The Role of Visual Shape Perception in the Relationship Between the Approximate Number System and Computational Fluency

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Abstract

Extensive research has revealed the correlation between the approximate number system and computational fluency, yet systematic examination and verification of the reasons underlying this relationship remain lacking. The visual shape perception hypothesis diverges from traditional domain-specific explanations of numerical processing, proposing that rapid shape perception constitutes a shared cognitive mechanism for both the approximate number system and computational fluency; that is, the capacity for rapid visual shape perception can account for the correlation between the two. Both the approximate number system and computational fluency depend on rapid shape perception during processing, with both involving the rapid processing of complex visual stimuli. The visual shape perception hypothesis has garnered support from a series of empirical findings; however, such research has been confined to examining the associations between visual shape perception and the two constructs, leaving the processing mechanisms through which visual shape perception operates in this relationship poorly understood. Future investigations should integrate multiple research methods and techniques to comprehensively explore, from diverse perspectives, the cognitive and neural mechanisms underlying the role of visual shape perception in this relationship, and subsequently apply these findings to mathematics classroom pedagogy and interventions for mathematical difficulties.

Full Text

The Role of Visual Form Perception in the Relationship Between the Approximate Number System and Arithmetical Fluency

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Abstract: Numerous studies have revealed a correlation between the approximate number system (ANS) and arithmetical fluency, yet systematic examination and theoretical justification for the underlying causes of this relationship remain lacking. The visual form perception hypothesis offers an alternative to traditional domain-specific explanations of numerical processing, proposing that rapid form perception constitutes a shared cognitive mechanism underlying both the approximate number system and arithmetical fluency. According to this view, the capacity for rapid visual form perception can explain the observed correlation between these two constructs. Both the approximate number system and arithmetical fluency depend on rapid form perception during processing, as both involve the swift processing of complex visual stimuli. While a series of studies have supported the visual form perception hypothesis, this research has been limited to exploring the relationship between visual form perception and the two constructs individually, leaving the processing mechanisms through which visual form perception operates in their relationship unclear. Future research should integrate multiple methodologies and techniques to investigate the cognitive and neural mechanisms of visual form perception' s role in the relationship between the approximate number system and arithmetical fluency from multiple perspectives, and apply these findings to mathematics classroom instruction and interventions for calculation difficulties.

Keywords: Approximate number system, Visual form perception, Arithmetical fluency

In daily life, people frequently encounter situations requiring rapid quantity processing. The cognitive system underlying this ability to process numerical information without relying on language or counting is known as the Approximate Number System (ANS) [?, ?]. This fundamental numerical processing capacity is shared by humans and other species [?, ?, ?] and emerges during human infancy [?, ?, ?, ?]. Arithmetical fluency refers to the ability to solve simple symbolic arithmetic problems quickly and accurately, representing a crucial component of mathematical competence [?, ?, ?]. As the foundation for mathematical problem-solving, higher arithmetical fluency enables individuals to conserve cognitive resources for more complex problem-solving and reasoning tasks [?, ?]. The relationship between basic numerical processing capacity and arithmetical fluency has long been a central concern for researchers, with numerous studies investigating the importance of the approximate number system in arithmetical fluency (e.g., [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]).

In light of this correlation, increasing attention has focused on why the approximate number system plays a role in arithmetical fluency, with researchers attempting to explain their relationship through mediating factors. However, these explanations remain diverse and lack sufficient direct evidence, while some experimental findings cannot be reasonably accounted for under previously proposed hypotheses. Recently, researchers have proposed the "visual form perception hypothesis," suggesting that rapid form perception represents an important cognitive factor shared by both approximate number system and arithmetical fluency processing [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. This paper therefore examines the relationship between the approximate number system and arithmetical fluency, compares the visual form perception hypothesis with alternative explanations, summarizes research findings on the role of visual form perception in the approximate number system, arithmetical fluency, and their relationship, discusses current limitations in research on this hypothesis, and outlines future research directions.

