Mathematical Psychology Second Postulate: Cognitive Symmetry

Authors: Gao Chuang, Ma Anran, Wei Wei, Delidaer, Gao Chuang

Date: 2021-08-15T00:00:00+00:00

Abstract

Humans in natural society, physical objects, and biological entities interact and induce resulting effects, which constitute the "events" governing the operation of the material world. The mathematical expression of event structure, linking the "structural encoding logic" of human cognitive functions, constitutes the first postulate of mathematical psychology. The information structure of an event carries its attributes (spatiotemporal attributes, motion attributes, dynamic attributes, causal attributes) and characteristic values. The set of event attributes constitutes material space. After undergoing cognitive processing "operations" such as sensation, perception, and thinking, they are correspondingly mapped into a set of psychological attributes, constituting psychological space. "Cognitive symmetry"posits that symmetry exists between material space and psychological space, wherein event characteristic values are correspondingly transformed into psychological quantities, and each transformation corresponds to a conservation law. This remarkably coincides with the constancy discovered in classical experiments. It constitutes the second postulate of mathematical psychology. "Cognitive symmetry" constitutes the "content logic" of human cognitive functions, addressing the structural construction of human cognitive functions and the logic among these functions. The structural units of human cognitive functions, as the cognitive level increases, exhibit gradually increasing dimensions of event attributes under symmetry transformations, continuously improving degrees of symmetry, and satisfy the "cognitive entropy increase principle" . The content to be reviewed, extended, and further revised in this paper is: the "Cognitive Symmetry Postulate" , which will weave classical local discoveries in psychology, colorimetry, perspective theory, etc., into a logical entity within a unified mathematical theoretical framework, and expand and extend the content of the two postulates of "mathematical psychology" . This may provide foundational exploration for the advent of "unified psychology" .

Full Text

The Second Postulate of Mathematical Psychology: Cognitive Symmetry

GAO Chuang[1], MA Anran[1], WEI Wei[2], DE Li Da Er[1]
[1]School of Psychology, Central China Normal University, Wuhan 430079, China
[2]College of Science, Huazhong Agricultural University, Wuhan 430070, China

Abstract

In the natural world, humans, physical objects, and organisms interact with each other, inducing resultant effects that constitute the "events" through which the physical world operates. The mathematical expression of event structure connects the "structure-coded logic" of human cognitive function, which constitutes the first postulate of mathematical psychology. The information structure of events carries their attributes (spatiotemporal attributes, motion attributes, dynamic attributes, causal attributes) and characteristic values. The collection of event attributes constitutes material space. After cognitive processing "operations" such as sensation, perception, and thinking, these are correspondingly mapped into a collection of psychological attributes that constitute mental space. "Cognitive symmetry" holds that symmetry exists between material space and mental space, where the feature values of events are correspondingly transformed into psychological quantities, with each transformation corresponding to a conservation law. This aligns surprisingly well with the constancies discovered in classical experiments, constituting the second postulate of mathematical psychology. "Cognitive symmetry" forms the "content logic" of human cognitive function, addressing the logical relationships between the structural construction of human cognitive functions and their interrelationships. As the level of cognitive function increases, the dimensions of event attributes undergoing symmetry transformation increase incrementally, the degree of symmetry continuously improves, and the "cognitive entropy increase principle" is satisfied. This paper will review, extend, and further refine the "cognitive symmetry postulate," connecting localized classical discoveries in psychology, colorimetry, perspective theory, and other fields into a logical body within a unified mathematical theoretical framework, while extending the content of the two postulates of "mathematical psychology." This may provide foundational exploration for the advent of "unifying psychology."

Keywords: cognitive symmetry, cognitive entropy increase principle, mathematical psychology, unifying psychology

The root of physical world operation lies in interactions between objects. The interactions between objects and their induced effects constitute events (GAO, 2021). As the origin of cognition (referred to as "stimulus" S in psychology), "events" represent the fundamental relationships underlying the information expression of physical operation, psychophysical relationships, mind-body relation-

ships, and human knowledge systems. The "event structure formula," as the first postulate of mathematical psychology, aligns surprisingly well with classical discoveries in psychological experimental phenomenology (GAO et al., 2021).

