Postprint: Analysis of Photometric Variability Characteristics of the BL Lac Object CGRaBS J0141-092

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Abstract

Observations from the Owens Valley Radio Observatory (OVRO) 40-meter telescope have revealed intense flux variations of the BL Lac object CGRaBS J0141-0928 in the 15 GHz radio band. We analyzed the variability period of CGRaBS J0141-0928 using the Lomb-Scargle Periodogram (LSP) method, the Weighted Wavelet Z-transform (WWZ) method, and the Jurkevich method. The results indicate that this object exhibits a Quasi-Periodic Oscillation (QPO) with a period of approximately 649 days at a confidence level of 4.4σ, which may be produced by the helical motion of the jet. We fitted two outburst processes with a double exponential function to derive their variability timescales, and subsequently estimated its average Doppler factor to be 3.8, suggesting that CGRaBS J0141-0928 has a significant beaming effect in the radio emission band. Employing the Discrete Correlation Function (DCF) method, we analyzed the variability correlations between the radio band and the Gamma-ray and optical R-band emissions, respectively. We found a strong correlation between optical and radio variability, with the optical variations leading the radio variations by 66$±$40 days.

Full Text

Analysis of the Optical Variability Characteristics of the BL Lac Object CGRaBS J0141-0928

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Abstract: Observation data from the Owens Valley Radio Observatory (OVRO) 40-meter telescope reveal dramatic variability in the 15 GHz radio band of the BL Lac object CGRaBS J0141-0928. We employ the Lomb-Scargle Periodogram (LSP) method, Weighted Wavelet Z-transform (WWZ) method, and Jurkevich method to analyze the variability period of CGRaBS J0141-0928. The results indicate that this object exhibits a quasi-periodic oscillation (QPO) of approximately 649 days with a confidence level of 4.4σ. This quasi-periodic oscillation may be generated by helical motion of the jet. We fit two flare processes using a double exponential function to obtain their variability timescales, from which we estimate an average Doppler factor of 3.8, suggesting significant beaming effects in CGRaBS J0141-0928 at radio wavelengths. Using the Discrete Correlation Function (DCF) method to analyze the correlation between radio and gamma-ray bands, as well as radio and optical R-band variability, we find strong correlation between optical and radio variability, with the optical variations preceding the radio variations by 66 ± 40 days.

Keywords: CGRaBS J0141-0928; LSP method; weighted wavelet Z-transform method; Jurkevich method; Doppler factor

Classification: P141.5

Blazars are a special subclass of Active Galactic Nuclei (AGNs) with relativistic jets oriented nearly directly toward Earth. They exhibit extreme observational properties, including high luminosity, high polarization, rapid variability, and non-thermal continuum radiation from radio to high-energy gamma rays [1]. The two subclasses of blazars are Flat Spectrum Radio Quasars (FSRQs) and BL Lac objects [2]. BL Lac objects have spectra with only weak emission lines or no emission lines at all, but possess strong X-ray and gamma-ray radiation [3]. Research has shown that long-term variations in some BL Lac objects are periodic, and different wavebands may exhibit different connections. Through observations and studies of BL Lac variability, important information about the physical mechanisms, radiation processes, and internal structural parameters of these objects can be obtained [4], which is of great significance for the exploration and understanding of active galactic nuclei.

CGRaBS J0141-0928 is a blazar with a redshift of 0.733 [5]. Numerous methods exist for analyzing the variability periods of BL Lac objects, with the most commonly used being autocorrelation function analysis, period fitting methods, power spectrum analysis of time series, and the Jurkevich method. These methods have been widely applied to long-period variability observations and studies of BL Lac objects [6]. In this paper, we employ the Lomb-Scargle periodogram method, weighted wavelet Z-transform method, and Jurkevich method to investigate the variability period of CGRaBS J0141-0928 in the 15 GHz radio band.

We fit the flare processes using a double exponential function and estimate the Doppler factor. The Discrete Correlation Function (DCF) method is used to analyze the correlation between gamma-ray and radio bands, as well as optical and radio bands [7]. The LSP method and weighted wavelet Z-transform method are applied for the first time to study the variability period of CGRaBS J0141-0928.