2. The Relationship Between the Approximate Number System and Arithmetical Fluency

The dot comparison paradigm is commonly used to assess the approximate number system (e.g., [?, ?, ?, ?]). In this task, two dot arrays are briefly presented, and participants must judge which array contains more dots without being able to count them individually. Mixed dot arrays [?, ?] and separated dot arrays [?, ?] represent the two primary stimulus presentation modes. Task difficulty is typically manipulated by the ratio between the numbers of dots in the two arrays; as the ratio approaches 1, the numerical difference decreases, making the judgment more difficult.

Numerous studies have demonstrated the importance of the approximate number system in arithmetical fluency. Initial evidence emerged from research on populations with mathematical learning disabilities, where researchers found that individuals with dyscalculia performed worse on dot comparison tasks compared to typically developing groups (e.g., [?, ?, ?, ?, ?, ?]). For instance, Piazza et al. (2010) found that individuals with dyscalculia exhibited poorer ANS acuity than their typically developing peers, manifesting as longer reaction times and higher error rates on dot comparison tasks [?, ?, ?]. The correlation between the approximate number system and arithmetical fluency has also been observed in typically developing children and adults across various grade levels [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?] as well as in adult populations [?, ?, ?, ?]. A large-scale study of 1,857 elementary students in grades three through six demonstrated that ANS acuity correlates with arithmetical fluency, even after controlling for intelligence, spatial processing ability, processing speed, and numerical magnitude comparison ability [?, ?]. Neuroscientific research has further confirmed the close relationship between the approximate number system and arithmetical fluency: researchers assessed children' s calculation abilities at age 10 and measured their ANS acuity two years later, finding that children with higher arithmetical fluency at age 10 showed greater activation in the intraparietal sulcus (reflecting higher ANS acuity) during the later assessment [?, ?].

Training studies have additionally shown that ANS-based training can effectively improve calculation performance in both children and adults [?, ?, ?, ?, ?]. For example, Hyde et al. (2014) trained 6- to 7-year-old children on dot comparison and approximate addition tasks, resulting in improved arithmetical fluency. Similarly, adults who received training on approximate arithmetic tasks with dot arrays showed significant improvements in both calculation performance and ANS acuity [?, ?, ?], with enhanced calculation performance significantly correlating with changes in ANS acuity [?, ?].

However, some studies have failed to find a correlation between the approximate number system and arithmetical fluency [?, ?, ?, ?, ?, ?, ?, ?]. These discrepancies may reflect influences from different measurement paradigms and statistical indices used in dot comparison tasks [?, ?]. For instance, when using the numeric distance effect (NDE) as a statistical index, no relationship between the approximate number system and arithmetical fluency emerges [?, ?, ?, ?]. Researchers have emphasized that accuracy, reaction time, and Weber fraction represent more appropriate and stable statistical indices for dot comparison tasks [?, ?]. Additionally, factors such as participants' age and whether general cognitive abilities are controlled can affect the relationship between the approximate number system and arithmetical fluency [?, ?, ?]. For example, Wilkey and Ansari (2020), in their review of non-significant relationships between the approximate number system and mathematical ability, noted that individuals might perceive continuous visual properties of dot arrays rather than numerical information per se, and that dot comparison tasks may involve additional cognitive components affecting mathematical processing, such as inhibitory control over visual information.

3. Theoretical Explanations for the Relationship Between the Approximate Number System and Arithmetical Fluency

In response to the observed correlation between the approximate number system and arithmetical fluency, researchers have increasingly focused on understanding why the approximate number system plays a role in calculation. This essentially concerns the cognitive mechanisms underlying calculation processes. Addressing this question will help clarify the function of the approximate number system in calculation and provide a theoretical foundation for using ANS training to promote children' s arithmetical fluency development. Our review of previous literature categorizes these theoretical explanations into two groups: domain-general explanations and domain-specific explanations of number.