Information about events arising from interactions between objects is loaded into information media (such as optical media) or acts directly on neural channels—that is, event signal modulation (see main text). Through information media, physical events interact with human sensory organs, allowing physical world event information to be input into the human cognitive system and form mental representations (GAO, 2021).

The human cognitive processing system consists of functionally independent yet interrelated cognitive structural units (such as sensation and perception). Through the functional operations of these cognitive structures, information about the structured attribute quantities of events and their relationships and characteristic values can be represented and computed. In other words, the cognitive system can perform operations (cognitive operation transformations) such as sensing, perceiving, and thinking about "events." Specifically, by comparing historical events and future events with current events, providing feedback evaluation, conducting causal analysis, and executing behavioral events, humans achieve interaction with the physical world. Cognitive operation transformations thus enable humans to perceive the world, understand its principles, and transform it.

Cognitive operations implement the mapping from physical world events to mentally represented events (i.e., mental events). Does a symmetric mapping exist between physical events and mental events, or to what degree does symmetry (invariance) exist? Asymmetry is termed "symmetry breaking."The issue of "cognitive symmetry" becomes the core for revealing the logical nature of cognitive functions.

The first postulate, the "event structure formula postulate," can connect the "structure-coded logic" of human cognitive functions (GAO et al., 2021). The second postulate, the "cognitive symmetry postulate," can connect the "content logic" of human cognitive functions. The establishment of the "structure-coded logic" and "content logic" of human cognition exposes the two mathematical logic lines of "information structural form" and "information content" processed by the human cognitive system. This paper will review how, under the second postulate of mathematical psychology—"cognitive symmetry"—localized classical discoveries in psychology, colorimetry, perspective theory, and other fields become logical bodies within a unified mathematical theoretical framework, while extending the content of these two postulates as much as possible.

This may provide foundational exploration for cognitive science' s development toward unifying psychology. That is, based on the two postulates and using the "structure-coded logic" and "content logic" of human cognitive functions as logical threads, classical discoveries in human cognitive function can be deduced through mathematical deduction to form a unified mathematical theoretical

framework.

1. The First Postulate of Mathematical Psychology: Event Structure Formula

Events in the physical world can be classified according to object type into physical events, biological events, mental events of humans or animals, and social events. These four categories can be merged into two fundamental expression forms:

$$E_{phy} = w_1 + w_2 + i + e + t + w_0 + c$$

where w_1 and w_2, i, e, t, w_0, c represent material objects, the medium of interaction between objects, event results, time, location, and initial values of the above elements, respectively. The objects in this type of event lack mental attributes—that is, they do not have purpose in any event. This includes physical events and non-mental events in biology.

Distinguished from the above events, w_1 and w_2 must include at least one human or animal with mental attributes, who are the initiators of purposeful events. b represents the behavioral motivation target object, and m represents the internal motivation target object. These constitute psychological or social events of humans, animals, and society, represented by E_{psy}. Their operations satisfy Boolean algebra rules.

Event information entering the brain has a unified event structure: the event structure formula. This thereby resolves a series of classical discoveries in localized fields:

(1) **Event information decoding stage**: The natural correspondence between perceptual function types and elements of the event structure formula (GAO, 2021).

(2) **Unity of psychological encoding**: The mathematical expression of Chomsky' s innate grammatical structure (Chomsky, 1957).

(3) **Learning experiments controlling event structural elements**: Behaviorist learning experiments collectively constitute a "complete set" of event structure formula elements as control conditions (GAO, 2021).

(4) **Structural system of knowledge and experience**: The physical event structure formula determines the four fundamental attributes of the physical world and the basic science systems of human knowledge extended from them (GAO et al., 2021).

(5) **Completeness of event information channels**: The result of human information channel capacity evolution—7 ± 2 (Miller, 1956).

The first postulate, the "event structure formula," summarizes the universal mathematical expression form of "physical world events" entering the cognitive system, connecting the underlying logic of material information structure expression and information cognitive processing mechanisms. Using "material event information structure" as a reference, the "event structure formula postulate" can connect the "structure-coded logic" of human cognitive functions. That is, with each independent element of the "event structure formula" as a reference, perceptual function types, semantic symbol event elements of psychological encoding, behaviorist learning experiments controlling event structural elements, and the structural system of knowledge and experience all correspond to the independent elements of the "material event structure formula." The number of items stored in working memory corresponds one-to-one with the total number of independent elements in the event structure formula.

