Mathematical and statistical modeling of morphometric and planar parameters of barchans in Pashoeyeh Erg in the west of Lut Desert, Iran (Postprint)

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Date: 2021-09-08T00:00:00+00:00

Abstract

Barchan dunes are among the most common accumulative phenomena made by wind erosion, which are usually formed in regions where the prevailing wind direction is almost constant throughout the year and there is not enough sand to completely cover the land surface. Barchans are among the most common windy landscapes in Pashoueyeh Erg in the west of Lut Desert, Iran. This study aims to elaborate on morphological properties of barchans in this region using mathematical and statistical models. The results of these methods are very important in investigating barchan shapes and identifying their behavior. Barchan shapes were mathematically modeled by simulating them in the coordinate system through nonlinear parabolic equations, so that two separate equations were calculated for barchan windward and slip-face parabolas. The type and intensity of relationships between barchan morphology and mathematical parameters were determined by the statistical modeling. The results indicated that the existing relationships followed the power correlation with the maximum coefficient of determination and minimum error of estimate. Combining the above two methods is a powerful basis for stimulating barchans in virtual and laboratory environments. The most important result of this study is to convert the mathematical and statistical models of barchan morphology to each other. Focal length is one of the most important parameters of barchan parabolas, suggesting different states of barchans in comparison with each other. As the barchan' s focal length decreases, its opening becomes narrower, and the divergence of the barchan' s horns reduces. Barchans with longer focal length have greater width, dimensions, and volume. In general, identifying and estimating the morphometric and planar parameters of barchans is effective in how they move, how much they move, and how they behave in the environment.

These cases play an important role in the management of desert areas.

Full Text

Preamble

Mathematical and Statistical Modeling of Morphometric and Planar Parameters of Barchans in Pashoeyeh Erg, Western Lut Desert, Iran

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Abstract: Barchan dunes are among the most common accumulative phenomena created by wind erosion, typically forming in regions where the prevailing wind direction remains nearly constant throughout the year and sand supply is insufficient to completely cover the land surface. These crescent-shaped dunes are particularly prevalent in Pashoeyeh Erg in the western Lut Desert of Iran. This study investigates the morphological properties of barchans in this region using mathematical and statistical models, as these methods are crucial for understanding dune shapes and predicting their behavior. Barchan shapes were mathematically modeled by simulating them in a coordinate system through nonlinear parabolic equations, yielding two separate equations for the windward and slip-face parabolas. Statistical modeling determined the type and intensity of relationships between barchan morphology and mathematical parameters. The results indicated that these relationships follow power correlations with maximum coefficient of determination and minimum estimation error. Combining these two methods provides a robust foundation for simulating barchans in virtual and laboratory environments. The most important outcome of this study is the conversion between mathematical and statistical models of barchan morphology. Focal length emerges as one of the most important parameters of barchan parabolas, revealing different states among barchans. As focal length decreases, the dune’ s opening becomes narrower and horn divergence reduces. Conversely, barchans with longer focal lengths exhibit greater width, dimensions, and volume. Overall, identifying and estimating the morphometric and planar parameters of barchans is essential for understanding their movement rates, migration patterns, and environmental behavior, which plays a vital role in desert area management.

Keywords: barchan dunes; desert; parabolic equations; statistical model; wind erosion; Pashoeyeh Erg; Lut Desert

1 Introduction

The shape of aeolian sand dunes is determined by the availability of sand and the directionality of wind \cite{Cooke_{{{et}}{{al}}}{1993}, Alvarez_{{{and}}{{Franklin}}}{2019}}. Wind shapes sand dunes through erosion and deposition processes \cite{Hersen_{2005}}. When sand supply is limited and winds are unidirectional, crescentic dunes known as barchans form and migrate along the wind direction \cite{Engel_{{{et}}{{al}}}{2018}}. Barchans exhibit a crescent-shaped morphology with two horns pointing downwind \cite{Zhang_{{{et}}{{al}}}{2018}}. Natural barchans display considerable size variation and typically show an asymmetric shape where width exceeds length \cite{Tsoar_{{{and}}{{Parteli}}}{2016}}.

