on port voltage/current phase angle stability [8]; sequence impedance criteria focusing on positive/negative sequence loop stability [9-11]; dominant loop methods focusing on phase-locked loop (PLL) stability [12]; and generalized torque coefficient methods focusing on inertia and synchronization dynamics [13].

Following this approach, many stability analysis methods can be derived for the same problem. Since different derivation methods originate from the same system, they yield consistent stability judgments as long as the mathematical derivations are rigorous. Taking converter impedance methods as an example, impedance criteria under various coordinates can be transformed into one another [14], producing identical stability assessment results. However, different stability criteria emphasize different physical characteristics and thus reflect different physical meanings. For instance, impedance criteria in polar coordinates focus on phase angle information of port voltage/current, reflecting synchronization characteristics between equipment and the grid, whereas sequence impedance criteria extract voltage/current sequence components, reflecting sequence circuit resonance characteristics between equipment and the grid.

Moreover, different criteria exhibit varying capabilities in characterizing system stability degrees. For example, stability margin characteristics differ across criteria, and the robustness of controllers designed based on different criteria varies significantly [15]. Consequently, the effectiveness of different analysis methods or criteria differs, and their applicable scenarios vary, raising a critical question: What are the differences among these stability analysis methods or criteria, which scenarios are they respectively suitable for, and why?

This paper investigates whether various stability analysis methods and criteria are applicable and whether their derived physical interpretations are reasonable. We propose quantitative methods and indices for evaluating criterion performance and further apply these assessment methods to analyze multiple equipment types and criteria, clarifying their relationships and corresponding physical interpretations for classification. The overall logical structure is illustrated in Fig. 1. The paper is divided into two parts: Part I discusses the principles and analysis methods for determining the applicability of equipment stability criteria, while Part II examines the stability mechanisms of typical equipment and their classification methods.

**Part I is organized as follows:** First, we review the main stability analysis approaches and criteria for power system equipment. Second, we explain the mathematical derivation processes and physical interpretations of various stability criteria, demonstrating that different criteria possess distinct physical meanings. Third, we propose selection principles for stability criteria and quantify them through loop gain sensitivity indices. Finally, using grid-connected converters as an example, we discuss the characteristics of several impedance-based analysis methods in frequency-domain analysis and explore their scopes of application.

The nomenclature used in this paper is provided in Table 1.

**Table 1 Nomenclature in this article**

| Symbol | Description |
|---|---|
| OL(s) | Loop gain sensitivity |
| S$\delta$(s) | Uncertainty sensitivity |
| det( · ) | Matrix determinant |
| arg( · ) | Complex argument |
| | · |
| $\Delta$a | Small increment of variable a |

## 2 Review of Stability Criteria Derivation

### 2.1 Modeling of Grid-Connected Equipment

The single-machine infinite-bus system models for synchronous machines, converters (representing PV, direct-drive wind turbines, etc.), and doubly-fed induction generators are shown in Fig. 2, with linearized models expressed in either time-domain or frequency-domain form.

**Fig. 2 Grid-connected system of power equipment**

The synchronous machine grid-connected system can be described by time-domain state equations:

$$\begin{cases} \Delta\dot{x} = A\Delta x + B\Delta u \\ \Delta y = C\Delta x \end{cases} \tag{1}$$

where Δx typically represents a state vector including multiple state variables such as rotor frequency $\Delta\omega$, rotor angle $\Delta\delta$, excitation voltage ΔE_f, and q-axis transient voltage ΔE' _q; Δu and Δy are input and output vectors; and A, B, C are the system state matrix, input matrix, and output matrix, respectively.

Applying Laplace transform to (1) yields the frequency-domain model of the synchronous machine grid-connected system:

$$\Delta y(s) = G(s)\Delta u(s) \tag{2}$$

where $G(s) = C(sI - A)^{-1}B$ is the system transfer function matrix.

Similarly, renewable energy grid-connected systems such as PV and wind turbines can be modeled in dq, sequence, or polar coordinates:

$$\begin{bmatrix} \Delta U_a \\ \Delta U_b \\ \Delta I_a \\ \Delta I_b \end{bmatrix} = Z(s) \begin{bmatrix} \Delta I_a \\ \Delta I_b \\ \Delta U_a \\ \Delta U_b \end{bmatrix} \tag{3}$$

where ΔU_a, ΔU_b, ΔI_a, and ΔI_b represent small-signal quantities of equipment port voltage and current in arbitrary coordinates (ab), such as rectangular coordinates (dq), sequence coordinates (pn), or polar coordinates (M ); and Z(s) is the equipment complex impedance matrix.

### 2.2 Review of Typical Stability Criteria

As shown in (1)-(3), whether for synchronous machines or renewable energy equipment, single-machine infinite-bus systems are MIMO systems. To facilitate analysis and control, researchers typically perform mathematical transformations or physical equivalences on complex systems to obtain simple, applicable stability criteria, which fall into two main categories.

**Category 1: Constructing equivalent SISO systems based on equipment physical characteristics to derive corresponding stability criteria.** This includes: 1. Using Schur complement transformation to eliminate partial variables (corresponding algebraically to transfer function transformations in physics), such as damping torque method, complex torque coefficient method, selective modal analysis (SMA) [4-5], modified sequence impedance criterion, and generalized impedance criterion, as well as directly calculating the determinant of MIMO system characteristic equations in impedance methods [16]. 2. Diagonalizing matrices through similarity transformations to convert MIMO systems into multiple equivalent SISO systems for analysis, such as generalized Nyquist criterion based on eigenvalue locus decomposition [7], dq impedance criterion [17], unified impedance criterion [18], and diagonalization transformation [19]. 3. Based on physical understanding, ignoring coupling or secondary loops to obtain simplified SISO systems, such as equivalent impedance method for synchronous machine SSO [5] and sequence impedance method without considering coupling [9].

**Category 2: Directly deriving stability criteria or quantitative stability margin indices from MIMO systems**, such as impedance criteria based on forbidden regions [20-21] or norms [22-23], and criteria based on short-circuit ratio [24-25].