2. The Second Postulate of Mathematical Psychology: Cognitive Symmetry

2.1 Psychological Space

Any attribute of an event' s elements will be mapped into the cognitive system and form a mental representation. The characteristic quantity of a material attribute is transformed into a psychological characteristic quantity (i.e., psychological quantity). Since material attribute quantities are independent, this means that within the observable range, the collection of material attributes is correspondingly mapped into a collection of psychological attributes. The material attribute space R_s is correspondingly mapped into a "psychological space," denoted as R_m. That is, their relationship satisfies: $R_s \to R_m$. Then, the characteristic value of any element' s attribute can be represented by a vector: $(v_1, v_2, ..., v_j, ..., v_n)$, where the number of attributes equals the number of components. Its corresponding psychological quantity can be expressed as:

2.2 Symmetry of Event Structural Elements

The human cognitive system completes its work through multi-level cognitive operations (sensation, perception, reasoning, etc.). Any level of cognitive operation can be mathematically understood as a transformation operation. When represented by a matrix, the following relationship is satisfied:

$$\begin{pmatrix} v'_1 \\ v'_2 \\ \vdots \\ v'_j \\ \vdots \\ v'_n \end{pmatrix} = T_C \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_j \\ \vdots \\ v_n \end{pmatrix}$$

where T_C represents the cognitive operation. This matrix is called the cognitive

transformation matrix.

2.3 Cognitive Symmetry Principle

As information, "events"are the carriers connecting the objective world (physical world) and the subjective world (mental world). The attribute quantities of events in physical space contain objective attributes of physics, biology, and society (physical quantities). After a series of cognitive processing operations (cognitive transformations), they are converted into psychological quantities, enabling humans to cognize the entire physical world (GAO, 2021).

The functional units of human cognition include sensation, perception, reasoning, decision-making, learning, and innovation. Each independent functional unit can be understood as an operational unit with independent cognitive processing capacity. Mathematically, this transforms the input event information structure into "mental events" sequentially output by each perceptual functional unit, as shown in Figure 1 [Figure 1: see original paper]. E_i is the input event, and E_o is the output event.

Figure 1 Cognitive symmetry transformation. Any functional link in cognition takes the structural information of lower-level events as input, processes it, and outputs new event information. This operation is understood as a "transformation." Cognitive transformations satisfy the symmetry principle and Noether' s theorem. That is, each level of transformation has a corresponding conservation law, equivalently converting lower-level causal relationships to higher-level events.

According to cognitive function, symmetry theory, group theory, and Noether' s theorem, GAO (2021) proposed the cognitive symmetry principle: Any functional unit participating in cognitive processing (sensation, perception, thinking) follows "symmetry transformation" in cognitive transformation.

Upper and lower levels satisfy complementary relationships. Lower-level functional units are asymmetric (i.e., broken) at higher dimensions, compensated by higher-level functional units to jointly form a cognitive closed loop. The total function of all cognitive links satisfies infinite approximation to knowability of physical world cognitive information (as shown in Figure 2 [Figure 2: see original paper]). In this process, the information entropy of human individuals cognizing external physical events satisfies:

Cognitive Entropy Increase Principle.

2.4 Cognitive Entropy Increase Principle

Each cognitive transformation of event information, after decoding by higher-level cognitive units, increases the information cognized by humans, satisfying the cognitive entropy increase principle (GAO, 2021).

$$\Delta S = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})} = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})} = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})} = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})}$$

where the joint probability of the i-th structural element is denoted as p_{ij} (p is joint probability), the total probability of event E_i is p_{ij}, K represents the k-th cognitive transformation, the sum of attributes of event structural elements is denoted as T, the dimension of the lower-level space is n_0, l represents the new space dimension, and the increased dimension from lower-level to higher-level transformation is m. Moreover, since event structural elements are encoded by symbols, the encoding probability of a single attribute element satisfies:

$$\Delta S = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})} = \log \frac{T_{lmn}(p_{jik})}{T_0(p_{jik})}$$

This term indicates that due to improved symmetry, new independent dimensions are introduced, causing the number of information items to increase.