1. Sample and Light Curve

The 40-meter telescope at the Owens Valley Radio Observatory (OVRO) is an ideal facility for global blazar observations (https://sites.astro.caltech.edu/ovroblazars/). Figure 1 [Figure 1: see original paper] shows the 15 GHz radio band light curve of the BL Lac object CGRaBS J0141-0928 from OVRO, containing 575 data points. The light curve reveals that CGRaBS J0141-0928 exhibits very intense activity in the radio band, with flux density fluctuations over time. In addition to five major outbursts in 2010, 2013, 2015, 2017, and 2019, several smaller flares are also visible (peaks in the light curve). We calculate the variability amplitude Amp [8] to assess the activity level of the object:

$$\mathrm{Amp} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \times 100\%$$

where $A_{\max}$ and $A_{\min}$ represent the maximum and minimum flux densities, respectively. Larger Amp values indicate more active objects. The calculated radio-band variability amplitude for CGRaBS J0141-0928 is 62.7, demonstrating that it is a very active object. Due to observational constraints, the light curve data are not continuous, which limits periodicity analysis.

2.1 Lomb-Scargle Periodogram (LSP) Method

The Lomb-Scargle periodogram (LSP) is a widely used method for identifying quasi-periodic oscillations (Lomb 1976; Scargle 1982; Press et al. 1992), developed by Lomb and subsequently improved by Scargle. The LSP method not only reduces spurious signals caused by uneven temporal sampling to some extent but also effectively extracts weak periodic signals from time series [9]. Consequently, the LSP method is well-suited for finding quasi-periodic variability hidden in noise. The fundamental principle of the LSP method is based on Fourier transform, fitting a time series with a linear combination of trigonometric functions $y = a\cos\omega t + b\sin\omega t$ through least squares, thereby transforming signal characteristics from the time domain to the frequency domain. The basic formula is as follows [10-11]:

$$P_{LS}(f) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_{i=1}^{N} x_i \cos\omega(t_i - \tau)\right]^2}{\sum_{i=1}^{N}\cos^2\omega(t_i - \tau)} + \frac{\left[\sum_{i=1}^{N} x_i \sin\omega(t_i - \tau)\right]^2}{\sum_{i=1}^{N}\sin^2\omega(t_i - \tau)}\right\}$$

where τ is the phase correction corresponding to time t, calculated as:

$$\tan(2\omega\tau) = \frac{\sum_{i=1}^{N} \sin 2\omega t_i}{\sum_{i=1}^{N} \cos 2\omega t_i}$$

The LSP method provides good results when analyzing periodicities in sinusoidal-type time series with small data gaps. To verify the correctness of the quasi-period calculated by the LSP method, we first perform a power-law fit to the periodogram to obtain the power-law index β, as shown in Figure 2 [Figure 2: see original paper]. We then calculate the confidence level for CGRaBS J0141-0928, with results presented in Figure 3 [Figure 3: see original paper]. In Figure 3, the green line represents the quasi-periodogram, with its peak indicating the quasi-period result; the blue, red, and purple lines represent the 95%, 99%, and 99.7% confidence levels from Monte Carlo simulations, respectively. The green line in Figure 3 shows a prominent peak at 649 days, with the peak confidence exceeding 99.7%, demonstrating that the result is reliable. Therefore, we adopt 649 days (approximately 1.78 years) as the quasi-period of CGRaBS J0141-0928.

2.2 Weighted Wavelet Z-Transform Method

Wavelet analysis is a novel analytical method that combines pure and applied mathematics. The Morlet wavelet is a single-frequency complex sinusoidal function under a Gaussian envelope, representing a complex wavelet. Using the Morlet wavelet as the mother wavelet for transformation, based on the complex Morlet wavelet [12]:

$$\psi(t) = e^{i\omega_0 t} e^{-t^2/2}$$

where ω_0 is the decay factor. When ω_0 takes larger values, the Morlet wavelet simplifies to:

$$\psi(t) = e^{i\omega t} e^{-t^2/2}$$

After scaling by a and translation by b, equation (5) becomes:

$$\psi\left(\frac{t-b}{a}\right) = e^{i\omega_m (t-b)} e^{-c\omega_m^2 (t-b)^2}$$

where $W_m = W_0/a$ and $c = 1/2W_0$. Based on the vector projection method, equation (7) can be considered a weighted mapping with $\phi(t) = e^{i\omega_m (t-b)}$ as the basis function and $w_\alpha = e^{-c\omega_m^2 (t_\alpha - b)^2}$ as the statistical weight factor. Additionally, introducing a constant function $L(t) = 1$, we obtain three basis functions in vector space:

$$\phi_1(t) = L(t)$$
$$\phi_2(t) = \cos(\omega_m(t-b))$$
$$\phi_3(t) = \sin(\omega_m(t-b))$$