In summary, the mathematical and physical methods for forming stability criteria are illustrated in Fig. 3. Although different stability criteria generation methods exist, the fundamental approach is to simplify complexity by leveraging the Nyquist stability criterion for SISO systems to derive physically meaningful stability criteria:

**Fig. 3 Mathematical and physical method of deriving stability criterion**

1. **Damping Torque Method:** Used for analyzing low-frequency oscillations in synchronous machines, this approach retains $\Delta\omega$ and $\Delta\delta$ variables related to rotor motion equations and simplifies the full-order model represented by (1) into a pseudo "second-order" system through Schur complement transformation of state equations [5], thereby forming a damping-based stability criterion using SISO system analysis techniques.

2. **Complex Torque Coefficient Method:** This method simplifies frequency-domain models by retaining oscillation modes strongly related to the shaft system [4]. It can be extended to converter electrical inertia analysis, yielding the generalized torque coefficient method [13]. In the frequency domain, stability criteria formed by damping torque method and complex torque coefficient method are equivalent to the Nyquist criterion [26].

3. **Frequency-Domain Impedance Method:** When analyzing induction generator effects formed by synchronous machines with series compensation, the synchronous machine and series compensation are equivalenced using one-dimensional impedance and circuit [5], forming impedance criteria with circuit mechanism interpretation. For grid-connected converters, impedance models are generally two-dimensional matrices. For dq impedance criteria, the generalized Nyquist criterion is typically used to convert MIMO systems into two equivalent SISO systems [17]; modified sequence impedance [10-11] and polar impedance criteria [8] utilize Schur complement transformation of closed-loop characteristic equation matrices to convert systems into SISO systems for analysis.

In summary, these criteria share the common feature of transforming complex systems into relatively simple systems to form convenient stability criteria. Different criteria focus on different key links, naturally resulting in different physical meanings. This issue can be analogized to a mechanical system: different criteria correspond to different perspectives or coordinate systems, and a good observation perspective better quantifies stability. For example, in the equilibrium system shown in Fig. 4, ignoring disturbance terms, the system appears balanced from any perspective. However, only the side view can capture the "essence" that determines system balance (stability) or imbalance (instability) when considering disturbances, making it more scientific and mechanistic.

**Fig. 4 Effect of observation perspective on the understanding of physical mechanism**

Similarly, numerous stability analysis methods and criteria can be derived through mathematical transformations or physical equivalences. However, different criteria adopt different descriptive perspectives, resulting in varying effectiveness for mechanism analysis. The following sections will discuss the rationality of perspectives, analysis methods, and stability criteria from a control theory perspective in conjunction with specific scenarios.

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## 3 Principles for Stability Criterion Selection

### 3.1 Mathematical Derivation Process of Stability Criteria

Without loss of generality, this paper uses Schur complement transformation of transfer function matrices as an example to illustrate the mathematical derivation approach for constructing equivalent systems and obtaining criteria, along with their physical interpretations.

Consider a grid-connected system described by the transfer function matrix in (2) or (3). For simplified derivation, we select an equal number of input and output variables for modeling, yielding the system frequency-domain model:

$$\Delta y(s) = G(s)\Delta u(s) \tag{4}$$

where $G(s)$ is an $n \times n$ transfer function matrix; $\Delta u(s) = [\Delta u_1, \cdots, \Delta u_n]$ and $\Delta y(s) = [\Delta y_1, \cdots, \Delta y_n]$ are $n$-dimensional input and output vectors, respectively.

Let the system feedback transfer function matrix be $K(s)$:

$$\Delta u(s) = -K(s)\Delta y(s) \tag{5}$$

The closed-loop characteristic equation of this multivariable system is:

$$\det(I + L(s)) = 0 \tag{6}$$

where $L(s) = G(s)K(s)$ is the system open-loop transfer function matrix.

Eliminating partial input/output variables yields an equivalent model. Let the retained $r$ output variable vector be $\Delta y_r = [\Delta y_1, \cdots, \Delta y_r]$, and the eliminated variable vector be $\Delta y_e = [\Delta y_{r+1}, \cdots, \Delta y_n]$. Then (6) can be written in block matrix form:

$$\det\left(\begin{bmatrix} I_{rr} + L_{rr}(s) & L_{re}(s) \\ L_{er}(s) & I_{ee} + L_{ee}(s) \end{bmatrix}\right) = 0 \tag{7}$$

After eliminating output variables $\Delta y_e(s)$, the characteristic equation becomes:

$$\det(I_{rr} + L_r(s)) = 0 \tag{8}$$

where $L_r(s)$ is the equivalent model' s open-loop transfer function matrix:

$$L_r(s) = L_{rr}(s) - L_{re}(s)(I_{ee} + L_{ee}(s))^{-1}L_{er}(s) \tag{9}$$

When the equation $\det(I_{ee} + L_{ee}(s)) = 0$ has no right-half-plane (RHP) zeros, the unstable modes of the equivalent system remain unchanged from the original system [5]. This equivalent process can also be achieved by eliminating partial input variables in the frequency-domain model or partial state variables in the time-domain state-space model; see [5] for details.

When retaining one set of input/output variables ($r = 1$), a stability criterion (and characteristic equation of the equivalent SISO model) for the grid-connected system can be obtained, as shown in (10). In this criterion, the open-loop transfer function matrix $L_r(s)$ reduces to the open-loop transfer function $L_1(s)$:

$$1 + L_1(s) = 0 \tag{10}$$

where $L_1(s)$ is the open-loop transfer function of the equivalent system obtained by retaining the first output variable $\Delta y_1$:

$$L_1(s) = L_{11}(s) - L_{12,1n}(s)(I_{22,nn} + L_{22,nn}(s))^{-1}L_{21,n1}(s) \tag{11}$$

Here, $L(s)$ can be rearranged through row/column exchanges to retain any output variable $\Delta y_x$ (or input variable $\Delta u_x$), yielding different equivalent characteristic equations $1 + L_x(s) = 0$ and open-loop transfer functions $L_x(s)$.

Analyzing $L_x(s)$ using frequency-domain theory reveals the stability properties of the corresponding characteristic equation. For convenience, the equivalent characteristic equation $1 + L_x(s) = 0$ is hereinafter referred to as the stability criterion. Notably, although some classical criteria mentioned in Section 1.2, such as damping torque method and complex torque coefficient method, are not obtained through matrix Schur complement transformation, they can similarly be converted into the form of (10). The applicability analysis methods proposed herein remain applicable, though detailed elaboration is omitted due to space constraints.