During barchan transformation and evolution, initial sand accumulations are assumed to form parabolic dunes. When these reach a critical height, they become unstable and develop a slip-face. As height increases further, the dune stabilizes \cite{Kocurek_{{{et}}{{al}}}{1992}}, though barchans continue to grow and migrate \cite{Sauermann_{{{et}}{{al}}}{2000}}. Throughout these movements and transformations, barchans maintain their parabolic three-dimensional shape while increasing in volume and size until achieving complete stability. Each barchan can be conceptualized as the intersection of three separate parabolas with distinct characteristics, each possessing its own focus, vertex, directrix, and equation. The outer parabola indicates maximum windward slope expansion, the middle parabola represents the barchan brink, and the third parabola shows maximum leeward slope expansion. A parabola is defined as the set of points equidistant from a fixed point (focus) and a fixed line (directrix) \cite{Leithold_{1992}}.

Natural barchans display various geometric forms whose appearance and morphology can be modeled mathematically through linear and nonlinear functions. Barchan dunes represent real manifestations of parabolic geometric shapes to which mathematical parabola rules can be applied. This analogy helps researchers better identify these phenomena and their performance in nature while accurately understanding their behavior for desert area management based on a systemic approach. Mathematical modeling using parabolic equations enables determination of the exact area under the parabolic curve, which can be used to calculate surface components such as environment and area.

Morphometric calculations of barchan dunes were first performed by Bagnold (1941) and Finkel (1959) in southern Peru. Lettau and Lettau (1969) developed a geometric pattern consisting of elliptical and parabolic shapes with straight lines to simulate a standard barchan dune using height, width, and length parameters to calculate dune volume. Numerous researchers have employed numerical simulations to predict barchan movement, evolution, and morphology \cite{Landsberg_{1956}, Howard_{{{and}}{{Morton}}}{1978}, Anton_{{{and}}{{Vincent}}}{1986}, Wippermann_{{{and}}{{Gross}}}{1986}, Anthonsen_{{{et}}{{al}}}{1996}, Sauermann_{{{et}}{{al}}}{2000}, Hersen_{2004}, Mousavi_{{{et}}{{al}}}{2010}, Duran_{{{et}}{{al}}}{2011}, Hamdan_{{{et}}{{al}}}{2016}, Yang_{{{et}}{{al}}}{2019}}. Hesp and Hastings (1998) examined relationships between morphological parameters, identifying them as factors controlling the three-dimensional shape of barchans. Sauermann et al. (2000) studied Moroccan barchans using three-dimensional parabolic equations, proposing a shape model to define differences and stability among barchans. Wang et al. (2007) investigated Chinese barchans through geometric theories, finding that barchan height was proportional to width and that the brink could be described by a parabolic shape. Zhang et al. (2018) studied asymmetric barchan migration in Northwest China, demonstrating that morphometric relations predicted from steady-state symmetric models could be applied to transitional asymmetric morphologies.

This study investigates the morphometric and planar features of barchan dunes in the selected region using mathematical modeling, quantitative measurement methods, and parabolic equations. Quadratic equations for barchan parabolas are presented to better identify their behavior, performance, and morphometric properties in nature. Barchan horizontal sections are first examined to verify if they follow parabolic equations, after which equation components are modeled using geometric properties. A combination of statistical and mathematical modeling is employed to study barchan shapes. These equations are useful for defining morphometric and planar indices, simulating barchans in virtual and laboratory environments, and accurately identifying their behavior and morphodynamic features.

2.1 Study Area

The Pashoeyeh Erg is located in the western Lut Desert, eastern Kerman Province, Iran, oriented northwest-southeast along the eastern margin of the Lut Desert' s Kalouts, at 970 m elevation. The study area lies between latitudes 29°55 -30°21 N and longitudes 57°50 -58°10 E. The Lut Desert is one of Iran' s most important deserts, ranking among the driest and hottest deserts globally, with annual precipitation below 50 mm and potential for several consecutive years without rainfall. Table 1 presents climatic parameter characteristics for the study area.

According to Table 1, no rainfall occurred during July, August, September, and October. The highest precipitation was 9.90 mm in February. July was the warmest month with an average temperature of 41.20°C, while January was the coldest at 13.70°C. The highest humidity was 36.90% in January and the lowest was 12.00% in July. Analysis of the wind-rose diagram from Shahdad synoptic station (northwest of the study area) shows that the prevailing wind direction is from the north, with a maximum wind speed of 11.11 m/s in this direction.