Figure 2 Schematic diagram of cognitive transformation invariance and breaking. Symmetry breaking at lower-level cognitive processing units is realized as symmetry at higher-level cognitive functional units. The direction of human evolution and development satisfies the cognitive entropy increase principle (GAO, 2021).

3. Cognitive Symmetry Transformations Deduce Cognitive Content Logic

Based on the cognitive symmetry principle, we can address the mathematical logic of cognitive functions as a whole. As shown in Figure 2, cognitive transformations beginning with sensation involve multiple sensory channel transformations. Event information achieves multi-channel integration at the perceptual stage and sequentially undergoes reasoning transformations, essentially enabling humans to process on-site events, historical events, and future events.

3.1 Sensory Transformation Invariance and Breaking

Structured information about events from interactions between material objects is input into human sensory organs through modulation of information media by objects, as shown in Figure 3 [Figure 3: see original paper]. Objects interact with each other to constitute events. Simultaneously, objects interact with sensory organs through communication media, transmitting structured event information by changing the characteristics of the information medium (such as amplitude modulation and frequency modulation of optical media). Sensory organs receive the changed signals, achieving loading of event signals onto the medium for collection and neural transmission by the sensory organs.

For a single sensory organ (single eye, single ear, etc.), it primarily implements the transduction relationship of material signals—that is, converting physical energy and component signals (chromaticity for light, frequency for vibration) into neural pulse signals.

Figure 3 Sensory modulation model schematic (GAO, 2021). Through optical media, objects in events interact with visual receptors. The optical attribute information of events performs amplitude modulation (fluctuating energy) and frequency modulation (color components) on the optical medium. Visual receptors collect the optical attribute information of events and convert it into neural carrier signals transmitted to the brain.

For the visual channel, monocular relational transformations satisfy colorimetric attribute transformations, which colorimetry has established as a fundamental relationship, as shown in Equation 4. According to cognitive symmetry theory, its essence is a symmetry transformation. X, Y, Z represent psychological quantities corresponding to colors R, G, B, respectively. This is a classical phenomenological discovery. Through this transformation, the monocular eye correspondingly transforms light component information into the human cognitive system, allowing us to see the colorful world.

Simultaneously, the monocular retina has the ability to transform spatial information—that is, through the eye' s convex lens, converting two-dimensional signals from physical space into cognized two-dimensional signals. In this transformation, depth information is missing. That is, two-dimensional spatial information is symmetric, but three-dimensional spatial information is asymmetric (symmetry breaking). Julesz (1960) used random dot stereogram experiments to reveal that monocular vision cannot perceive depth information (see Figure 4 [Figure 4: see original paper]). In other words, monocular vision cannot achieve three-dimensional transformation. From the perspective of cognitive symmetry, the Julesz experiment is a classic spatial information breaking experiment revealing the asymmetry of monocular processing of three-dimensional spatial information.

Figure 4 Bela Julesz' s 1960 random dot stereogram experiment. The left and middle images are randomly generated dot patterns. When observers' left and right eyes fixate on the left and middle images respectively, binocular disparity fusion occurs, producing the right image with stereoscopic vision. This experiment found that as long as binocular disparity exists, three-dimensional spatial information emerges. From the cognitive symmetry perspective, monocular vision cannot generate three-dimensional spatial information—that is, monocular processing of the physical world exhibits spatial information breaking.

Through binocular mechanisms, humans acquire depth information. The x, y, z information of three-dimensional space, after binocular transformation, becomes psychological quantities that humans can perceive: x', y', z'. Their depth relationships were initially established by perspective methods in art—that is, humans used geometric relationship measurements to obtain the mapping of depth

relationships in the perceived three-dimensional world onto a two-dimensional plane. GAO (2021) developed this relationship and restored its psychological essence. The relationship between three-dimensional spatial information and perceived psychological quantities satisfies:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \frac{m}{z} & 0 & \frac{x}{z} \\ 0 & \frac{m}{z} & \frac{y}{z} \\ 0 & 0 & \frac{m}{z} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where m represents the distance between the cyclopean eye retina and the eye' s optical center. Thus, binocular vision achieves the transformation from physical three-dimensions to psychological three-dimensions, achieving completeness in spatial position attribute transformation.

