

Two-dimensional hydrodynamic robust numerical model of soil erosion based on slopes and river basins Postprint

Authors: KANG Yongde, HUANG Miansong, HOU Jingming, TONG Yu, PAN Zhanpeng, HUANG Miansong, Jingming Hou

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Abstract

Erosion is an important issue in soil science and is related to many environmental problems, such as soil erosion and sediment transport. Establishing a simulation model suitable for soil erosion prediction is of great significance not only to accurately predict the process of soil separation by runoff, but also improve the physical model of soil erosion. In this study, we develop a graphic processing unit (GPU)-based numerical model that combines two-dimensional (2D) hydrodynamic and Green-Ampt (G-A) infiltration modelling to simulate soil erosion. A Godunov-type scheme on a uniform and structured square grid is then generated to solve the relevant shallow water equations (SWEs). The highlight of this study is the use of GPU-based acceleration technology to enable numerical models to simulate slope and watershed erosion in an efficient and high-resolution manner. The results show that the hydrodynamic model performs well in simulating soil erosion process. Soil erosion is studied by conducting calculation verification at the slope and basin scales. The first case involves simulating soil erosion process of a slope surface under indoor artificial rainfall conditions from 0 to 1000 s, and there is a good agreement between the simulated values and the measured values for the runoff velocity. The second case is a river basin experiment (Coquet River Basin) that involves watershed erosion. Simulations of the erosion depth change and erosion cumulative amount of the basin during a period of 1–40 h show an elevation difference of erosion at 0.5–3.0 m, especially during the period of 20–30 h. Nine cross sections in the basin are selected for simulation and the results reveal that the depth of erosion change value ranges from -0.86 to -2.79 m and the depth of deposition change value varies from 0.38 to 1.02 m. The findings indicate that the developed GPU-based hydrogeomorphological model can reproduce soil erosion processes. These results are valuable for rainfall runoff and soil erosion predictions on rilled hillslopes and river basins.

Full Text

Preamble

Two-Dimensional Hydrodynamic Robust Numerical Model of Soil Erosion Based on Slopes and River Basins

KANG Yongde¹, HUANG Miansong², *HOU Jingming*¹, TONG Yu¹, PAN Zhan-peng¹

¹ State Key Laboratory of Eco-hydraulics in Northwest Arid Region of China, School of Water Resources and Hydroelectric Engineering, Xi'an University of Technology, Xi'an 710048, China

² Beijing Capital Eco-Environment Protection Group Co. Ltd., Beijing 100028, China

Abstract: Soil erosion represents a critical challenge in soil science with far-reaching environmental implications, including sediment transport and land degradation. Developing robust simulation models for soil erosion prediction is essential not only for accurately forecasting runoff-driven soil detachment but also for advancing physically-based erosion theory. This study presents a GPU-accelerated numerical model that integrates two-dimensional (2D) hydrodynamic modeling with Green-Ampt (G-A) infiltration theory to simulate soil erosion processes. A Godunov-type scheme on a uniform structured square grid is employed to solve the shallow water equations (SWEs). The key innovation lies in leveraging GPU-based acceleration technology to enable efficient, high-resolution simulation of slope and watershed-scale erosion. Model validation demonstrates strong performance in reproducing soil erosion dynamics across multiple scales. The first validation case examines slope surface erosion under artificial rainfall conditions from 0 to 1000 s, showing excellent agreement between simulated and measured runoff velocities. The second case involves watershed-scale erosion in the Coquet River Basin, where simulations of erosion depth and cumulative erosion over 1–40 h reveal elevation differences of 0.5–3.0 m, particularly during the 20–30 h period. Analysis of nine cross-sections shows erosion depth changes ranging from –0.86 to –2.79 m and deposition depth changes from 0.38 to 1.02 m. These results demonstrate that the developed GPU-based hydrogeomorphological model can effectively reproduce soil erosion processes, providing valuable insights for rainfall-runoff and soil erosion predictions on rilled hillslopes and in river basins.

Keywords: soil erosion; two-dimensional modeling; rainfall runoff; Green-Ampt model; gully erosion

1 Introduction

Soil erosion constitutes a fundamental concern for soil and water conservation, watershed management, and non-point source pollution control, with direct linkages to soil degradation, ecological restoration, dam construction, flood forecast-

ing, and water resources management. Soils must be actively managed as they respond dynamically to rainfall variations. Storm events represent the primary driver of soil erosion, with most erosion occurring during intense rainfall. These factors significantly influence both land use patterns and erosion rates. Furthermore, the potential impacts of climate change on rainfall frequency and magnitude, coupled with land use transformations, warrant serious attention. Previous studies have investigated relationships between rainfall characteristics and gully erosion dynamics (Qin et al., 2018a, b), demonstrating that individual heavy rainfall events can trigger severe soil erosion (Wang et al., 2016; Bryndal et al., 2017). Researchers have extensively examined how rainfall event characteristics and intensity affect erosion processes (Xu et al., 2013; Wang et al., 2016; Cerdà et al., 2017; dos Santos et al., 2017; Chen et al., 2018; Panagos and Katsoyiannis, 2019), with severe erosion typically resulting from multiple intense storms. The Loess Plateau in northwestern China's arid region exhibits particularly prominent erosion processes, and scholars have quantified erosion amounts, especially during rill erosion stages, which proves crucial for understanding regional erosion patterns (e.g., Shen et al., 2016).