Since input/output selection is not unique, repeating this process yields multiple stability criteria and methods–i.e., different $L_1(s)$ exist. However, the following questions must be addressed:

**Question 1:** What are the differences among various stability analysis methods/criteria, which scenarios are they respectively suitable for, and why are they suitable?

### 3.2 Physical Interpretation of Stability Criterion Derivation

The process of transforming complex systems into equivalent simplified systems physically corresponds to merging transfer function loops of partial secondary loops.

As shown in Fig. 5(a), the open-loop transfer function matrix between retained input/output variables is $L_{11}(s)$, representing the dominant loop (shown in blue) composed of extracted key links. The process of eliminating other variables to obtain the equivalent transfer function matrix $L_r(s)$ can be viewed as 折算 (folding) other secondary loops/links into the dominant loop, yielding the equivalent system shown in Fig. 5(b).

**Fig. 5 The frequency domain model of equivalent system**

The equivalent SISO system is obtained based on physical understanding of instability and extraction of key links, and the resulting stability criterion has clear physical meaning: (10) represents the criterion formed by retaining output variable $y_1$, reflecting stability issues dominated by variable $y_1$; or, from a transfer function loop perspective, reflecting physical problems dominated by the link corresponding to the dominant loop $L_{11}(s)$. For example, when retained input/output are current/voltage, the criterion describes system impedance characteristics; when phase angle variables are retained, it reflects equipment synchronization characteristics. Therefore, the difference among various stability criteria and equivalent models lies in different understandings of instability, and different input/output variables forming corresponding stability criteria reflect different physical mechanisms. Consequently, another question must be addressed:

**Question 2:** What mechanism or physical interpretation do different stability analysis methods/criteria reflect, and is it reasonable and scientific?

In summary, grid-connected systems can form different stability criteria and corresponding equivalent models through mathematical transformations. Even though the stability assessment results of various criteria are consistent, the reflected physical characteristics and mechanism interpretations differ. Continuing with the Fig. 4 analogy, under the side view (e.g., a certain criterion), the disturbance force has a smaller projection than the balancing force, whereas under the top view (another criterion), the disturbance force "overwhelms" the balancing force. Thus, when considering uncertainty factors, the so-called "balance mechanism" obtained from the top view cannot be considered the true mechanism. Therefore, each criterion' s focus on key links reflects physical characteristics, and applicable scenarios differ. How to interpret and evaluate the applicability scope of each criterion will be discussed in Chapter 3.

### 3.3 Example: Stability Criteria for Renewable Energy Grid-Connected Systems

Taking PLL-based converters as an example, the renewable energy grid-connected system has a polar coordinate impedance model [15]:

$$\begin{bmatrix} \Delta U \\ \Delta I \end{bmatrix} = \begin{bmatrix} Y_{UI}(s) & Y_{U\theta}(s) \\ Y_{\theta I}(s) & Y_{\theta\theta}(s) \end{bmatrix} \begin{bmatrix} \Delta U \\ \Delta \theta \end{bmatrix} \tag{12}$$

where expressions for each element and operating conditions are given in [15]. The system' s transfer function model is shown in Fig. 6(a), representing a MIMO system.

Through coordinate transformation, polar coordinate impedance can be converted to dq rectangular coordinates or pn sequence coordinates, with equivalent stability discrimination among the three [14].

The generalized impedance criterion eliminates $\Delta E/\Delta U$ variables from the transfer function model in Fig. 6(a) and retains $E\Delta\theta_E/U\Delta\theta_U$ variables—i.e., phase angle variables—with the characteristic equation:

$$1 + L_G(s) = 0 \tag{13}$$

where the open-loop transfer function $L_G(s) = \frac{Y_{\theta\theta}(s)}{Y_{\theta\theta,net}(s)}$, representing the Nyquist stability criterion for impedance. See [15] Eq. (24) for details.

The generalized impedance model can be viewed as taking the blue phase angle loop in Fig. 6(a) as the dominant loop while folding the orange magnitude loop, yielding the equivalent system shown in Fig. 6(b).

**Fig. 6 Generalized impedance equivalent model of grid-connected converter**

Sequence impedance is established in the positive/negative sequence coordinate system, comprising two loops as shown in Fig. 7. Directly ignoring coupling terms between positive and negative sequences simplifies the MIMO system to an SISO system, but analysis results may be incorrect when PLL participation is significant [10-11]; the two are not equivalent. The modified sequence impedance model does not ignore coupling terms. By eliminating negative sequence voltage component $\Delta U_n$ and retaining positive sequence voltage component $\Delta U_p$, the modified sequence impedance characteristic equation is obtained (negative sequence is similar):

$$1 + L_{P/N}(s) = 0 \tag{15}$$

where positive/negative sequence open-loop transfer functions $L_{P/N}(s) = \frac{Z_{p,net}(s)}{Z_p(s)}$; see [15] Eqs. (25)-(26) for details. The modified sequence impedance model folds the converter' s negative sequence loop impact into the positive sequence. For convenience, hereinafter "sequence impedance" refers to the sequence impedance of the modified sequence impedance model.

**Fig. 7 Sequence impedance equivalent model of grid-connected converter**

In addition to stability criteria focusing on converter phase angle and circuit characteristics, there are criteria focusing on DC-side dynamics, such as the equivalent model obtained by retaining $\Delta I_{dc}/\Delta U_{dc}$ [27], with the characteristic equation:

$$1 + L_{DC}(s) = 0 \tag{16}$$

where $L_{DC}(s) = \frac{Z_{DC,net}(s)}{Z_{DC}(s)}$; see [27] for specific expressions. This model can also be transformed with other impedance models [28]; herein, we refer to it as the DC voltage stability criterion and DC equivalent model.

These three models retain different input/output variables and derive different stability criteria. Although these three stability criteria share the same base model and produce consistent stability assessment results, their physical meanings differ. The generalized impedance criterion retains the system' s phase angle loop, describing the phase angle impedance matching characteristics between equipment and the grid, which can explain synchronization stability dominated by phase angle. The sequence impedance criterion uses positive/negative sequence impedance resonance conditions as the stability criterion, describing equivalent impedance resonance characteristics, which can be interpreted as sequence circuit electrical resonance problems. The DC voltage stability criterion retains DC voltage/output power and can be interpreted as AC current/voltage magnitude stability issues (DC current has a strong relationship with AC current magnitude; see Section 4.1).