3.1. Domain-General Explanations

Domain-general explanations propose that general cognitive factors can account for the approximate number system' s role in arithmetical fluency. The primary accounts include the inhibitory control hypothesis [?, ?, ?] and the visual form perception hypothesis [?, ?, ?, ?, ?]. The inhibitory control hypothesis suggests that inhibitory control may constitute an important factor influencing the relationship between ANS acuity and arithmetical fluency [?, ?, ?]. ANS acuity is typically measured using dot comparison tasks. To address the influence of visual perception on dot array judgments, researchers generate two types of dot patterns: congruent conditions, where visual features positively correlate with numerosity, and incongruent conditions, where visual features negatively correlate with numerosity. The incongruent condition requires participants to inhibit visual features and focus on numerical information, closely resembling the classic Stroop paradigm. Since Stroop tasks commonly measure inhibitory control, and children' s inhibitory control abilities correlate strongly with their calculation abilities [?, ?, ?, ?], Gilmore et al. (2013) proposed the inhibitory control hypothesis, suggesting that inhibitory control mediates the relationship between ANS acuity and arithmetical fluency. Using a task involving visual shape processing [?, ?] to measure inhibitory control in 7- to 11-year-old children, they presented a series of black-and-white circles and squares for rapid naming, then required opposite naming (e.g., saying "circle" when viewing a square). Results showed that inhibitory control could explain the relationship between children' s ANS acuity and arithmetical fluency.

The visual form perception hypothesis posits that visual form perception represents the underlying cognitive mechanism linking the approximate number system and arithmetical fluency, such that controlling for visual form perception acuity eliminates the correlation between ANS acuity and arithmetical fluency [?, ?, ?, ?, ?, ?]. This hypothesis emphasizes the role of rapid form perception in their relationship. Specifically, both the approximate number system and arithmetical fluency depend on rapid form perception during processing, as both involve the swift processing of complex visual stimuli. Arithmetical fluency involves rapid perception of mathematical symbol shapes (including digits and operators) and rapid search and retrieval of arithmetic facts. Dot comparison relies on structural relationships between dots during rapid numerosity judgments. For example, visual properties such as convex hull size (the geometric shape formed by peripheral dots) and dot density (the degree of clustering between dots) influence ANS acuity. Therefore, rapid visual perception of structural relationships between dots and (invisible) lines is crucial during dot array processing. Similarly, arithmetical fluency depends on visual form perception, which may share similar cognitive processing with extracting numerical information from dot array or other visual object configurations. Indeed, research on the relationship between the approximate number system and different mathematical abilities has found that dot comparison ability correlates with arithmetical fluency but not with more complex, less fluent mathematical abilities such as problem-solving and estimation [?, ?, ?, ?].

Although both the visual form perception hypothesis and the inhibitory control hypothesis emphasize the role of general cognitive factors in the relationship between the approximate number system and arithmetical fluency, they differ in their focus: the inhibitory control hypothesis stresses the explanatory role of inhibitory control abilities, whereas the form perception hypothesis emphasizes rapid form perception—that is, the speed of shape perception—in explaining their relationship. These hypotheses are not mutually exclusive; the inhibitory control task used by Gilmore et al. (2013) may have involved visual form perception abilities. However, several studies have yielded results inconsistent with the inhibitory control hypothesis [?, ?, ?]. A review of these studies reveals that inhibitory control was typically measured using the Head-Toes-Knees-Shoulders task, which involves minimal shape perception processing. Moreover, some evidence contradicting the inhibitory control hypothesis failed to control for general cognitive factors related to visual form perception.

3.2. Domain-Specific Explanations

Domain-specific explanations of number propose that numerical information from dot arrays can be extracted directly, and that both the approximate number system and arithmetical fluency involve quantity processing, making numerical processing the shared mechanism [?, ?]. According to this view, the approximate number system forms the foundation for acquiring arithmetical fluency (e.g., [?, ?, ?, ?, ?, ?]). Additionally, both the approximate number system and arithmetical fluency activate core brain regions for numerical processing, particularly the intraparietal sulcus [?, ?, ?].