This exposes a puzzling question for biology and neuroscience: Why do human eyes need (evolve) binocular vision? What is its mathematical essence? The answer is to achieve symmetric transformation of three-dimensional spatial information. The same principle can answer other sensory channels such as binaural hearing (acquiring three-dimensional stereo sound), dual nostrils (acquiring three-dimensional stereo olfaction). The tongue obtains three-dimensional information through rotation. In terms of physical attributes, binocular vision completes symmetric transformations of both energy attributes and spatial attributes of visual information. This mechanism can be generalized to other neural channels—that is, other neural channels also satisfy symmetry mechanisms.

Thus, to enable symmetric transformation from the "physical world" to the "mental world," the long-term evolution of physiological structures such as binocular vision and binaural hearing ensures symmetric transformation of spatial attribute information in sensory symmetry transformations. Experimental phenomena of "psychophysical transformation" between the physical world and mental world have long been unequivocally present. These classical psychological phenomena constitute solid phenomenological evidence for symmetry breaking in sensory transformations within cognitive symmetry transformations.

When an object' s depth value z increases, the object' s size in mental space decreases and may even disappear—that is, the "vanishing point" in aesthetic perspective drawings. This mechanism can be used to calculate various illusions. The Wundt-Hering illusion (Bondarko, Bondarko, Solnushkin, & Chikhman, 2019), Ponzo illusion (Leibowitz, Brislin, Perlmutrer, & Hennessy, 1969), Ebbinghaus illusion (Eymond, Malkinson, & Naccache, 2020), Müller-Lyer illusion (Santacà, Petrazzini, Agrillo, & Wilkinson, 2020), and moon illusion (Holway, & Boring, 1940) are all key evidence for cognitive symmetry in visual two-dimensional to three-dimensional transformations.

Taking the Wundt illusion as an example, two parallel lines segmented by a set of diamond-shaped lines appear to bend inward, as shown in Figure 5 [Figure 5: see original paper].

Figure 5 Wundt illusion. Two originally parallel lines segmented by a set of diamond-shaped lines no longer appear parallel but seem to bend inward. Taking the currently observed surface as the projection plane and establishing a coordinate system, let any two points A and B on the line be selected, with point A coordinates (x_1', y_1') and point B coordinates (x_2', y_2'). In physical space, according to psychophysical space transformation, point A coordinates are $(\frac{m}{z_1} \cdot x_1, \frac{m}{z_1} \cdot y_1, \frac{m}{z_1} \cdot z_1)$ and point B coordinates are $(\frac{m}{z_2} \cdot x_2, \frac{m}{z_2} \cdot y_2, \frac{m}{z_2} \cdot z_2)$. In areas with dense lines, perception is of greater distance, therefore:

$$z_1 > z_2$$

In the vertical direction, the height relationship between the two points is:

$$\frac{m}{z_1} \cdot y_1 > \frac{m}{z_2} \cdot y_2$$

Thus, in real space, point B' s height from the horizontal axis is smaller, while point A shows the opposite pattern, being closer to the vertical axis, creating the illusion that the middle parts of the lines are closer together. This reasoning can be generalized to all illusions listed above—that is, visual illusions are not truly illusions. These "visual illusion" phenomena we observe are perspective drawings, or mental events in psychological space.

3.2 Perceptual Transformation Invariance and Breaking

Attributes obtained through any neural information channel are collections of single sensory channel attributes of object events. Only by merging all channel attribute information collections (forming a union) can on-site information cognized by humans be complete and symmetric, while single sensory channels exhibit asymmetry (breaking) in this aspect. Perception can integrate attributes from all channels, further improving cognitive symmetry. According to the Boolean operation equation for multi-channel integration derived from the first postulate (GAO et al., 2021), perceptual events consistent in time and space satisfy the additive principle for object attributes (or features)—that is, the event integration principle, also called the perceptual integration transformation principle (GAO, 2021). Mathematically expressed as:

$$E_p = \sum_{i=1}^{n} E_{s_i}$$

where i represents the i-th neural channel, n represents the total number of neural channels, p is the abbreviation for perception in English, and the plus sign satisfies Boolean algebra rules. Since each element' s attribute set satisfies symmetry in sensory transformation, spatial transformation still satisfies symmetry

after Boolean addition. Structured event information is equivalently transferred to perception. Perception satisfies symmetric information transformation.