2.2 Parameter Measurements and Data Analysis

The study area was investigated using Google Earth satellite imagery. Field sampling was conducted along ten transects covering nearly the entire study area, with morphometric and planar parameters measured for each barchan along these transects (Table 2). The sample size depended on the number of barchans intersecting the transects, totaling 50 evaluated barchans.

Morphometric and planar parameters were measured to statistically investigate barchan characteristics. Figure 1 [Figure 1: see original paper] illustrates these parameters and measurement methods. Total length and width were calculated using Equations 1 and 2:

$$L = L_o + L_s$$

$$W = W_a + W_b$$

where L is total barchan length (m), L_a is right horn length (m), L_b is left horn length (m), L_o is windward side length (m), L_s is slip-face length (m), W is total barchan width (m), W_a is right side width (m), and W_b is left side width (m).

To calculate barchan perimeter and area, a hypothetical coordinate system was established for each dune on the ground, with 21 points determined on its perimeter. These points were plotted on graph paper at 1:100 scale, and area and perimeter were calculated using a planimeter and curvimeter, respectively. Assuming barchan volume equals half of a pyramid’s volume \cite{Hesse_{2008}}, sediment volume was determined using Equation 3:

$$V = \frac{1}{3}SH$$

where V is barchan volume (m^3), S is area (m^2), and H is height (m). Volumes were calculated for all 50 barchans, forming a volume vector. Thus, height, length parameters, width parameters, perimeter, area, and volume of wind-eroded sediments were calculated for all barchans, with data matrices prepared for modeling.

To model barchan parabolas, 20 barchans were selected from the 50-member population using simple random sampling. Based on 21 ground points, these 20 barchans were plotted on 1:100 graph paper. For mathematical modeling using parabolic equations, a hypothetical coordinate system was established for each barchan, with the x-axis parallel to wind direction and the symmetry line, and the y-axis perpendicular to wind direction. The system was oriented so that the barchan opening and its parabolas opened to the right. The x-axis corresponded to the barchan symmetry line and windward direction, with the windward slope parabola vertex at the coordinate origin (Fig. 2 [Figure 2: see original paper]).

After grading coordinate axes in millimeters at 1:100 scale, horn endpoint coordinates were determined. Given that parabolas opened to the right, the windward slope parabola was modeled using:

$$y^2 = 4px$$

where $4p$ is the length perpendicular to the parabola' s focus, x and y are length and width coordinates of parabola points. This equation applies when the parabola vertex is at (0, 0), with focus at $(p, 0)$, directrix at $x = -p$, and symmetry line along the x-axis. These conditions describe the windward slope parabola whose equation can be easily calculated (Fig. 2).

The slip-face slope parabola was modeled in the same coordinate system by transferring the axis to the slip-face parabola vertex using Equation 5:

$$y^2 = 4p(x - h)$$

where h is the x-coordinate of the slip-face parabola vertex. This equation is applicable when the parabola vertex is not at (0, 0). In this case, the axis must be transferred by subtracting h from x in the parabolic equation, locating the leeward parabola vertex at $(h, 0)$. The focus coordinates become $F(h+p, 0)$, and the directrix equation is $x = h - p$. Using this method, the slip-face parabola equation and related properties can be modeled. While the windward slope parabola vertex corresponds to the coordinate origin (making $h = 0$), the slip-face parabola vertex lies on the x-axis away from the y-axis, so $k = 0$ and h equals its distance from the y-axis (Fig. 2).

Overall, parabola vertex coordinates and horn endpoints were determined for all 20 barchans, with parabolic equations, focus coordinates, directrix equations, and p values calculated for each. After mathematical modeling and calculating p values and coefficient x to identify relationship types and effects on morphometric parameters, relationships were evaluated using SPSS software and regression analysis. Simple linear and nonlinear regression methods were tested first, assessing various relationships including linear, power, logarithmic, and cubic forms. Those with higher preference values (higher coefficient of determination and lower standard error) were selected, with power functions proving optimal. Therefore, only simple power correlations are presented below, with correlation coefficient, coefficient of determination, adjusted coefficient of determination, standard error of estimate, and significance level reported to identify relationship types and impact extent.