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# 4 Principles for Selecting Equipment Stability Criteria

## 4.1 Qualitative Principles and Quantitative Indices

As discussed above, a time-invariant system can form multiple stability criteria and analysis methods by selecting different input/output variables, corresponding to multiple equivalent simplified systems and open-loop transfer functions $L_1(s)$, as well as multiple physical interpretations. For a given oscillation mode, the problem becomes how to identify the most suitable criterion for stability analysis and control from numerous possibilities.

For convenience, we first define the following terms: 1. **Definition (Mechanism Criterion and Mechanism Model):** For a given system oscillation mode, the stability criterion and equivalent model suitable for its analysis and control are called the mechanism criterion and mechanism model for that problem. 2. **Definition (Dominant Output Variable and Derived Mechanism):** The output variable retained by the mechanism model and criterion is the dominant output variable for that problem. Combining equipment physical characteristics with the mechanism model to derive the cause of system instability and its physical interpretation constitutes the derived mechanism for that problem.

According to these definitions, determining whether a method/criterion is suitable for analyzing stability in a given scenario (Question 1) or whether a stability mechanism is scientific (Question 2) can be transformed into analyzing whether the corresponding stability criterion (or equivalent model) is reasonable—i.e., whether the selected open-loop transfer function $L_1(s)$ in (10) is reasonable. If the stability criterion selection is reasonable, its corresponding equivalent model can be considered the mechanism model, and its derived physical interpretation of instability should be scientific.

Furthermore, considering that the purpose of obtaining mechanism models/criteria is to facilitate system stability margin assessment and control design for achieving good dynamic performance, analyzing whether $L_1(s)$ is reasonable can be transformed into analyzing whether it satisfies multiple necessary constraints required by control theory, thereby converting the subjective question of "whether the method is suitable and the mechanism scientific" into an objective problem solvable by control theory.

According to classical control theory, to ensure good dynamic performance, dynamic systems must satisfy not only stability requirements but also nominal performance (NP) and robust stability (RS) requirements [29], as shown in Fig. 8. NP describes the ability of the nominal system' s Nyquist curve to stay away from the $(-1, j0)$ point (not entering the blue circle), i.e., the system possesses certain stability margins. RS describes the ability of the nominal system to remain stable after considering uncertainty factors, i.e., the uncertainty region represented by red circles does not encircle the $(-1, j0)$ point.

**Fig. 8 Schematic diagram of equivalent system satisfying the selection principles**

Therefore, to ensure effective analysis and control design, mechanism criteria and models must satisfy the following three qualitative principles:

1. **Principle 1 (Stability Equivalence Principle):** Ensure stability equivalence between the stability criterion and the original system without "misjudgment" (see Section 3.2).
2. **Principle 2 (Nominal Performance Principle):** Ensure the stability margin represented by the stability criterion is valid; e.g., the criterion must not contain open-loop RHP poles (see Section 3.3).
3. **Principle 3 (Robustness Principle):** Ensure the stability criterion possesses high robustness, with stability margin sensitivity to uncertainty factors not being excessive (see Section 3.4).

To quantitatively analyze the nominal performance and robustness of equivalent models, this paper defines the following loop gain sensitivity index:

**Definition (Loop Gain Sensitivity/Margin Change Rate):** The loop gain sensitivity at oscillation mode $s_1$, defined as the product of the open-loop transfer function' s sensitivity to oscillation mode $s_1$ and the unit direction vector of the open-loop transfer function at oscillation frequency $\omega_1$, is:

$$OL(s_1) = \left.\frac{dL(s)}{ds}\right|_{s=s_1} \times \frac{L(s_1)}{|L(s_1)|} \tag{17}$$

where the prime denotes derivative of the transfer function.

Loop gain sensitivity is a complex number reflecting the sensitivity of the open-loop transfer function (margin function) to oscillation modes, capable of characterizing changes during model tuning or perturbation, as shown in Fig. 9. Its angle and magnitude have the following physical meanings: the angle $\arg(OL)$ characterizes the difference between the tuning angle and oscillation mode increment angle, enabling analysis of whether the stability criterion satisfies the nominal performance principle; the magnitude $|OL|$ characterizes the change magnitude of the transfer function under perturbation, useful for analyzing the robustness of stability criteria. When oscillation mode $s_1$ is a weakly damped mode, $OL(j\omega_1) \approx OL(s_1)$.

**Fig. 9 Physical significance of loop gain sensitivity**

### 4.2 Qualitative Principle 1: Stability Equivalence Principle

Ensuring system stability is a basic control requirement. Therefore, stability criteria must first guarantee correct analysis results. For example, during the Schur complement transformation discussed earlier, when the eliminated transfer function contains unstable poles, the equivalent system’ s transfer function may lose some unstable poles of the original system due to zero-pole cancellation, leading to incorrect stability judgment results [30].

A sufficient condition for this principle is that the ignored or eliminated transfer function loop does not contain unstable modes. Notably, the stability equivalence principle is generally satisfied because equipment control design typically ensures each transfer function loop contains no unstable modes [7]. This principle receives the most attention; for example, the common assumption in impedance methods is that “grid-connected equipment is stable.”

### 4.3 Qualitative Principle 2: Nominal Performance Principle

#### 4.3.1 Stability Margin Failure and Nominal Performance Principle

To ensure system nominal performance, the Nyquist curve must stay away from the $(-1, j0)$ point [29], i.e., the system must possess adequate stability margins. Therefore, nominal performance analysis presupposes that stability margins are meaningful or valid.