Rainfall plays a pivotal role in soil erosion and sediment transport, as eroded particles form gullies that impede land use (Woodward, 1999; Borrelli et al., 2017). Moreover, increased flow energy converts to kinetic energy for particle detachment and transport, thereby accelerating erosion rates (Wirtz et al., 2010). Over the past decade, numerous laboratory and field experiments have investigated soil erosion processes (González-Hidalgo et al., 2009; Taye et al., 2013; Wu et al., 2017, 2019; Qin et al., 2018a, b; Wang et al., 2018; Zhao et al., 2019; An et al., 2020). Empirical models such as the Universal Soil Loss Equation (USLE) (Wischmeier and Smith, 1978) and Revised Universal Soil Loss Equation (RUSLE) (Renard et al., 1997) remain widely used for field-scale research. However, these empirical models rely on measured datasets and primarily consider slope length and gradient while neglecting spatial characteristics between rills and slope morphology. To address gully erosion specifically, researchers have developed physically-based models of rainfall erosion processes (An and Liu, 2009; Qin et al., 2018a, b). The Water Erosion Prediction Project (WEPP) model, built on physical processes, defines erosion as the sum of material transport between rills. Similar physically-based models include the Kinematic Runoff and Erosion Model and the European Soil Erosion Model (Smith et al., 1995; Morgan et al., 1998). Additional factors such as discharge and slope affect gully evolution (Bingner et al., 2016), with studies showing that increased slope and inter-gully flow rates progressively enlarge rill width and widening rates (Wells et al., 2013). The collapse processes and physical mechanisms of slope sidewalls under erosion remain complex and widely studied (Chaplot et al., 2011; Chen et al., 2013; Bingner et al., 2016). Despite extensive research, predicting gully erosion damage across regions remains challenging with existing models, requiring localized parameterization. While numerical simulation techniques can represent these processes, quantitative simulation of erosion depth still presents difficulties (Abderrezzak et al., 2013; Wells et al., 2013; Bingner

et al., 2016), necessitating further field experiments and modeling to enhance understanding of gully retreat (Chaplot et al., 2011). Improved comprehension of gully expansion is essential for developing water erosion prediction modules (Wells et al., 2013; Bingner et al., 2016; de Lima et al., 2019) to protect vulnerable gullies.

Several researchers have developed two-dimensional runoff and soil erosion models capable of simulating rill erosion spatial characteristics and determining rainfall-runoff processes (Govindaraju et al., 1992; Gottardi and Venutelli, 1993; Howes et al., 2006; Chen et al., 2013; Aksoy et al., 2016). However, these models often lack physical modules to characterize soil erosion processes. Consequently, one-dimensional (1D) and 2D coupled models were developed for hill-slope rainfall-runoff and erosion simulation (Tayfur, 2007; An and Liu, 2009), idealizing slopes as combinations of 1D aligned rills separated by 2D interrill areas. While these 2D slope erosion models represent advances, they cannot distinguish erosion between different rill configurations (Nord and Esteves, 2005). Despite progress over the past decade, significant challenges remain, including the lack of distributed erosion information that reflects both slope and channel erosion processes. Although modeling approaches may incorporate quantitative erosion prediction, the interaction mechanisms between rill spatial structure and slope morphology on rainfall-runoff-erosion processes remain unclear.

Distributed models still lack research on rill spatial characteristics and nonplanar slopes at landscape scales. Studies show that both rainfall patterns and concave hillslopes dramatically affect erosion processes (Qin et al., 2018a, b; Tian et al., 2017; Batista et al., 2019). Hillslopes can generate depressions that provide channels for runoff and sediment sources (Shen et al., 2016; Tian et al., 2017), with rill direction angles, local density, and width-depth spatial characteristics proving crucial for slope runoff and erosion (Appels et al., 2011; Di Stefano et al., 2017, 2018; Wu et al., 2019). Slope shape significantly affects flow and erosion patterns (Sweeney et al., 2015), with different natural slope forms creating varying runoff and erosion characteristics that substantially increase local rill shear stress and erosion (Gordon et al., 2012; Batista et al., 2019). Due to erosion process complexity (Stroosnijder, 2005) and unevenly distributed rainfall, accurate runoff forecasting requires sufficient representativeness (Beven, 2012). Failure to account for complex soil surface microtopography changes may introduce large errors in erosion models (Wu et al., 2019). However, laboratory experiments typically feature short rainfall durations, small flume scales, and insufficient time for distinct topographic features like rills to develop (Aksoy et al., 2016). Currently, no distributed modeling method adequately represents complex gully networks and slope shapes while providing soil erosion prediction errors. No consensus has emerged regarding the spatial structure of rills and slopes concerning rainfall, runoff, and erosion (Wu et al., 2017, 2019), hindering development of targeted soil conservation strategies that account for spatial variability.

This study proposes a novel soil erosion model coupling 2D hydrodynamics

with the Green-Ampt (G-A) infiltration model to improve understanding of rainfall-erosion interactions. Theoretically, slope flow represents a complex three-dimensional problem, making traditional shallow water equations (SWE) inappropriate for steep, uneven terrain. The model therefore incorporates GPU-based acceleration technology to enhance computational efficiency. Through simulations of soil erosion, transport, and deposition on slopes and small watersheds, the spatial distribution characteristics of rainfall-induced erosion can be evaluated.

2 Modelling Approach and Development

Based on a spatial-temporal dynamic model coupling physical and empirical laws, we designed a framework comprising soil infiltration, erosion, and sediment transport modules to facilitate solution of the 2D sediment transport equation.