Projecting the data vector $x(t)$ onto these three basis functions yields:

$$y_a = \sum_{i=1}^{3} a_i \phi_i$$

where a_i is calculated as:

$$a_i = \frac{\langle x, \phi_i \rangle}{\langle \phi_i, \phi_i \rangle}$$

Based on the above process, Foster defined the Weighted Wavelet Transform (WWT) [13]:

$$\mathrm{WWT} = \frac{\sum_{\alpha,\beta} w_\alpha w_\beta x_\alpha x_\beta \phi_\alpha \phi_\beta}{\sum_{\alpha,\beta} w_\alpha w_\beta \phi_\alpha \phi_\beta}$$

Since the wavelet shape changes, the effective number of data points N_{eff} in the low-frequency portion exceeds that in the high-frequency portion, causing WWT values to be biased toward high frequencies and resulting in deviations. Therefore, Foster defined the Weighted Wavelet Z-transform based on Z-statistics as [14]:

$$Z = \frac{(N_{\text{eff}} - 3)}{2} \frac{|\langle x, \phi \rangle|^2}{\langle x, x \rangle - |\langle x, \phi \rangle|^2}$$

which follows an F-distribution with degrees of freedom 3 and $2(N_{\text{eff}} - 3)$, and has an expected value of 1. Using equation (14), we obtain the periodicity diagram shown in Figure 4 [Figure 4: see original paper]. In the WWZ diagram, the WWZ value on the frequency axis represents the periodicity of the data vector, while the WWZ value on the time axis indicates fluctuations of the data vector over time [15]. The period of the object’ s light curve can be determined from the maximum value in the WWZ diagram. The blue, red, and purple lines in the figure represent the 95%, 99%, and 99.7% confidence levels, respectively. From Figure 4, we obtain a radio-band variability period of approximately 636 days for CGRaBS J0141-0928, with confidence exceeding 99.7%.

2.3 Jurkevich Method

The Jurkevich method [35], proposed by Jurkevich in 1971, is a period algorithm based on the mean square error of expected values. It folds data according to test periods and is suitable for analyzing variability periods of objects with unevenly sampled data. Assuming N observed sample data points, where each measurement is X_i, $\bar{X}$ is the mean of all samples, V^2 is the variance of the measurement data sample, and S^2 is the standard deviation, the data sample is divided into m groups according to phase near the test period. The statistical parameter for group l is:

$$V_l^2 = \frac{1}{m_l - 1} \sum_{i=1}^{N_l} (X_i - \bar{X}_l)^2$$

The total variance corresponding to m groups is:

$$V_m^2 = \sum_{l=1}^{m} V_l^2$$

When the test period approaches the true period, V_m^2 reaches a minimum value. Additionally, Kidger et al. [36] provided a method for judging period reliability based on the Jurkevich method, namely:

$$f = \frac{V^2 - V_m^2}{V^2}$$

where V^2 is the normalized value. Typically, $f \geq 0.25$ indicates possible periodicity, while $f \geq 0.5$ indicates a highly significant period.

Applying the Jurkevich method to analyze the radio-band light curve of CGRaBS J0141-0928 yields the results shown in Figure 5 [Figure 5: see original paper]. In Figure 5, the area below the green line represents the confidence level for $f \geq 0.25$, while the area below the red line represents the confidence level for $f \geq 0.5$. When searching for periods, those satisfying two conditions are generally considered reliable: first, the time span of the analyzed data sample exceeds six times the period, and second, the resulting curve shows obvious amplitude. From Figure 5, we can see that corresponding to the minimum value of V_m^2, CGRaBS J0141-0928 may have a relatively reliable period of 650 days in the radio band, with an f value of 0.37. The 650-day period obtained by the Jurkevich method is very close to the results from the LSP and WWZ methods. Period results from minima after 650 days clearly do not satisfy the above two conditions and are therefore not adopted.