According to frequency-domain theory, the system margin function $M(\omega)$ is [29]:

$$M(\omega) = |1 + L(j\omega)| \tag{18}$$

The value of the margin function at oscillation mode $s_1$ frequency $\omega_1$ is the vector margin (VM):

$$VM = |1 + L(j\omega_1)| \tag{19}$$

The magnitude of the dominant mode’s vector margin approximates the shortest distance from the Nyquist curve to the $(-1, j0)$ point, capable of characterizing stability margins [31]:

$$M \approx \min_{\omega\in[0,\infty)} |1 + L(j\omega)| \tag{20}$$

The vector margin angle is commonly used to guide control design. The direction angle $\alpha$ for fastest increase satisfies:

$$\alpha = \begin{cases} \arg(M) & \text{if } \arg(M) < 0 \\ \arg(M) - \pi & \text{if } \arg(M) > 0 \end{cases} \tag{21}$$

If the system open-loop transfer function is tuned to $\tilde{L}(s) = L(s) + k(s)$, the tuning angle at the oscillation mode frequency is:

$$\beta = \arg(\tilde{L}(j\omega_1)) - \arg(L(j\omega_1)) \tag{22}$$

When tuning angle $\beta$ satisfies $\beta \in (\alpha - \pi/2, \alpha + \pi/2)$, the system' s vector margin increases; otherwise, it decreases.

However, when the eliminated transfer function loop is improperly selected, the equivalent system may contain open-loop RHP poles, potentially causing the phenomenon where margin increases but characteristic roots shift rightward, leading to stability margin failure. For example, consider an open-loop transfer function $L(s) = \frac{2s^2 - 10(s-20)}{(s+85)(s^2+4s+32)}$ with RHP poles. Its Nyquist curve is shown in Fig. 10(a), encircling the $(-1, j0)$ point once, corresponding to system stability. After parameter tuning, the vector margin decreases to $VM_2$ and increases to $MV_3$ from $MV_1$, yet the system' s dominant characteristic roots shift rightward in both cases, as shown in Fig. 10(b).

**Fig. 10 Stability margin failure caused by RHP poles of the open loop transfer function**

At this point, the stability margin loses meaning because tuning parameters according to the vector margin increase direction actually makes the system less stable. This phenomenon is termed "margin reversal" caused by open-loop RHP poles, which can be used to identify open-loop RHP pole issues.

**Nominal Performance Principle:** The mechanism criterion (mechanism model) must have valid stability margins within the analysis frequency band, generally simplified to the absence of margin failure caused by open-loop RHP poles.

**4.3.2 Nominal Performance Discrimination Method: Angle of Loop Gain Sensitivity** This paper utilizes the angle of loop gain sensitivity to identify the "margin reversal" phenomenon caused by open-loop RHP poles, excluding equivalent models that do not satisfy the nominal performance principle.

**Theorem (Angle Property of Loop Gain Sensitivity):** Consider open-loop transfer function variation $k(s)$ caused by uncertainty perturbation, where the perturbed transfer function is $\tilde{L}(s) = L(s) + k(s)$. The angles of characteristic root increment $\Delta s$, vector margin increment $\Delta VM$, and loop gain sensitivity $OL$ satisfy:

$$\arg(\Delta s) \approx \arg(\Delta VM) - \arg(OL) \tag{23}$$

**Proof:** The perturbed characteristic equation and vector margin are:

$$1 + \tilde{L}(s) = 0, \quad VM = |1 + L(j\omega_1)| \tag{24}$$

Differentiating both sides of (24) with respect to $s$ yields:

$$\tilde{L}'(s) = L'(s) + k'(s) \tag{25}$$

At the oscillation mode $s = s_1$, $1 + L(s_1) = 0$, giving:

$$\left.\frac{d(1 + L(s))}{ds}\right|_{s=s_1} = L'(s_1) \tag{26}$$

Differentiating both sides of (25) with respect to $k$ yields:

$$\frac{dVM}{dk} \approx \frac{d|1 + L(j\omega_1)|}{dk} \tag{27}$$

Since vector multiplication adds angles, the oscillation mode movement direction satisfies $\arg(\Delta s) \approx \arg(\Delta VM) - \arg(OL)$. QED.

Based on the angle property of loop gain sensitivity and the above analysis, the open-loop transfer function of mechanism criteria and models should satisfy:

**Angle Condition of Loop Gain Sensitivity:** When the vector margin increases microscopically (tuning angle $\beta \in (\alpha - \pi/2, \alpha + \pi/2)$), if the difference between the tuning angle and loop gain sensitivity angle ($\gamma = \beta - \arg(OL)$) falls within $(-\pi/2, \pi/2)$, the stability criterion exhibits "margin reversal," fails to satisfy the nominal performance principle, and cannot serve as a mechanism criterion.

**Proof:** When the vector margin increases microscopically, tuning angle $\beta \approx \arg(\Delta VM)$. According to the angle property, $\gamma = \arg(\Delta s)$. Therefore, if $\beta \in (\alpha - \pi/2, \alpha + \pi/2)$ and $\gamma \in (-\pi/2, \pi/2)$, the vector margin increase causes the characteristic root to shift rightward, resulting in "margin reversal."

When the equivalent model contains no open-loop RHP poles, increasing the vector margin always shifts characteristic roots leftward, enhancing stability, with tuning angle $\beta \in (\alpha - \pi/2, \alpha + \pi/2)$ having a fixed range. When the equivalent model contains open-loop RHP poles, "margin reversal" may or may not occur, making it impossible to determine the tuning range that enhances system stability. Thus, "margin reversal" identified by the angle condition is a sufficient condition for the existence of open-loop RHP poles in the stability criterion.

### 4.4 Qualitative Principle 3: Robustness Principle

**4.4.1 Robustness Issues and Robustness Principle** Literature [15] points out that different equivalent impedance models exhibit inconsistent robustness for the same oscillation mode and uses condition number indices to measure uncertainty amplification in different equivalent models. Without loss of generality, using the equivalent process in (8)-(11) as an example, when retaining the first output variable, the equivalent system open-loop transfer function is (11); when retaining the $n$-th output variable, the equivalent system open-loop transfer function is:

$$L_n(s) = L_{nn}(s) - L_{n1,n(n-1)}(s)(I_{11,(n-1)(n-1)} + L_{11,(n-1)(n-1)}(s))^{-1} L_{1n,(n-1)n}(s) \tag{30}$$

If the closed-loop transfer functions corresponding to $L_{11}(s) \cdots L_{nn}(s)$ contain no unstable poles, the equivalent systems described by (11) and (30) produce consistent stability analysis results with the original system.