Overland flow is calculated using 2D SWEs, consisting of one continuity equation and two momentum equations for the x and y coordinate directions. Rainfall energy drives sediment detachment, while runoff energy transports eroded material. Raindrops enhance solute transport from soil to surface runoff, and detached particles may be transported through raindrop-water interactions. When suspended sediment is no longer supported by flow, deposition occurs due to reduced transport capacity.

Assuming an initially dry soil layer, rainfall must be incorporated into the model, with flow depth, velocity, and sediment concentration initialized to zero at computational points. Soil surfaces may dry again after rainfall events, creating numerical complexity that requires specialized treatment. One challenge involves very shallow water flows requiring special handling (Hou et al., 2013a, b). The corrected program in subsequent time steps employs a wetting/drying front and implements a flexible negative water depth method as needed (Hou et al., 2013a, b). The numerical domain is represented as a uniform square grid with longitudinal and transverse slopes in x and y directions, featuring closed boundaries at the upper end and open boundaries at the lower end. Based on this rectangular grid, we coupled a sediment transport module with a hydrodynamic numerical model to establish a 2D water-sediment coupling model for slope erosion under moving bed conditions using the finite volume method, simulating unsteady water flow and cohesive sediment transport during slope erosion.

This fully coupled water-sediment model accounts for sediment density and topographic effects on flow movement. The central upwind scheme solves interface fluxes with linear reconstruction of interface variables to ensure second-order spatial accuracy. The model employs an unbalanced sediment transport formula to calculate sediment transport processes while guaranteeing nonnegative water depth throughout calculations, ensuring strong stability and robustness. The erosion rate is considered positive during detachment and negative during deposition. Soil detachment rate increases with flow shear stress when the acting shear stress exceeds critical shear stress, enabling rill detachment prediction.

2.1 Hydrology Model of Soil Infiltration

The Green-Ampt (G-A) infiltration model, derived from capillary theory (Green and Ampt, 1911; Chu, 1978), offers a simplified formulation that has been further improved for specific applications. The governing equation is as follows:

where f_p represents the instantaneous infiltration rate (cm/s); R denotes rainfall intensity (cm/min); K_s is saturated vertical hydraulic conductivity (cm/min); i and s are initial and saturated water content (cm^3/cm^3), respectively; S_f is wetting front suction (cm); I_p is cumulative infiltration after ponding (cm); t is time (min); and t_p is time to ponding (min).

The ponding time function t_p is defined as:

2.2 Soil Erosion Sediment Transport Submodule

As an effective approximation of Saint-Venant's equations (Hou et al., 2020), the governing equation employs 2D SWEs (Equation 3) with vertical averaging, assuming hydrostatic pressure distribution from Reynolds averaging while neglecting viscosity and turbulent pressure. The equation is expressed in conservative form (Chen et al., 2013):

where:

In these equations, q represents the flow variable vector containing h ; F and G are flux vectors of conserved variables in x and y directions; S is the source vector; h is flow depth (m); q_x and q_y are unit-width discharges in Cartesian directions (m^2/s); c is volumetric concentration in sheet flow (dimensionless); z_b is bed elevation (m); β denotes velocity difference between sediment and fluid phases (Greimann et al., 2008), equal to 1 (Guan et al., 2014); v is average flow velocity in y direction (m/s); C is volumetric concentration in flow depth (dimensionless); P is sediment material porosity (dimensionless); S_b represents bed slope in source terms; S_f represents friction slope in source terms; i is rainfall intensity (m/s); f_p is infiltration rate through soil surface (cm/s); γ is a coefficient; α_f is the angle between S and x ($^\circ$); g is gravitational acceleration (m/s^2); D is sediment deposition rate calculated as $D = \omega_s C_a$, where ω_s is sediment particle settling velocity (m/s) and C_a is near-bed equilibrium concentration at reference level; and E is sediment entrainment rate calculated as $E = \omega_s C_{ae}$, where C_{ae} is equilibrium near-bed sediment concentration at reference height (Smith and Mclean, 1977).

2.3 Numerical Scheme

This study employs the finite volume method in Godunov format, using the Harten-Lax-van Leer contact (HLLC) approximate Riemann solver to obtain coupled solutions for water flow and solute transport fluxes. All test cases utilize two boundary condition types: open and closed boundaries, controlled through flux calculations at boundaries (Hou et al., 2013a, b). Solute advection simulation adopts a second-order explicit total variation diminishing (TVD)

scheme with a limiter to control numerical errors, reducing numerical dissipation and oscillation caused by advection terms (Horton, 1993; Hu and Cao, 2009; Liang and Marche, 2009; Hou et al., 2013a, b, 2015).

Computational efficiency is quantitatively evaluated using runtime on specific hardware: an Intel i7-6700 CPU (3.40 GHz) with Windows 10 operating system. The GPU-accelerated parallel computing process is programmed using C++ and CUDA (Computer Unified Device Architecture). In CUDA programming, the CPU serves as host for logical and serial operations while the GPU acts as coprocessor for parallel computing tasks. Data are initially read into CPU memory, where grid information, boundary conditions, and calculation parameters are initialized for each grid. Data are then allocated to corresponding GPU device space and copied to GPU memory, with results synchronized back to CPU after computation.