3. Doppler Factor Analysis

We selected two flare processes containing both rising and falling phases from the radio light curve, corresponding to MJD ranges 56167.4-56410.7 and 56762.8-57228.5. We used the double exponential function flare fitting formula [33,34] to fit these two flare processes:

$$F(t) = F_c + F_0 \left[\exp\left(\frac{t_0 - t}{t_r}\right) + \exp\left(\frac{t - t_0}{t_d}\right) \right]^{-1}$$

where F_c represents the baseline flux, t_0 is the time corresponding to the peak, t_r and t_d are the exponential rise and decay timescales, respectively, and F_0 measures the flare amplitude.

The double exponential function fits to the two flare processes are shown in Figure 6 [Figure 6: see original paper], with fitting parameters for each flare process listed in Table 1 . Column 1 gives the Modified Julian Date range; Column 2 shows the reduced minimum sum of squared residuals from fitting; Column 3 provides the peak MJD and error; Column 4 gives the baseline flux and error; Column 5 shows the flare amplitude and error; Columns 6 and 7 list the exponential rise and decay timescales with errors; and Column 8 gives the Doppler factor for each flare process. Additionally, an extra small peak is visible in the right panel of Figure 6, likely caused by a flare from a shock in the relativistic jet [43].

The Doppler factor (δ) is related to the bulk flow velocity and viewing angle of the jet, but both quantities are not directly observable. Therefore, indirect methods for estimating the Doppler factor are necessary, with radio variability Doppler factors (δ_R) being relatively accurate [27-29]. Assuming the variability is intrinsic, and based on the source size constrained by variability timescales, the brightness temperature calculation formula for blazars is given in [30]:

$$T_b = 1.05 \times 10^{12} \frac{\Delta F \lambda^2}{t_{\text{ob}}^2 D^2} (1 + z)$$

where T_b is the brightness temperature, ΔF is the flux variation in Jy, t_{ob} is the variability timescale in days, λ is the observed wavelength in cm, D is the luminosity distance in Mpc (using cosmological parameters $H_0 = 72$ km s^{-1} Mpc^{-1} and $\Omega_m = 0.3$), and z is the redshift.

It is generally accepted that the brightness temperature T_b of jet components in blazars cannot exceed the equipartition brightness temperature $T_{\text{eq}} = 5 \times 10^{10}$ K [31]. Our calculated brightness temperatures are $T_b = 2.574 \times 10^{12}$ K and $T_b = 2.923 \times 10^{12}$ K, exceeding the equipartition temperature by several orders of magnitude and indicating significant Doppler boosting effects. Following [28], we select T_{eq} as the intrinsic brightness temperature and further estimate the radio-band variability Doppler factor using:

$$\delta_R = \left(\frac{T_b}{T_{\mathrm{eq}}}\right)^{1/3}$$

From equation (25), we obtain Doppler factor δ_R values of 3.72 and 3.88, with an average of approximately 3.8. Fan et al. [32] estimated a gamma-ray Doppler factor $\delta = 5.50$ for this object, which is comparable to our radio-band estimate.

4. Correlation Analysis

The Discrete Correlation Function (DCF) method can be used to analyze correlations between two sets of discrete data [16-18]. This method can determine correlations without any processing of the data samples. The DCF method was introduced by Edelson for studying time delays and can investigate source structure and other properties through time delay calculations [19].

The discrete correlation function is defined as follows: for any two discrete data sequences a_i and b_j, the unbinned discrete correlation function is:

$$\mathrm{UDCF}_{ij} = \frac{(a_i - \bar{a})(b_j - \bar{b})}{\sigma_a \sigma_b}$$

where $\bar{a}$ and $\bar{b}$ are the mean values of data sequences a_i and b_j, and σ_a and σ_b are the corresponding standard deviations. Each value UDCF_{ij} is associated with a time lag $\tau = \Delta\tau = t_j - t_i$. For noisy data, we can use:

$$\sigma_a^2 = \frac{1}{N-1} \sum (a_i - \bar{a})^2$$

instead of σ_a and σ_b in the above equation. For a given τ, if there are M UDCF_{ij} values satisfying $\tau - \Delta\tau/2 \leq \Delta t_{ij} < \tau + \Delta\tau/2$, we average these M data points to obtain:

$$\mathrm{DCF}(\tau) = \frac{1}{M} \sum \mathrm{UDCF}_{ij}$$

$\mathrm{DCF}(\tau)$ is the discrete correlation function. The time lag domain is divided into bins, each with interval $\Delta\tau$, yielding a useful $\mathrm{DCF}(\tau)$. If no data points fall within a certain interval, $\mathrm{DCF}(\tau)$ takes no value. When the two correlated sequences are different, we obtain the discrete cross-correlation coefficient. When they are identical, we obtain the discrete autocorrelation coefficient. In DCF analysis plots, larger DCF peaks indicate stronger correlations, while smaller peaks indicate weaker correlations. When the DCF peak is on the positive side, it means data a leads data b; when on the negative side, it means data a lags data b.