Through sensory-perceptual symmetric transformation, human individuals obtain on-site characteristic information of events, enabling observation of the vivid physical world. In addition to perceiving the complete set of attribute information of the physical world, humans need to process information from on-site physical events transcoded into semantic symbol events. The correspondence between physical event structure and semantic symbol event structure implies correspondence and equivalence between event attribute collections. Set operations satisfy Boolean algebra, and semantic symbol events constitute the material content basis for thinking operations. Therefore, the mathematical rules of thinking operations follow Boolean algebra.

Chomsky' s (1957) statistics on thousands of existing human languages indicate that language has structural characteristics and proposed innate grammar. GAO discovered that language structure is naturally determined by event structure and exhibits correspondence (GAO, 2021). This means that the semantic space expressed by symbol space based on semantics corresponds to physical attribute space. According to Boolean algebra rules, this correspondence implies symmetry of attribute space. Similarly, symbol events encoded by semantics also correspond to physical events (GAO, 2021). These two correspondences imply correspondence between semantic space based on symbols and physical space.

Human thinking is based on symbols. The correspondence between physical event structure and language structure enables equivalent symbolic expression of physical space attribute expressions. This directly implies that physical space and human semantic space naturally correspond. This allows humans to rely on symbols for offline symbolic operations after leaving on-site characteristic quantity expression following perception. This constitutes the basic symmetric relationship from physical space to symbolic space.

3.3 Thinking Transformation

Thinking is human mental activity conducted through linguistic symbols. It uses symbols as codes, primarily for reasoning, decision-making, learning, and innovation. If sensation and perception are on-line information, thinking can work off-line. It includes four key stages: reasoning, decision-making, learning, and innovation. This reflects human subjective agency.

Since event signals cannot be fully collected into the sensory system, sensory-perceptual transformations exhibit breaking. "Reasoning" as a new cognitive function compensates for this gap. Reasoning has the function of making uncertain factors determinate through inference about perceptual events. After this cognitive processing unit of reasoning transformation, a corresponding relationship exists between input and output information (GAO, 2021).

Taking the reasoning phenomenon of sound source perception as an example (see Figure 6 [Figure 6: see original paper]), when an aircraft moves at high speed, because light propagation speed far exceeds sound propagation speed, the seen position and heard position of the aircraft separate—that is, perceptual transformation exhibits breaking. Through reasoning with physical knowledge, we discover that the perceived visual event and auditory event are actually the same event, achieving symmetry between the reasoned event and the external event (GAO, 2021).

Figure 6 Temporal breaking. (a) When the same observer observes a distant sound source, auditory and visual events can be merged into one. (b) For high-speed moving sound sources, the different arrival times of visual and auditory signals cause separation of perceived spatial signals, preventing integration of auditory and visual events. Perception breaks. This breaking is resolved through reasoning.

Sensory transformation, perceptual transformation, and reasoning transformation implement processing of on-site event information, satisfying at different levels. There are two forms of reasoning: inductive and deductive. Its essential results produce two forms: supplementing non-on-site information about event elements, and inputting obtained regularities into human experience systems, which becomes learning transformation. Its implemented transformations are shown in Table 1 (see GAO et al., 2021 for details). On this basis, humans acquire the essential attributes of events with non-on-site information and the relationships between event elements, which are causal relationships. Once causal relationships of certain events are understood through breakthroughs, this constitutes innovation. Its essence remains transformation of event structural element attributes and relationships. Thus, event attribute and characteristic information continuously increase, satisfying the entropy increase principle.

Table 1 Correspondence between control conditions in behaviorist learning experiments and event elements (GAO et al., 2021)

Two types of control events	Event consistency	Spatiotemporal consistency	Spatial positional consistency	Pedal-box door-food	Action relationship consistency	Motivation target object consistency	Behavioral motivation consistency
Pavlov's dog experiment (Pavlov, 1927)							

Two types of control events	Event consistency	Spatiotemporal consistency	Spatial position consistency	Pedal-box door-food	Action relationship consistency	Motivation target object consistency	Behavioral motivation consistency
Tolman's experiment (Tolman, 1948)							
Thorndike's cat experiment (Thorndike, 1998)							
Skinner box experiment (Skinner, 1938)							
Seligman's experiment (Maier & Seligman, 1976)							

3.4 Cognitive Conservation Laws

According to symmetry principles or Noether' s theorem (Noether, 1918), each symmetry transformation corresponds to a conserved quantity. In physics, for example, time translation symmetry corresponds to energy conservation in classical mechanics, spatial rotation invariance corresponds to angular momentum conservation, and charge gauge transformation invariance corresponds to charge conservation in quantum mechanics.