3 Model Verification

Model evaluation was conducted using data from the State Key Laboratory of Soil Erosion and Dryland Farming on the Loess Plateau, Northwest A&F University, Shaanxi Province, China (Liu et al., 2006). Experimental parameters included a flume measuring 3.2 m length $\times$ 1.0 m width $\times$ 0.3 m depth with a 20° slope. Artificial rainfall was generated using a side-spray automatic simulator with nozzle height approximately 16.0 m, rainfall uniformity >90%, and controllable intensity of 150–200 mm/h. The experiment employed artificial filling with local loess (25 cm thickness, median particle size $\sim$ 0.02 mm, bulk density 1.33 g/cm³, initial water content 0.2206 cm³/cm³, saturated water content 0.5027 cm³/cm³). Soil effective hydraulic conductivity was 1.67×10^{-6} m/s with matric suction of 0.15 m. Runoff samples were collected at intervals using 300 mL beakers, with sampling times recorded. Instantaneous runoff flow and sediment concentration were determined by the drying method to approximate cumulative runoff volume and erosion amount (Liu et al., 2006).

[Figure 1: see original paper] illustrates relationships between time and runoff velocity, flow discharge, and cumulative erosion. During the first 10 minutes, simulated average flow velocity shows the same sharp increasing trend as measured values, gradually stabilizing after 10 minutes at approximately 0.23 m/s—slightly higher than measured values until 50 minutes, when simulated and measured values converge. Flow rate changes mirror this trend, showing sharp initial increases. Cumulative sediment yield shows consistent trends between simulated and measured values, with both velocity and discharge increasing over time followed by increasing sediment yield, consistent with erosion physics. Overall, simulated flow velocity, discharge, and cumulative erosion closely match measured values, demonstrating good agreement.

The model effectively reproduces rainfall-runoff process changes and erosion dynamics, enabling erosion modeling efforts to be extended to slopes using this method. For quantitative analysis, three performance indicators evaluate model

effectiveness: root mean square error (RMSE), coefficient of determination (R^2), and mean relative error (MRE), defined as:

where M is measured value; S is simulated value; $\bar{M}$ is mean measured value; $\bar{S}$ is mean simulated value; and n is sample number. R^2 approaching 1 indicates proximity to exact solution, while RMSE approaching 0 indicates perfect fit.

As shown in Table 1, the proposed model reliably simulates slope soil erosion and produces good slope morphology predictions. Comparative analysis confirms that RMSE, R^2 , and MRE reflect simulation accuracy, demonstrating strong advantages of the developed prediction model.

4.1 Numerical Simulation Example of an Indoor Slope

The first test case involves slope erosion under artificial rainfall conditions (Wang et al., 2018) conducted at the Rain Test Hall of the State Key Laboratory of Ecological Water Conservancy Engineering in Northwest Arid Areas, Xi' an University of Technology, Shaanxi Province, China. The experiment used an adjustable steel channel (15° slope, $6.0\text{ m} \times 1.5\text{ m} \times 0.6\text{ m}$). Seepage holes at the bottom simulated natural soil permeability. Soil samples were local loess from Ansai District ($36^\circ 45'\text{ N}$, $109^\circ 11'\text{ E}$), air-dried before use. Particle components are shown in Table 2.

The procedure involved covering loess with 20 cm of natural sand, controlling average dry bulk density at 1.09 g/cm^3 . Clay loess quantified effects of up-slope runoff and sediment on downslope rill erosion under varying sediment concentrations and rainfall intensities. A downward sprinkler system simulated continuous rainfall for 60 minutes at 50 mm/h intensity, with sediment samples collected at 5-minute intervals.

Under rainfall and runoff action, slope rill erosion evolves through three processes: (1) runoff formation and rill development along flow paths; (2) soil particle transport and collapse of uneroded rill sidewalls under shear forces; and (3) sudden, unpredictable toe erosion from rill sidewalls due to gravity. While narrow grooves form, cracks develop on side surfaces from shear forces, with small collapses occurring when gravity exceeds soil cohesion and friction, creating rills. Martinez-Casasnovas et al. (2004) and Chen et al. (2013) found that continuous rainfall and flow repetition destroy overhanging layers through rapid concentrated flow, accelerating collapse and gully erosion.

As shown in Figure 2a [Figure 2: see original paper], at $t = 0\text{ s}$, runoff initiates and gradually forms as rainfall continues, with rills developing on the slope. During $t = 10\text{--}50\text{ s}$, runoff velocity increases rapidly while energy consumed by erosion decreases at the slope bottom, accompanied by rill formation. During $t = 100\text{--}1000\text{ s}$, rill sidewalls widen and scour, forming erosion channels that transport sand particles and create sequential small rills, transforming the entire slope shape. These three processes interact and influence each other over time. Flow depth changes between rills are relatively small compared to erosion effects.

As shown in Figures 2c-d, spatial changes become detectable during $t = 100$ - 1000 s, primarily because rill erosion intensity depends on rainfall intensity and duration. The middle slope section experiences the strongest erosion due to high erosive capacity at this location. Despite uniform slope length, erosion intensity varies significantly among different rills, with steeper, more continuous slopes exhibiting higher runoff velocities and erosion capacities.