We analyze the correlation between the radio band and gamma-ray data from the Fermi Gamma-ray Space Telescope, as well as optical R-band data from the KAIT Fermi AGN Light-Curve Reservoir (http://herculesii.astro.berkeley.edu/kait/agn/), using the DCF method. Fortran programs were used to calculate the results shown in Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper]. The top panels show flux light curves for radio vs. gamma-ray/optical R bands, while the bottom panels show correlation results (DCF values) between gamma-ray/optical R bands and the radio band. Peaks closer to 1 indicate better correlation. From Figure 7, we see the maximum DCF value between gamma-ray and radio bands is 0.3, indicating very weak correlation, suggesting different emission regions and radiation processes. Figure 8 shows the maximum DCF between optical and radio bands reaches 0.71, with the optical band leading the radio band by 16-110 days, indicating consistent radiation processes.

5. Discussion and Conclusions

By collecting variability data for the BL Lac object CGRaBS J0141-0928 and analyzing the period of its radio-band light curve using the Lomb-Scargle periodogram method, weighted wavelet Z-transform method, and Jurkevich method, we obtain mutually consistent results. The LSP method yields a period of approximately 649 days, the WWZ method gives approximately 636 days, and the Jurkevich method gives approximately 650 days. The results from the WWZ and Jurkevich methods further support the existence of a reliable ~649-day variability period in the radio band of CGRaBS J0141-0928. Using the double exponential function, we fit two flare processes and estimate a Doppler factor $\delta_R = 3.8$. The results indicate significant beaming effects in the radio emission from CGRaBS J0141-0928, supporting the relativistic jet model. DCF analysis of radio vs. gamma-ray and radio vs. optical R-band correlations shows weak correlation between gamma-ray and radio bands, but strong correlation between optical R-band and radio bands, with the optical R-band leading the radio band by 16-110 days. The physical models for long-term variability in blazars remain unclear, with common models including binary black hole models [20-21], helical jet models [22-25], and thermal instabilities in thin disks.

The quasi-periodicity observed in CGRaBS J0141-0928 may be produced by helical jet motion [37]. Jet helical precession is driven by the orbital motion of a supermassive black hole binary (SMBHB) system. Rieger provided the relationship between the actual physical driving period P_d and the observed quasi-period P as:

$$P_d \approx \frac{P}{(1+z)\gamma_b^2}$$

where γ_b is the bulk Lorentz factor, approximately 7.5 [38], and z is the redshift. Using the 649-day quasi-period, we obtain a physical driving period $P_d \approx 57.71$

years. Alternatively, using $\gamma_b = 15$ [39], we obtain $P_d \approx 230.85$ years. When the mass ratio between the primary and secondary black holes is $R \leq 1/3$, it is called a "major merger" ; if $3 \leq R \leq 10^4$, it is called a "minor merger" [40]. Regardless of the mass ratio, the mass of the primary black hole can be estimated using [41]:

$$M \approx 10^8 \left(\frac{P_d}{1\ \mathrm{yr}}\right)^{5/3} M_\odot$$

where P_d is in years. For an SMBHB system undergoing a major merger, the mass ratio can be assumed as $R = 3/2$. Substituting parameters into equation (29) yields a primary black hole mass of $M \approx 10^{8.93} M_\odot$ for CGRaBS J0141-0928. Using $\gamma_b = 15$ [42] for a minor merger SMBHB system gives a primary black hole mass of $M \approx 10^{9.89} M_\odot$. Wu et al. [26] reported a black hole mass of $M \approx 10^{9.63\pm0.70} M_\odot$ for CGRaBS J0141-0928, consistent with our estimated primary black hole mass.

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Note: Figure translations are in progress. See original paper for figures.