On the surface, the visual form perception hypothesis appears to conflict with the domain-specific hypothesis. However, two points warrant consideration. First, Halberda et al. (2008) found that children' s ANS acuity correlated with their calculation performance even after controlling for numerous general cognitive abilities including intelligence, visual working memory, spatial reasoning, reading, and executive function. Notably, except for dot comparison, the cognitive tasks in this study did not involve rapid presentation—that is, they rarely depended on shape perception processing speed—which may explain the unique predictive power of the approximate number system for calculation performance. Furthermore, in other studies supporting the domain-specific hypothesis [?, ?, ?, ?, ?, ?, ?, ?, ?], most lacked processing speed requirements (e.g., the Test of Early Mathematics Ability, TEMA), meaning participants did not need to process and respond to problems quickly. In contrast, the visual form perception hypothesis emphasizes the close relationship between the approximate number system and arithmetical fluency, as well as the explanatory role of rapid visual form perception. Second, studies revealing correlations between ANS acuity and arithmetical fluency have often lacked adequate control for general cognitive factors, particularly visual form perception [?, ?, ?, ?, ?, ?, ?, ?, ?]. For example, some studies have found that ANS acuity no longer correlates with calculation after controlling for visual processing ability [?, ?].

4. The Role of Visual Form Perception in the Approximate Number System and Arithmetical Fluency

Based on the above review, direct evidence regarding the cognitive mechanisms underlying the relationship between the approximate number system and arithmetical fluency remains insufficient. Although some experimental results directly or indirectly support both domain-specific and domain-general inhibitory control hypotheses, certain findings are difficult to explain under these theoretical frameworks [?, ?, ?, ?]. Recently, researchers have directly proposed and tested the "visual form perception hypothesis" concerning the relationship between the approximate number system and arithmetical fluency. Below we summarize direct and indirect evidence for the role of visual form perception in the approximate number system and arithmetical fluency.

4.1. The Role of Visual Form Perception in Arithmetical Fluency

Research has shown that children with dyscalculia exhibit deficits on visual form perception tasks [?, ?, ?, ?]. For example, after controlling for choice reaction time, spatial ability, visual tracking ability, and intelligence in children with dyscalculia, Zhou and Cheng (2015) found deficits in their visual form perception abilities (measured using a rapid figure matching task). Compared to children with reading disabilities and typically developing children, those with dyscalculia performed worse on inhibitory control tasks involving shape processing [?, ?]. In typically developing children, visual form perception processing abilities correlate with calculation performance, and this relationship persists even after controlling for other general cognitive factors such as intelligence and auditory perception [?, ?, ?, ?, ?]. For instance, Cui et al. found that visual form perception ability constitutes a potential cognitive mechanism for calculation ability in third- through fifth-grade children [?, ?, ?].

Furthermore, the occipital lobe represents a key brain region for calculation processing [?, ?, ?, ?, ?, ?]. Tinelli et al. (2015) found that occipital lobe damage impairs the ability to discriminate symbolic digits. A patient with bilateral occipital lobe damage (D.F.) who developed visual form agnosia could not distinguish simple geometric shapes, Arabic numerals, or letters. Although her auditory counting ability remained intact, she had difficulty with dot enumeration and simple calculation [?, ?, ?]. Compared to multiplication, which relies more on memorized multiplication tables, addition operations depend more on visual imagery processing of numbers and activate more visuospatial processing brain regions, including the middle occipital gyrus, superior occipital gyrus, and right intraparietal sulcus [?, ?].

4.2. The Role of Visual Form Perception in the Approximate Number System

Research has confirmed that dot array processing is influenced by visual properties, including cumulative surface area [?, ?], average dot size [?, ?], density [?, ?], and convex hull (the smallest convex polygon containing all dots) [?, ?]. Studies have found that individuals process dot array shapes automatically and holistically [?, ?, ?, ?], sometimes relying on visual properties to judge dot array numerosity. This occurs not only in children with dyscalculia [?, ?] but also in typically developing children [?, ?, ?] and adults [?, ?, ?]. For example, children with dyscalculia struggle to judge dot array numerosity when visual features are incongruent with quantity [?, ?]. Clayton et al. (2015) found that the level of control over dot array visual properties affects the measurement of ANS acuity.