In natural environments, slope lengths range from several meters to hundreds of meters with varying gradients. Precipitation distribution is highly uneven seasonally, with rainstorm intensities varying considerably. Most storms have short durations and uncertain locations (Wang and Jiao, 1996; Jiang, 1997). These topographic and rainfall extremes create different slope erosion characteristics. Rainfall is often random and non-uniform, with intensity variations impacting erosion differently. Studies have examined rainfall pattern effects on slope erosion (Cerdà et al., 2017). Under constant rainfall intensity, permeability decreases and stabilizes over time while excess rainfall rate increases, particularly when rainfall peaks later, enhancing runoff erosion capacity at peak times (Liu et al., 2006).

Erosion between slope rills does not change continuously because each grid element connects to others only in x or y directions. Slope gullies undergo major pattern changes and substantial shifts during erosion, transport, and deposition, altering sediment load and water discharge. The y-direction gradient on each grid is steeper than the x-direction gradient, making soil more susceptible to vertical erosion (y-direction).

As shown in Figure 3 [Figure 3: see original paper], sediment yield and rill erosion width increase over time with similar trends. At 800 s, rill erosion width begins exceeding sediment yield rate, possibly indicating changed sand production sources on the slope. Regression analysis reveals an extremely linear positive relationship:

where Y is cumulative sediment yield (g); X is rill erosion width (mm); and coefficients a and b are 0.8743 and 2.4012, respectively. This study constructed a prototype rill along the flow direction at mid-slope, assuming a 2.4012 mm width to guide rill structure development, enabling modeling for different slope lengths and gradients based on actual conditions.

As shown in Figure 4 [Figure 4: see original paper], runoff velocity decreases over time, with four sharp decreases at 0-100, 100-140, and 320-510 s. Two rapid velocity decreases correspond to rapid increases in narrow groove width. However, at experiment beginning and end, concentrated flow rate and rill width changes are inconsistent. During the first 100 s, runoff velocity decreased 40% while rill width remained unchanged, primarily because concentrated flow shear force at the ditch toe increased runoff width and reduced depth, decreasing velocity as wash-out depth increased. During 320-360 s, narrow groove width increased 17% while concentrated flow rate decreased only 9.5%, with more ditch wall expansion occurring while toe wash-out erosion rate remained essentially

constant.

Regression analysis (Table 3) shows significant negative correlation between runoff velocity and rill erosion width, expressed by an exponential function:

where Y is runoff velocity (cm/s), X is narrow groove width (cm), with $R^2 = 0.9591$ and $p < 0.005$.

The GAST (GPU Accelerated Surface Water Flow and Transport Model) applicability and accuracy are evaluated using R^2 and ENS indicators. R^2 is calculated through linear regression between measured and simulated values, representing simulation consistency in trend reproduction. ENS represents similarity between simulated and measured values at the 1:1 level (e.g., ENS = 0.8 indicates 80% similarity). ENS is calculated as:

where M is measured value, O is simulated value, $\bar{M}$ is mean measured value, $\bar{O}$ is mean simulated value, and n is observation count.

As shown in Figure 5 [Figure 5: see original paper], the GPU-accelerated, high-efficiency, high-resolution numerical model coupling hydrodynamics and morphology proves reliable with good simulation results. Calculations yield $R^2 = 0.97$ and ENS = 0.98, demonstrating strong, broad applicability and excellent capability to simulate inter-ditch erosion spatial distribution characteristics.

4.2 Numerical Simulation Example of the Erosion Process in a Small Watershed

The second test case examines watershed erosion in the Coquet River Basin (Hou et al., 2020), selected for its historic channel instability and morphology changes under rising river water and rainfall conditions. Figure 6 [Figure 6: see original paper] shows simulated erosion depth changes and cumulative erosion over 1–40 h, with elevation differences of 0.5–3.0 m. Average erosion depth increases over time, particularly during $t = 20$ –30 h (Figs. 6e–f).

For significant erosion sites and rainfall events, key simulations focus on model capabilities and erosion patterns. Hourly output data (not shown) represent spatial erosion location changes across the basin, illustrating watershed erosion morphological changes including sediment erosion and deposition in important river sections, providing insight into relationships between morphological changes and erosive energy (Fig. 7 [Figure 7: see original paper]). Energy appears concentrated in the riverbed due to erosion and sediment exchange (Ham, 2005; Fuller et al., 2003). Besides substantial sediment migration causing channel deposition, channels migrate during relaxation periods through erosion and redeposit in lower reaches. Extensive siltation in river sections also results from increased sediment inflow at higher flow rates combined with riverbank erosion input, with low river section slope being a key factor affecting flood response (Danks, 1998). Low slope decreases sediment transport capacity and flow power, while high bank retreat rates relate to excessive flow energy concentration on

banks, causing greater lateral erosion, particularly on bend outer sides (Newson, 1980; Hickin, 1984).

Figure 8 [Figure 8: see original paper] compares measured old riverbed and simulated new eroded riverbed elevations at nine Coquet River Basin cross-sections. Analyzing each section improves understanding of topographic changes under different erosion intensities and key deposition processes. More stable subranges resist morphological changes and sediment flux, having not undergone fundamental landform changes in response to high-intensity erosion. In terms of erosion sediment mobility, strong responses occur to either high-intensity erosion or frequent low-amplitude events.

At 20.00 m distance, the new eroded riverbed is approximately 0.45 m lower than the old riverbed due to strong watershed erosion and sediment “over-transport,” changing upper basin structure and transferring large sediment volumes to middle and lower reaches. Across Figures 8b-i, old and new riverbed erosion changes range 0.20–1.00 m. Continuous erosion and increased lateral erosion width cause further migration at Cross-section 9 (Fig. 8i), with bank retreat potentially driven by frequent left-bank failures through cantilever failure—progressive removal of riparian material by flow erosion of non-cohesive sand.