3 Results and Discussion

The study area in the western Lut Desert contains numerous barchan dunes. Table 3 presents statistical characteristics of the sampled barchans. Maximum and minimum heights were 9.00 m and 2.00 m, respectively. Maximum and minimum lengths were 815.94 m and 110.87 m, while maximum and minimum widths were 1100.02 m and 115.93 m, respectively.

The parabolic shape represents one of the most important morphometric properties of barchans, influenced by their crescentic and aerial morphology \cite{Sauermann_{{{et}}{{al}}}{2000}}. Parabolic quadratic equations can thus model barchan two-dimensional surfaces (ignoring spatial features) \cite{Anthonsen_{{{et}}{{al}}}{1996}}. For mathematical modeling, the coordinate system was designed so that barchan parabolas opened to the right in the wind direction, with the windward slope parabola vertex at the origin and the x-axis parallel to wind direction and barchan symmetry line. Consequently, vertex and focus coordinates were positive and constant across all barchan parabolas, while p values and coefficient x varied among adjacent barchans. Directrix lines were parallel to the y-axis in all cases.

The p value (focal length) and coefficient x are important parameters for calculating barchan parabolic equations and studying morphology. These components represent different parabolic states: as p decreases, focal length and coefficient x reduce, bringing the focus closer to the vertex, narrowing the barchan opening, and decreasing horn divergence. Conversely, longer focal lengths increase width, dimensions, and volume.

In a barchan composed of three separate parabolas, the windward slope parabola has the lowest p value, which increases in the other two parabolas. Thus, the windward slope parabola shows the lowest horn divergence while the slip-face parabola shows the highest. The degree of divergence increases from windward slope to slip-face with increasing p value and coefficient x. Generally, barchans with longer focal lengths have greater width, dimensions, and volume.

Table 4 presents mathematical modeling results using parabolic equations, including p values, directrix equations, parabolic equations, and focus and vertex coordinates. Table 5 shows correlations between windward slope parabola focal length and barchan morphometric parameters.

Analysis revealed maximum significant power correlations between windward slope focal length and barchan area and volume, with determination coefficients of 0.989 and 0.984, and estimation errors of 0.148 and 0.250, respectively (Table 5). Width showed the third highest correlation with focal length (determination coefficient 0.970, error 0.156). The minimum significant power correlation occurred between windward slope focal length and barchan length (determination coefficient 0.952, error 0.311). Perimeter showed the second lowest correlation (determination coefficient 0.957, error 0.247).

Analysis of slip-face parabola focal length correlations showed maximum significant power correlations with barchan width and area (determination coefficients 0.797 and 0.785, errors 0.497 and 0.650, respectively) (Table 6). Volume ranked third (determination coefficient 0.772, error 0.934). The minimum significant power correlation occurred with barchan length (determination coefficient 0.705, error 0.887). Height showed the second lowest correlation (determination coefficient 0.732, error 0.288) (Fig. 4 [Figure 4: see original paper]).

Table 7 indicates correlation coefficients between windward slope focal length and morphometric parameters, with significance levels. The focal length-volume correlation showed the lowest estimation error (0.010), while focal length-area showed the highest standard error (0.542).

Table 8 presents correlation coefficients between slip-face focal length and morphometric parameters. The focal length-height correlation exhibited the lowest error (0.301), while focal length-area showed the highest standard error (288.796). Results indicated good linear relationships between slip-face height and horn width \cite{Hesp_{{{and}}{{Hastings}}}{1998}, Sauermann_{{{et}}{{al}}}{2000}}.

4 Conclusions

This study combined statistical and mathematical modeling methods to investigate barchan morphometric and planar parameters. Planar coordinates can be obtained through parabolic equations and mathematical modeling, while morphometric parameters can be calculated through statistical modeling of geometric properties and justified by regression analysis. The results enable accurate and rapid estimation of barchan parameters. Evaluation of focal lengths in sampled barchans shows maximum and minimum values of 72.22 m and 8.69 m, respectively. Correlation analysis indicates that windward side focal length and barchan area show the highest correlation (determination coefficient 0.989, error 0.148), while length shows the least correlation (determination coefficient 0.952, error 0.311). These simulation results are significant for calculating barchan area under the curve and perimeter and area occupied by barchans in nature.

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Note: Figure translations are in progress. See original paper for figures.