Considering uncertainty, the system input-output relationship satisfies:

$$\Delta y(s) = G_s(s)\Delta u(s), \quad G_s(s) = G(s)(I + \Delta_i(s)) \tag{31}$$

where $\Delta_i(s)$ represents input multiplicative uncertainty. The system open-loop transfer function matrix is:

$$L_s(s) = L(s)(I + \Delta(s)), \quad \Delta(s) = L(s)\Delta_i(s) \tag{32}$$

When retaining loops $L_{11}(s)$ and $L_{nn}(s)$, the equivalent system open-loop transfer functions are:

$$L_{s1}(s) = L_{11}(s)+\Delta_{11}(s)-[L_{12,1n}(s)+\Delta_{12,1n}(s)][I_{22,nn}+L_{22,nn}(s)+\Delta_{22,nn}(s)]^{-1}[L_{21,n1}(s)+\Delta_{21,n1}(s)] \tag{33}$$

$$L_{sn}(s) = L_{nn}(s)+\Delta_{nn}(s)-[L_{n1,n(n-1)}(s)+\Delta_{n1,n(n-1)}(s)][I_{11,(n-1)(n-1)}+L_{11,(n-1)(n-1)}(s)+\Delta_{11,(n-1)(n-1)}(s)]^{-1}[L \tag{34}$$

Comparing $L_{s1}(s)$ and $L_{sn}(s)$, when uncertainty is considered, equivalent models retaining different loops have different transfer functions before the perturbation term, resulting in different "amplification" effects on uncertainty factors.

Consequently, different equivalent models have different robustness, and their characterization of system stability margins also differs. In system control design, we typically need to select equivalent systems with higher robustness—i.e., stability margins should not change significantly after considering model uncertainty. Otherwise, the retained stability margins in control design are quickly "consumed" by uncertainty, leading to system instability [15].

**Robustness Principle:** The mechanism criterion (mechanism model) should have high robustness within the analysis frequency band.

**4.4.2 Robustness Discrimination Method: Magnitude of Loop Gain Sensitivity** The magnitude of loop gain sensitivity reflects the sensitivity of the stability criterion (or its equivalent model' s open-loop transfer function/margin function) to oscillation modes. A smaller magnitude indicates better robustness of the criterion and equivalent model. Therefore, the ideal model (with open-loop transfer function $L_i(s)$) has the minimum loop gain sensitivity magnitude:

$$|OL_i(s_1)| = \min_{\forall x} |OL_x(s_1)| \tag{35}$$

Since input/output variables can be linearly combined to produce infinite input/output combinations, corresponding to infinite equivalent models through different mathematical transformations, in practical problems we only need to consider several equivalent models with clear physical meaning that are convenient for measurement and practical analysis/control as candidates. The stability criterion with smaller loop gain sensitivity magnitude is selected as the mechanism criterion for the specific problem:

**Loop Gain Sensitivity Robustness Condition:** Among candidate stability criteria, the mechanism criterion should have the minimum loop gain sensitivity magnitude at the oscillation mode.

Notably, the magnitude of loop gain sensitivity defined herein is consistent with the transfer function condition number index defined in [15]. The condition number index refers to the maximum value of transfer function sensitivity to system perturbations, expressed as [15]:

$$\mathrm{cond}(L) = \max_{\delta} \frac{\|\partial L(s)/\partial \delta\|}{\|L(s)\|} \tag{36}$$

where $\delta$ represents system uncertainty perturbations, including parameter or structural uncertainty; $s_1$ denotes the system oscillation mode; $R_L(s_1)$ reflects the sensitivity of the transfer function (stability margin) to perturbations, with its maximum value being the condition number index. A smaller condition number indicates better model robustness. Combining (17) yields that the ratio of $R_L(s_1)$ to loop gain sensitivity magnitude $|OL(s_1)|$ is the sensitivity of the oscillation mode to perturbations:

$$S_\delta(s_1) = \frac{R_L(s_1)}{|OL(s_1)|} \tag{37}$$

When the system is determined, the sensitivity $S_\delta(s_1)$ of oscillation mode $s_1$ to a given uncertainty perturbation $\delta$ is fixed and does not change with modeling or criterion selection. Therefore, when $S_\delta(s_1) \neq 0$ (i.e., perturbation affects the mode), $|OL(s_1)|$ and $R_L(s_1)$ (or the condition number index) are consistent. However, the condition number index calculation depends on detailed system transfer function models and requires considering all system uncertainties when obtaining its maximum value, making the process cumbersome. In contrast, the loop gain sensitivity index only needs to be calculated once, which is more convenient.

It is also worth noting that, based on the equivalence between time-domain and frequency-domain, stability criteria derived from the time-domain should also follow this principle. For multi-machine systems, the issue of how to identify key units for analysis simplification also exists. Different simplification methods yield systems with different robustness, and the proposed principles can still be extended to such multi-machine problems. Meanwhile, the proposed principles and methods are essentially based on robust control theory and represent a relatively general approach for identifying key loops in complex dynamic systems.

### 4.5 Flowchart for Stability Criterion Adaptation Analysis

Based on the three qualitative principles and loop gain sensitivity quantitative index, the equipment stability criterion selection flowchart is shown in Fig. 11:

**Fig. 11 Flow chart of equipment stability criterion selection**

1. **Establish a detailed MIMO model** of the system, analyze its main physical links, retain different output variables to form different stability criteria and equivalent systems, and constitute a candidate criterion set.
2. **Calculate the loop gain sensitivity** for each criterion and select the mechanism criterion according to the three selection principles.
3. **Derive the instability mechanism** based on the physical meaning of the mechanism criterion and dominant output variables to obtain the derived mechanism.

# 5 Applicability Analysis of Converter Grid-Connected Impedance Criteria

## 5.1 Applicability Analysis

Grid-connected converters exhibit multiple stability modes [32]. The three stability criteria in Section 2.3 respectively describe different aspects of the system. We further analyze their applicability to different oscillation modes using the loop gain sensitivity index. The converter grid-connected system structure and parameters are provided in Appendix B, with calculation results shown in Table 2.