Quantitative analysis of riverbed erosion and deposition at nine cross-sections appears in Figure 9 [Figure 9: see original paper]. In the most severely eroded area, erosion depth reaches approximately 13.00 m. Generally, erosion exceeds deposition across sections, with erosion depth changes ranging from -0.86 to -2.79 m and deposition depth changes from 0.38 to 1.02 m. Most areas experience approximately 1.00 m erosion depth, primarily in flood-prone regions. Comparing erosion and deposition through sediment accumulation calculations reveals net erosion dominating the basin, increasing basin width. This process constitutes watershed erosion.

These changes result from erosion and deposition altering river channel course, bed elevation, and gradient through sediment accumulation. Besides lateral channel development, upstream river bends erode, increasing specific riverbed section depths: 2.40 m at Cross-section 3, 1.29 m at Cross-section 4, and 0.63 m at Cross-section 6. These factors clearly affect the active channel by altering width, gradient, and causing bank erosion. Eroded channels can form new channels during reciprocating erosion processes, changing width and slope to enhance erosion.

Erosion and deposition characterize the basin, determined by calculating cross-section balance changes. Net sediment gains and losses indicate excess erosion and downstream material export, which can occur even in stable rivers. This process links to sediment distribution, with increased sediment flux associated with higher rainfall magnitude and erosional bank sediment input, leading to watershed geometry changes. Estimating riverbed and bank material erosion along basin channels indicates dominant processes influencing morphologic change and meandering basin development. If eroded sediment is not removed

or transported, cross-section shape changes can be expected related to active basin width.

To demonstrate GPU code computational efficiency, a CPU model (four-core Intel Xeon E5-2609) simulated the same test case. Runtime comparisons across different grid resolutions show that GPU code performance improves as grids become finer. Therefore, GPU acceleration enables large-scale erosion simulations and plays a significant role in evaluating water and soil loss processes.

5 Conclusions

This study developed a novel soil erosion model coupling 2D hydrodynamics with the G-A rainfall infiltration model, applying GPU acceleration to increase computational speed. Model testing shows good consistency between simulated and measured values, with slope erosion patterns and spatial distribution characteristics at different times highly consistent with final gully morphology. The model enables accurate slope erosion predictions. A linear function can predict sediment transport amount and rill erosion width in slope erosion, with erosion rate depending primarily on soil resistance to erosion. Two turning points exist in erosion rate and rill width relationships: when velocity sharply decreases, erosion width gradually increases until reaching equilibrium, after which width decreases as velocity increases. Rill width depends mainly on groove opening development from small to large. Shearing force effects, though small, cannot be ignored. In large basins, bank incisions collapse progressively, eroding and washing away large riverbank areas.

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References

- Abderrezzak K E, Die Moran A, Mosselman E, et al. 2013. A physical, movable-bed model for non-uniform sediment transport, fluvial erosion and bank failure in rivers. *Journal of Hydro-environment Research*, 8(2): 95-114.
- Aksoy H, Gedikli A, Unal N E, et al. 2016. Rainfall runoff model considering microtopography simulated in a laboratory erosion flume. *Water Resources Management*, 30(15): 5609-5624.
- An J, Zhang Y Q, Wang Y Y. 2020. Rainstorm pattern effects on the size distribution of soil aggregate in eroded sediment within contour ridge systems. *Journal of Soils and Sediments*, 20(4): 2192-2206.
- An Y, Liu Q Q. 2009. Two-dimensional hillslope scale soil erosion model. *Journal of Hydrologic Engineering*, 14(7): 690-697.

- Appels W M, Bogaart P W, van der Zee S E. 2011. Influence of spatial variations of microtopography and infiltration on surface runoff and field scale hydrological connectivity. *Advances in Water Resources*, 34(2): 303–313.
- Batista P V G, Davies J, Silva M L N, et al. 2019. On the evaluation of soil erosion models: Are we doing enough? *Earth-Science Reviews*, 197: 102898, doi: 10.1016/j.earscirev.2019.102898.
- Beven K J. 2012. *Rainfall-Runoff Modelling: The Primer* (2nd ed.). Chichester: Wiley-Blackwell, 3–15.
- Bingner R L, Wells R R, Momm H G, et al. 2016. Ephemeral gully channel width and erosion simulation technology. *Natural Hazards*, 80(3): 1949–1966.
- Borrelli P, Robinson D A, Fleischer L R, et al. 2017. An assessment of the global impact of the 21st century land use change on soil erosion. *Nature Communications*, 8: 1–13.
- Bryndal T, Franczak P, Krocak R, et al. 2017. The impact of extreme rainfall and flash floods on the flood risk management process and geomorphological changes in small Carpathian catchments: a case study of the Kasiniczanka river (Outer Carpathians, Poland). *Natural Hazards*, 88(1): 95–120.
- Cerdà A, Borja M E L, Úbeda X, et al. 2017. *Pinus halepensis* M. versus *Quercus ilex* subsp *Rotundifolia* L. runoff and soil erosion at pedon scale under natural rainfall in Eastern Spain three decades after a forest fire. *Forest Ecology and Management*, 400: 447–456.
- Chaplot V, Brown J, Dlamini P, et al. 2011. Rainfall simulation to identify the storm-scale mechanisms of gully bank retreat. *Agricultural Water Management*, 98(11): 1704–1710.
- Chen A Q, Zhang D, Peng H, et al. 2013. Experimental study on the development of collapse of overhanging layers of gully in Yuanmou Valley, China. *CATENA*, 109: 177–185.
- Chen H, Zhang X P, Abia M, et al. 2018. Effects of vegetation and rainfall types on surface runoff and soil erosion on steep slopes on the Loess Plateau, China. *CATENA*, 170: 141–149.
- Chen L, Sela S, Svoray T, et al. 2013. The role of soil-surface sealing, microtopography and vegetation patches in rainfall-runoff processes in semiarid areas. *Water Resources Research*, 49(9): 5585–5599.
- Chu S T. 1978. Infiltration during an unsteady rain. *Water Resources Research*, 14(3): 461–466.
- Danks M. 1998. Contemporary and historic river channel change at Swinhope bum, Weardale: a study of river response to flood events in an upland, gravel-bed stream. MSc Thesis. Durham: Durham University.