In psychology, perceptual constancies are precisely the conserved quantities of psychophysical symmetry transformations. Size constancy (Correspondent, 1972), shape constancy (Bower, 1966), distance constancy (Engel & Dougherty, 1971), location constancy (Goolkasian & Bojko, 2001), and orientation con-

stancy (Day & Wade, 1969) are all conserved quantities of spatial attribute information symmetry transformations. Color constancy (Brainard, 2004) and brightness constancy (Gilchrist & Jacobsen, 1983) are conserved quantities of optical attribute information symmetry transformations.

These constancies constitute direct evidence for cognitive symmetry transformations and cognitive transformation invariance. In other words, the cognitive symmetry principle fundamentally reveals the content logic of classical psychological discoveries. The proof and prediction of constancies corresponding to cognitive symmetry transformations involve neurophysiological mechanisms and will be demonstrated in subsequent reports.

4. Implications for a Unified Theory of Psychology

The unification of psychology has long been a sensitive topic in the field, discussed for many years in the American Psychological Association (Staats, 1991; Kihlstrom, 2004; Henriques, 2003, 2011; Green, 2015; Núñez et al., 2019; TANG, 2007; GAO, 2021).

The first and second postulates of mathematical psychology, in a remarkably simple manner, connect the encoding logic and content logic of information in cognitive processes. This represents an original breakthrough of significant importance for theoretical unification in psychology. In terms of unification, it has in fact already completed two unifying tasks.

4.1 Universal Expression of Physical World Information

Physical world signals are the source of stimuli. The discovery of their universal form resolves the universal expression of S-stimulus. That is, in the S-O-R pattern, it completes the mathematical expression work of the first link.

4.2 Universal Expression of Operations

The physical representation of external information becomes mental representation through O-operation. Cognitive symmetry transformation proposes the basic norm for cognitive operations. It is the universal expression of the second link in S-O-R.

Starting from the first and second postulates, subjective cognition and practice constitute a "psychophysical-mind-body" cognitive feedback control verification closed loop, accumulating into human experiential knowledge systems of "physical events" : kinematics (phenomenology), mechanics, dynamics (or causality).

4.3 Fundamental Change in Psychological Research Nature

Beginning with the first and second postulates, mathematical expression can be used to deduce classical discoveries in psychological experimental phenomenology. This starting point fundamentally transforms the nature of psychological

research, shifting from "experimental phenomenology" to "unified mathematical theoretical framework" in nature. This is a watershed work.

4.4 Feasibility of Unification as Fact

Whether unification is factually feasible is a question that divides psychology into two camps (Staats, 1991; Kihlstrom, 2004; Henriques, 2003, 2011; Green, 2015; Núñez et al., 2019; TANG, 2007; GAO, 2021). The first and second postulates factually prove the feasibility of deducing and deriving classical experimental results in psychology and constructing a basic theoretical system based on mathematical methods, and this has in fact already begun. It transforms from a controversial approach to a 落地的 factual step. This will be the beginning of revolutionary work.

5. Future Work on Unification

Humans are spiritual dynamic systems that form their own behavior patterns driven by stable value quantities. Individual differences in stable concepts formed by value quantities constitute personality differences. This means that describing humans also requires establishing an axiomatic system for describing human value concepts, which includes two other core contents:

(1) **Behavior pattern postulate**: The descriptive relationship between any stable behavior and value concepts.

(2) **Causality postulate**: The essence of value concepts driving human behavior is various forms of causality, such as physical causality, biological causality, and social causality. They drive human behavior to become part of human attribution. This requires establishing postulates for the relationships between behavior, value concepts, personality, and spiritual dynamics.

We will publish and report on subsequent postulates in future work.