**Table 2 Adaptation analysis of three different stability criteria**

| Oscillation Mode $s_0$ | Generalized Impedance Criterion | Sequence Impedance Criterion | DC Voltage Criterion |
|---|---|---|---|
| $-1.76 \pm j9.04$ (outer loop) | $VM = 0.467\angle-38.23°$, $OL = 0.403\angle137.94°$, $\gamma = -176.17°$ | $VM = 0.373\angle-28.56°$, $OL = 0.356\angle154.59°$, $\gamma = -183.15°$ | $VM = 0.233\angle-64.78°$, $OL = 0.140\angle124.21°$, $\gamma = -188.99°$ |
| $-0.85 \pm j86.25$ (PLL) | $VM = 0.147\angle-9.16°$, $OL = 0.205\angle168.69°$, $\gamma = -177.85°$ | $VM = 0.061\angle169.19°$, $OL = 0.699\angle-12.31°$, $\gamma = 1.50°$ | $VM = 0.198\angle141.81°$, $OL = 0.538\angle-41.02°$, $\gamma = 2.83°$ |
| $-9.87 \pm j639.05$ (inner loop) | $VM = 0.069\angle-51.26°$, $OL = 0.0073\angle129.22°$, $\gamma = -180.48°$ | $VM = 0.037\angle-53.87°$, $OL = 0.0038\angle127.43°$, $\gamma = -181.30°$ | $VM = 0.703\angle-20.11°$, $OL = 0.181\angle162.74°$, $\gamma = -182.85°$ |

All three criteria can accurately assess system stability across the full frequency band [8,10,27]. However, in sub/super-synchronous frequency bands with high PLL participation, ignoring coupling in sequence impedance criteria may produce incorrect results [10-11], violating the stability equivalence principle and thus not considered herein.

**Case 1 (Current magnitude stability caused by DC voltage outer loop):** Based on vector margins, the generalized impedance, sequence impedance, and DC voltage criteria characterize stability margins differently, with tuning angles along the vector margin increase direction being $\beta_G = -38.2°$, $\beta_{PN} = -28.6°$, and $\beta_{DC} = -64.8°$. Consequently, the discrimination angles are $\gamma_G = -176.2°$, $\gamma_{PN} = -183.2°$, and $\gamma_{DC} = -190°$, all satisfying the loop gain sensitivity angle condition. However, according to the robustness condition, the three criteria have different $|OL|$ values, with the DC voltage criterion having the smallest $|OL|$, indicating better robustness and greater suitability for analyzing this problem.

Additionally, outer-loop-dominated stability issues may also cause voltage collapse when the equilibrium point reaches a saddle-node bifurcation. When converter output current increases to 1.17 pu, the system loses stable equilibrium. Approximate analysis at the pre-collapse operating point yields a real root at $s_1 = 1.67/-1.56$, where the angle condition is meaningless. According to the robustness condition, the generalized impedance, modified sequence impedance, and DC voltage criteria have $|OL|$ values of 0.023, 0.017, and 0.002, respectively, with the DC voltage criterion still being the mechanism criterion for this mode.

**Table 3 Adaptation of different impedance criteria (instability caused by outer loop)**

| Criterion | Stability Equivalence | Nominal Performance | Robustness |
|---|---|---|---|
| Generalized Impedance | Satisfied | Satisfied | Moderate |
| Sequence Impedance | Satisfied | Satisfied | Moderate |
| DC Voltage | Satisfied | Satisfied | **Good** |

**Case 2 (Oscillations caused by PLL):** $s_1 = -0.85 \pm j86.25$ is a weakly damped mode with vector margins below 0.2 across all criteria, indicating oscillation risk. Based on the angle condition, discrimination angles are $\gamma_G = -177.9^\circ$, $\gamma_{PN} = 1.5^\circ$, and $\gamma_{DC} = 2.8^\circ$. For sequence impedance and DC voltage criteria, vector margin increase leads to decreased oscillation mode real parts, causing "margin reversal" and violating the nominal performance principle. The open-loop transfer function pole distributions for the three criteria are shown in Fig. 12(a), where sequence impedance and DC voltage criteria contain 1 and 2 open-loop RHP poles, respectively, while the generalized impedance criterion has none. The Nyquist curves for the three criteria are plotted in Fig. 12(b), showing that sequence impedance and DC voltage criteria encircle the $(-1, j0)$ point 1 and 2 times, respectively, indicating correct stability judgment results but "margin reversal" due to open-loop RHP poles, which is unfavorable for nominal performance analysis and tuning.

**Fig. 12 RHP Pole analysis of open loop transfer function for three different impedance criteria**

Since "margin reversal" is related to operating conditions and parameters, this inadvertently increases the complexity of control parameter tuning based on these criteria. Further analysis reveals that the generalized impedance criterion has the smallest $|OL|$, indicating better model robustness and greater suitability for robust stability analysis and control.

**Table 4 Adaptation of different impedance criteria (oscillations caused by phase-locked loop)**

| Criterion | Stability Equivalence | Nominal Performance | Robustness |
|---|---|---|---|
| Generalized Impedance | Satisfied | **Satisfied** | **Good** |
| Sequence Impedance | Satisfied | RHP poles exist | Moderate |
| DC Voltage | Satisfied | RHP poles exist | Poor |

**Case 3 (Oscillations caused by inner loop):** $s_1 = -9.87 \pm j639.05$ is a weakly damped mode. All three criteria satisfy the angle condition based on discrimination angles. For the robustness condition, the sequence impedance criterion has the smallest $|OL|$, indicating better model robustness. Therefore, the sequence impedance criterion is applicable to oscillations caused by the inner loop, facilitating mechanism interpretation.

**Table 5 Adaptation of different impedance criteria (oscillations caused by inner loop)**

| Criterion | Stability Equivalence | Nominal Performance | Robustness |
|---|---|---|---|
| Generalized Impedance | Satisfied | Satisfied | Moderate |
| Sequence Impedance | Satisfied | Satisfied | **Good** |
| DC Voltage | Satisfied | Satisfied | Poor |

In summary: - **PLL-dominated oscillations** are best explained by the generalized impedance criterion retaining voltage/current phase angle variables, with the derived mechanism being phase angle impedance mismatch between converter and network, belonging to synchronization stability [15]. - **Inner-loop-dominated oscillations** are best explained by the modified sequence impedance criterion retaining sequence components, with the derived mechanism being impedance resonance in positive or negative sequence loops, belonging to electrical resonance. - **Outer-loop-dominated instability** should be explained by the DC voltage criterion retaining DC dynamics, where DC voltage instability transfers to the AC side through converter power conversion, causing output current and voltage magnitude instability, belonging to voltage stability.