- de Lima J L M P, Santos L, Mujtaba B, et al. 2019. Laboratory assessment of the influence of rice straw mulch size on soil loss. *Advances in Geosciences*, 48: 11-18.
- Di Stefano C, Ferro V, Palmeri V, et al. 2017. Flow resistance equation for rills. *Hydrological Processes*, 31(15): 2793-2801.
- Di Stefano C, Ferro V, Palmeri V, et al. 2018. Testing slope effect on flow resistance equation for mobile bed rills. *Hydrological Processes*, 32(5): 664-671.
- dos Santos J C N, de Andrade E M, Medeiros P H A, et al. 2017. Effect of rainfall characteristics on runoff and water erosion for different land uses in a tropical semiarid region. *Water Resources Management*, 31(1): 173-185.
- Fuller I C, Large A R G, Milan D J. 2003. Quantifying channel development and sediment transfer following chute cutoff in a wandering gravel-bed river. *Geomorphology*, 54(3-4): 307-323.
- González-Hidalgo J C, de Luis M, Batalla R J. 2009. Effects of the largest daily events on total soil erosion by rainwater. An analysis of the USLE database. *Earth Surface Processes and Landforms*, 34(15): 2070-2077.
- Gordon L M, Bennett S J, Wells R R. 2012. Response of a soil-mantled experimental landscape to exogenic forcing. *Water Resources Research*, 48: W10514, doi: 10.1029/2012WR012283.
- Gottardi G, Venutelli M. 1993. A control-volume finite-element model for two-dimensional overland flow. *Advances in Water Resources*, 16(5): 277-284.
- Govindaraju R S, Kavvas M L, Tayfur G. 1992. A simplified model for two-dimensional overland flows. *Advances in Water Resources*, 15(2): 133-141.
- Green W H, Ampt G A. 1911. Studies on soil physics I. The flow of air and water through soils. *Journal of Agricultural Science*, 4: 1-24.
- Greimann B, Lai Y, Huang J C. 2008. Two-dimensional total sediment load model equations. *Journal of Hydraulic Engineering*, 134(8): 1142-1146.
- Guan M, Wright N G, Sleight P A. 2014. 2D process based morphodynamical model for flooding by non-cohesive dyke breach. *Journal of Hydraulic Engineering*, 140(7): 44-51.
- Ham D G. 2005. Morphodynamics and sediment transport in a wandering gravel-bed channel: Fraser River, British Columbia. PhD Dissertation. British Columbia: University of British Columbia.
- Hickin E J. 1984. Vegetation and river channel dynamics. *The Canadian Geographer Le Géographe Canadien*, 28(2): 111-126.
- Horton R E. 1933. The role of infiltration in the hydrological cycle. *Transactions, American, Geophysical Union*, 14(1): 446-460.

- Hou J M, Liang Q H, Simons F, et al. 2013a. A 2D well balanced shallow flow model for unstructured grids with novel slope source term treatment. *Advances in Water Resources*, 52: 107-131.
- Hou J M, Liang Q H, Simons F, et al. 2013b. A stable 2D unstructured shallow flow model for simulations of wetting and drying over rough terrains. *Computers & Fluids*, 82(17): 132-147.
- Hou J M, Liang Q H, Zhang H B, et al. 2015. An efficient unstructured MUSCL scheme for solving the 2D shallow water equations. *Environmental Modelling & Software*, 66: 131-152.
- Hou J M, Kang Y D, Hu C H, et al. 2020. A GPU-based numerical model coupling hydrodynamical and morphological processes. *International Journal of Sediment Research*, 35(4): 386-394.
- Howes D A, Abrahams A D, Pitman E B. 2006. One- and two-dimensional modelling of overland flow in semiarid shrubland, Jornada basin, New Mexico. *Hydrological Processes*, 20: 1027-1046.
- Hu P, Cao Z X. 2009. Fully coupled mathematical modeling of turbidity currents over erodible bed. *Advances in Water Resources*. 32(1): 1-15.
- Jiang D S. 1997. Soil and Water Loss in Loess Plateau and the Regulation Mode. Beijing: China Water & Power Press, 105-110. (in Chinese)
- Liang Q, Marche F. 2009. Numerical resolution of well-balanced shallow water equations with complex source terms. *Advances in Water Resources*, 32(6): 873-884.
- Liu Q, Xiang H, Singh V P. 2006. A simulation model for unified interrill erosion and rill erosion on hillslopes. *Hydrological Processes*, 20: 469-486.
- Martinez-Casasnovas J A, Ramos M C, Poesen J. 2004. Assessment of side-wall erosion in large gullies using multi-temporal DEMs and logistic regression analysis. *Geomorphology*, 58(1-4): 305-321.
- Morgan R P C, Quinton J N, Smith R E, et al. 1998. The European Soil Erosion Model (EUROSEM): a dynamic approach for predicting sediment transport from fields and small catchments. *Earth Surface Processes and Landforms*, 23(6): 527-544.
- Newson M. 1980. The geomorphological effectiveness of floods—a contribution stimulated by two recent events in mid-wales. *Earth Surface Processes and Landforms*, 5(1): 1-16.
- Nord G, Esteves M. 2005. PSEM-2D: A physically based model of erosion processes at the plot scale. *Water Resources Research*, 41: 1-14.
- Panagos P, Katsoyiannis A. 2019. Soil erosion modelling: The new challenges as the result of policy developments in Europe. *Environmental Research*, 172: 470-474.