Neuroscientific research on approximate number system processing also demonstrates the role of visual form perception in dot comparison. Dewind et al.' s (2019) fMRI study found that individuals activate early visual cortex when encoding dot array numerical information. An ERP study controlling for dot array visual features revealed that individuals make numerosity judgments after weighing different visual cues present in numerical stimuli [?, ?]. Gebuis and Reynvoet (2014) found different neural responses under conditions where visual and numerical information were congruent versus incongruent, supporting the view that approximate number processing relies on visual cues.

4.3. The Role of Visual Form Perception in Their Relationship

Researchers have also directly confirmed that rapid visual form perception constitutes a cognitive factor shared by the approximate number system and arithmetical fluency. Using rapid figure matching tasks to assess visual form perception, studies have revealed its role in both the approximate number system and arithmetical fluency (e.g., [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]). In this task, each trial presents a target figure on the left and three candidate figures on the right for 400 milliseconds, requiring participants to judge quickly and accurately whether the target figure appears among the candidates (see Figure 1 [Figure 1: see original paper]).

Zhou et al. (2015) studied 424 third- through fifth-grade elementary students and found that visual form perception, measured by the rapid figure matching task, explained the relationship between ANS acuity and arithmetical fluency after controlling for nonverbal matrix reasoning, choice reaction time, visual tracking ability, working memory, and mental rotation. Subsequent studies have confirmed the role of visual form perception in the approximate number system and arithmetical fluency [?, ?, ?, ?, ?, ?]. For example, third-grade students' visual form perception predicted their calculation performance three years later [?, ?]. Moreover, this visual form perception processing ability serves as a cognitive foundation for both reading comprehension and calculation fluency [?, ?, ?]. Additionally, research has found that only visual processing abilities involving substantial shape perception (rapid visual perception, visual short-term memory) can explain the relationship between the approximate number system and arithmetical fluency, whereas spatial processing abilities involving less shape perception (three-dimensional mental rotation, spatial short-term memory) cannot [?, ?, ?, ?].

Research on dyscalculia has also directly supported the role of visual form perception in the relationship between the approximate number system and arithmetical fluency [?, ?, ?, ?]. Studies have found that children with reading disabilities and dyscalculia lag behind typically developing children in visual form perception ability, and their deficits in visual form perception lead to poorer ANS acuity and lower performance in calculation and reading [?, ?]. Training ANS acuity improved visual form perception acuity in children with dyscalculia, which in turn enhanced their arithmetical fluency [?, ?].

An ERP study has provided new evidence for the visual form perception hypothesis. Li et al. (2020) found that occipital regions responsible for visual form perception ability may support the link between ANS acuity and arithmetical fluency. This study revealed that the N1 component at occipital electrode sites correlated significantly with reaction times on simple addition/subtraction, dot comparison, and rapid figure matching tasks. Furthermore, during simple addition/subtraction processing, the N1 component at occipital sites correlated with reaction times on rapid figure matching and dot comparison tasks.

5. Summary and Outlook

In summary, this paper has focused on the recently proposed "visual form perception hypothesis," examining the correlation between the approximate number system and arithmetical fluency and summarizing evidence for the explanatory role of rapid visual form perception in their relationship as a shared cognitive factor. The visual form perception hypothesis diverges from traditional domain-specific hypotheses by suggesting that the approximate number system may influence calculation processes at the perceptual level rather than through domain-specific numerical processing. This implies that mathematics educators should place greater emphasis on developing students' basic cognitive abilities and strengthening their capacity for shape representation and processing of mathematical symbols.

Nevertheless, research on the visual form perception hypothesis faces several limitations. First, methodological approaches have been relatively singular, consisting primarily of behavioral studies with only one event-related potential study examining the hypothesis from a temporal perspective; more precise brain region localization studies are lacking. Second, previous studies validating the visual form perception hypothesis have relied predominantly on the rapid figure matching task to measure visual form perception, making it impossible to rule out task-specific effects on results. Third, previous visual form processing tasks may have confounded other general cognitive factors such as inhibitory control and visual attention, leaving it unclear whether rapid visual form perception alone plays a role or whether it operates in conjunction with inhibitory control and visual attention. Fourth, direct evidence testing the visual form perception hypothesis has focused exclusively on the correlation between the approximate number system and arithmetical fluency, without examining other important mathematical abilities such as counting, conceptual understanding, rule learning, and problem-solving. Finally and most importantly, current research has been limited to exploring relationships between visual form perception and the approximate number system and arithmetical fluency individually, while the processing mechanisms through which visual form perception operates in their relationship remain unclear, lacking both theoretical framework support and direct empirical testing.