References

GAO, C., MA, A., WEI, W., & DE, L. D. E. (2021). The first postulate of mathematical psychology: Event structure formula. [ChinaXiv:202107.00026]

GAO, C. (2021). Mathematical psychology: Psychological space geometry. Changchun, Jilin: Jilin University Press.

TANG, X. W. (2007). Unified framework of psychology and cognitive theory. Shanghai: Shanghai People' s Publishing House.

Bondarko, V. M., Bondarko, D. V., Solnushkin, S. D., & V. N. Chikhman. (2019). Relation between the Wundt-Hering Illusion, the Tilt Illusion, and Estimation of Length of Inclined Line Projections. *Human Physiology*, 45, 370-377.

Bower, T. (1966). Slant Perception and Shape Constancy in Infants. *Science*, 151, 832-834.

Brainard, D. H. (2004). Color Constancy. In L. Chalupa & J. Werner (Eds.), *The Visual Neurosciences* (Vol. 1, pp. 948-961). Cambridge, MA: MIT Press.

Correspondent, O. E. (1972). Perception: Size Constancy. *Nature*, 240, 74-75.

Chomsky, N. (1957). *Syntactic Structures*. The Hague/Paris: Mouton.

Green, C. D. (2015). Why psychology isn' t unified, and probably never will be. *Review of General Psychology*, 19(3), 207–214.

Day, R. H., & Wade, N. J. (1969). Mechanisms involved in visual orientation constancy. *Psychological Bulletin*, 71(1), 33–42.

Engel, G. R., & Dougherty, W. G. (1971). Visual-Auditory Distance Constancy. *Nature*, 234, 308-308.

Eymond, C., Malkinson, T. S., & Naccache, L. (2020). Learning to see the Ebbinghaus illusion in the periphery reveals a top-down stabilization of size perception across the visual field. *Scientific Reports*, 10.

Gilchrist, A. L., & Jacobsen, A. (1983). Lightness constancy through a veiling luminance. *Journal of Experimental Psychology: Human Perception and Performance*, 9(6), 936–944.

Goolkasian, P., & Bojko, A. (2001). Location constancy and its effect on visual selection. *Spatial Vision*, 14(2), 175-190.

Henriques, G. (2003). The tree of knowledge system and the theoretical unification of psychology. *Review of General Psychology*, 7(2), 150-182.

Henriques, G. (2011). *A New Unified Theory of Psychology*. New York: Springer.

Holway, A. H., & Boring, E. G. (1940). The moon illusion and the angle of regard. *The American Journal of Psychology*, 53(1), 109-116.

Julesz, B. (1960). Binocular depth perception of computer-generated patterns. *Bell System Technical Journal*, 39(5), 1125-1162.

Kihlstrom, J. F. (2004). Unity within psychology, and unity between science and practice. *Journal of Clinical Psychology*, 60(12), 1243-1247.

Leibowitz, H., Brislin, R., Perlmutrer, L., & Hennessy, R. (1969). Ponzo Perspective Illusion as a Manifestation of Space Perception. *Science*, 166, 1174-1176.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63(2), 81–97.

Maier, S. F., & Seligman, M. E. (1976). Learned helplessness: Theory and evidence. *Journal of Experimental Psychology: General*, 105(1), 3–46.

Noether, Emmy (1918). Invariante Variationsprobleme, *Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, 235–257.

Núñez, R., Allen, M., Gao, R., Miller Rigoli, C., Relaford-Doyle, J., & Semenuks, A. (2019). What happened to cognitive science? *Nature Human Behaviour*, 3(8), 782–791.

Pavlov, I. P. (1927). *Conditioned Reflexes*. Oxford University Press.

Skinner, B. F. (1938). *The Behavior of Organisms: An Experimental Analysis*. New York: Appleton-Century-Crofts.

Santacà, M., Miletto Petrazzini, M. E., Agrillo, C., & Wilkinson, A. (2020). Exploring the Müller-Lyer illusion in a nonavian reptile (*Pogona vitticeps*). *Journal of Comparative Psychology*, 134(4), 391–400.

Tolman, E. C. (1948). Cognitive maps in rats and men. *Psychological Review*, 55(4), 189–208.

Thorndike, E. L. (1998). Animal intelligence: An experimental study of the associative processes in animals. *American Psychologist*, 53(10), 1125-1127.

Note: Figure translations are in progress. See original paper for figures.