- Qin C, Zheng F L, Xu X M, et al. 2018a. A laboratory study on rill network development and morphological characteristics on loessial hillslope. *Journal of Soils and Sediments*, 18(4): 1679-1690.
- Qin C, Zheng F L, Zhang X C, et al. 2018b. A simulation of rill bed incision processes in upland concentrated flows. *CATENA*, 165: 310-319.
- Renard K G, Foster R, Weesies G, et al. 1997. Predicting soil erosion by water: A guide to conservation planning with the revised universal soil loss equation (RUSLE). USDA Agriculture Handbook 703. Washington, D.C.: United States Department of Agriculture, 1-404.
- Shen H O, Zheng F L, Wen L L, et al. 2016. Impacts of rainfall intensity and slope gradient on rill erosion processes at loessial hillslope. *Soil and Tillage Research*, 155: 429-436.
- Smith J D, Mclean S R. 1977. Spatially averaged flow over a wavy surface. *Journal of Geophysical Research*, 82(12): 1735-1746.
- Smith R E, Goodrich D C, Woolhiser D A, et al. 1995. KINEROS: A Kinematic Runoff and Erosion Model. [2021-04-03]. <https://www.tucson.ars.ag.gov/kineros/>.
- Stroosnijder L. 2005. Measurement of erosion: Is it possible? *CATENA*, 64(2-3): 162-173.
- Sweeney K E, Roering J J, Ellis C. 2015. Experimental evidence for hillslope control of landscape scale. *Science*, 349(6243): 51-53.
- Taye G, Poesen J, Van Wesemael B, et al. 2013. Effects of land use, slope gradient, and soil and water conservation structures on runoff and soil loss in semiarid northern Ethiopia. *Physical Geography*, 34(3): 236-259.
- Tayfur G. 2007. Modelling sediment transport from bare rilled hillslopes by areally averaged transport equations. *CATENA*, 70(1): 25-38.
- Tian P, Xu X Y, Pan C Z, et al. 2017. Impacts of rainfall and inflow on rill formation and erosion processes on steep hillslopes. *Journal of Hydrology*, 548: 24-39.
- Wang T, Li P, Hou J M, et al. 2018. Response of the meltwater erosion to runoff energy consumption on loessal slopes. *Water*, 10(11): 1522, doi: 10.3390/w10111522.
- Wang W H, Jiao J Y. 1996. Rainfall and Erosion Sediment Yield in Loess Plateau and Sediment Transportation in the Yellow River Basin. Beijing: Science Press, 142-144. (in Chinese)
- Wang Z J, Jiao J Y, Rayburg S, et al. 2016. Soil erosion resistance of "Grain for Green" vegetation types under extreme rainfall conditions on the Loess Plateau, China. *CATENA*, 141: 109-116.
- Wells R R, Momm H G, Rigby J R, et al. 2013. An empirical investigation of gully widening rates in upland concentrated flows. *CATENA*, 101: 114-121.

Wirtz S, Seeger M, Ries J B. 2010. The rill experiment as a method to approach a quantification of rill erosion process activity. *Zeitschrift Fur Geomorphologie*, 54(1): 47-64.

Wischmeier W H, Smith D D. 1978. Predicting rainfall erosion losses: A guide to conservation planning. The USDA Agricultural Handbook No. 537. Washington, D.C.: United States Department of Agriculture.

Woodward D E. 1999. Method to predict cropland ephemeral gully erosion. *CATENA*, 37(3-4): 393-399.

Wu S B, Yu M H, Chen L. 2017. Nonmonotonic and spatial-temporal dynamic slope effects on soil erosion during rainfall-runoff processes. *Water Resources Research*, 53(2): 1369-1389.

Wu S B, Chen L, Wang N L, et al. 2019. Two-dimensional rainfall-runoff and soil erosion model on an irregularly rilled hillslope. *Journal of Hydrology*, 580: 12346, doi: 10.1016/j.jhydrol.2019.124346.

Xu X X, Ju T J, Zheng S Q. 2013. Sediment sources of Yan' gou watershed in the Loess Hilly region China under a certain rainstorm event. *Springerplus*, 2(Supp1): S2, doi: 10.1186/2193-1801-2-S1-S2.

Zhao J L, Yang Z Q, Govers G. 2019. Soil and water conservation measures reduce soil and water losses in China but not down to background levels: evidence from erosion plot data. *Geoderma*, 337: 729-741.

Note: Figure translations are in progress. See original paper for figures.

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