Future research should therefore integrate multiple approaches to investigate the cognitive and neural mechanisms underlying the role of visual form perception in the approximate number system and arithmetical fluency. Specifically:

First, future studies should combine cognitive behavioral testing with neuroimaging techniques to explore the role of shape perception speed in different mathematical processing tasks from multiple angles. Based on the visual form perception hypothesis, the approximate number system is closely related to arithmetical fluency, and rapid shape perception may explain why visual form perception plays a role in their relationship. Indeed, the visual perception task paradigm (rapid figure matching) is also a classic measure of perceptual speed [?, ?]. According to the Cattell-Horn-Carroll (CHC) model of intelligence, perceptual speed constitutes a fundamental and important component of intelligence [?, ?] and plays a significant role in calculation [?, ?, ?, ?]. Future research could employ correlational designs and experimental manipulations using mediation analysis and variable control (e.g., processing speed, shape processing complexity) to reveal the cognitive mechanisms through which visual form perception operates in the approximate number system and arithmetical fluency. Previous research has found that calculation and mathematical rule processing rely on different brain regions, with mathematical rules depending more on connections between frontal and temporal lobes [?, ?], whereas calculation processing involves more extraction of mathematical symbols (Arabic digits) and relies more on parieto-occipital connections [?, ?, ?, ?]. Based on these findings, future research could explore at the neural level the brain mechanisms through which visual form perception operates in the approximate number system and different symbolic mathematical abilities.

Second, future research must consider the influence of other general cognitive factors such as inhibitory control and visual attention on this relationship, investigating which cognitive components truly play a role. Previous research has revealed the importance of inhibitory control in both calculation [?, ?, ?, ?] and dot comparison [?, ?, ?, ?]. Beyond inhibitory control, visual attention may also influence the role of visual form perception in this relationship, as attentional capacity is required for all three tasks (rapid figure matching, dot comparison, and calculation). Current research cannot determine whether rapid visual form perception alone plays a role or whether it operates together with inhibitory control or visual attention. Therefore, future studies must systematically disentangle the contributions of different cognitive factors to the visual form perception hypothesis.

Third, future research should investigate the relationship between visual form perception and the approximate number system and arithmetical fluency across different cultural contexts, as well as the processing mechanisms underlying its role. Unlike Western alphabetic languages, Chinese characters involve combinations of components and strokes in logographic script that depend more heavily on form perception; Chinese character form and visual complexity affect character recognition [?, ?, ?, ?, ?]. Alphabetic languages typically involve only phonological processing of words and letters. If shape perception is crucial for calculation processing, might arithmetical fluency differ among learners influenced by culturally different scripts of varying shape complexity? Chinese mathematics textbooks extensively use Chinese characters to explain mathematical knowledge and concepts. Based on the visual form perception hypothesis, Chinese children' s experience learning Chinese characters may facilitate their mathematics learning, potentially causing them to rely more on visual form perception in calculation processing. Future research should therefore test and analyze the visual form perception hypothesis through cross-cultural comparisons of arithmetical fluency between Chinese and Western children.

Fourth, the ultimate goal of psychological research is to better serve society and benefit humanity. Future research must test the visual form perception hypothesis in practical applications, establishing causal relationships to demonstrate the effectiveness of visual form perception in mathematical processing, developing effective intervention studies for children with dyscalculia, and promoting mathematical processing abilities in typical populations through visual form perception training. In mathematics education contexts, how to cultivate students' shape representation abilities and mathematical learning capacity based on the role of visual form perception processing warrants deeper consideration and investigation.

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