The instability modes for different stability issues in polar coordinates are shown in Appendix C. Although this paper analyzes criterion applicability and mechanisms for converter systems with typical parameters, the parameter selection follows physical characteristics, making the conclusions 具有一定普适性 (have certain universality).

### 5.2 Control Strategy Verification Based on Mechanism Criteria

To verify the above conclusions, control parameter tuning is performed under different equivalent models. Even with identical control objectives (e.g., stability margins), the tuned control performance is inconsistent. To ensure robust stability, the vector margin magnitude must be at least 0.5 according to control theory requirements [29].

Following the vector margin tuning method described in Section 3.3, Case 1 has a damping ratio of 0.19 with adequate margins across all three criteria, thus not discussed further. Since Case 2 exhibits "margin reversal" making tuning difficult, we perform control parameter tuning for Case 3 (inner-loop dominated) and Case 4 (PLL dominated), as shown in Table 6.

**Table 6 Parameter tuning of two impedance models**

| Parameter | Generalized Impedance | Sequence Impedance | DC Voltage |
|---|---|---|---|
| Inner-loop PI tuning | 0.42, 240 | 0.55, 250 | - |
| Post-tuning VM | $0.50\angle 25.33°$ | $0.50\angle 25.40°$ | - |
| Post-tuning mode | $-130.9 \pm j640.76$ | $-221.9 \pm j639.50$ | - |
| PLL PI tuning | 105, 17500 | 63, 18200 | - |
| Post-tuning VM | $0.50\angle 35.07°$ | $0.51\angle 7.34°$ | - |
| Post-tuning mode | $-100.6 \pm j144.72$ | $-3.21 \pm j155.36$ | - |

The original system in Case 3 has a damping ratio of 0.004. Since the DC voltage criterion yields excessively large stability margins contrary to reality, it is not considered for tuning. The generalized impedance and sequence impedance models achieve post-tuning damping ratios of 0.2 and 0.33, respectively, indicating good nominal performance. With a 0.1 pu disturbance applied at $t = 0.5$ s to the converter terminal voltage, the output d-axis current time-domain waveforms are shown in Fig. 13(a). Both converge rapidly, demonstrating adequate stability margins and nominal performance. At $t = 1$ s, with a 35% increase in line inductance, the sequence impedance-tuned system shows better convergence than the generalized impedance-tuned system, indicating higher control robustness. This aligns with the conclusion that generalized impedance satisfies nominal performance but violates robustness principles.

**Fig. 13 Output active current waveform in EMT simulation**

The original system in Case 4 has a damping ratio of 0.01. Since the DC voltage criterion yields a vector margin magnitude of 0.885 (excessively large and contrary to reality), it is not discussed. The generalized impedance and sequence impedance models achieve post-tuning damping ratios of 0.57 and 0.02, respectively, with the sequence impedance-tuned system having insufficient damping and poor nominal performance. With a 0.1 pu disturbance at $t = 0.5$ s, the port output d-axis current waveforms are shown in Fig. 13(b). Both converge, but the sequence impedance-tuned system converges poorly, failing nominal performance requirements. At $t = 1$ s, with a 25% increase in line and filter inductance, the generalized impedance-tuned system remains stable with robust control, while the sequence impedance-tuned system diverges into oscillation, demonstrating poor control robustness. This matches the judgment that sequence impedance fails to satisfy both nominal performance and robustness principles.

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## 6 Conclusion

This paper investigates the applicability of system stability analysis methods/criteria derived from key input/output variables and the scientific nature of stability mechanism interpretations. We propose three qualitative principles—stability equivalence, nominal performance, and robustness—to describe whether frequency-domain stability criteria are reasonable, along with quantitative indices based on loop gain sensitivity. The study reveals that although different stability analysis methods/criteria can be mathematically transformed into one another, their corresponding mechanism interpretations differ, as do their characterizations of system stability margins and robustness, leading to different applicable scenarios.

When applying or proposing a new stability analysis method, besides focusing on stability analysis results, attention must also be paid to nominal performance and robustness. Furthermore, the principles and methods proposed herein can provide theoretical foundations for reasonable modeling analysis, control, and selection of stability assessment indices for complex systems, such as the applicability issues of various short-circuit ratios. This will be the subject of future research.

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## Appendices

### Appendix A: Equivalent System Transfer Function Expressions with Perturbation

The expressions for equivalent system transfer functions with perturbation are provided in the full paper.

### Appendix B: Converter Grid-Connected Model Structure and Parameters

**Fig. B1 Converter grid-connected system model**

**Table B1 Parameters of grid-connected converter**

| Parameter | Case 1 | Case 2 | Case 3 [32] | Case 4 [15] |
|---|---|---|---|---|
| System base $S_b$ (kVA) | 1100 | 1100 | 1100 | 1100 |
| AC voltage base $U_b$ (V) | 1100 | 1100 | 1100 | 1100 |
| DC voltage base $U_{bdc}$ (V) | 1100 | 1100 | 1100 | 1100 |
| DC capacitor $C_{dc}$ (pu) | 0.038 | 0.038 | 0.038 | 0.0272 |
| Filter $L_f, C_f$ (pu) | 0.05, 0.05 | 0.05, 0.05 | 0.05, 0.05 | 0.15, 0.25 |
| DC voltage loop PI | 0.2, 5 | 1.2, 10 | 0.8, 6 | 0.25, 16 |
| Current loop PI | 0.6, 18 | 0.8, 18 | 0.25, 240 | 0.476, 3.28 |

| Parameter | Case 1 | Case 2 | Case 3 [32] | Case 4 [15] |
|---|---|---|---|---|
| PLL PI | 50, 3600 | 10, 7200 | 35, 3000 | 60, 18200 |
| Line inductance $L_{grid}$ (pu) | - | - | - | - |

*Note: Case parameters follow references [15,32].*

**Appendix C: Instability Waveforms**

**Fig. C1 Instability waveform of grid-connected converter**

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*Note: Figure translations are in progress. See original paper for figures.*