The Triple Numerical Processing System Hypothesis: A New Exploration of Numerical Processing Mechanisms

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Abstract

Mechanisms of numerical processing constitute one of the core scientific questions in the field of numerical cognition. The triple numerical processing system hypothesis elucidates the relationships among various numerical processing mechanisms from a novel perspective, proposing that the cognitive system rapidly analyzes the numerosity of non-symbolic stimuli via three distinct mechanisms: the subitizing mechanism precisely analyzes the numerosity of 1-4 stimuli; the numerosity mechanism analyzes the numerosity of moderately dense stimulus dot arrays, with processing error proportional to the numerosity under analysis, in accordance with Weber' s law; when stimulus density exceeds a certain range, the density mechanism infers numerical relationships by analyzing stimulus density, with processing error proportional to the square root of the numerosity being processed. A series of studies have confirmed that these three numerical processing mechanisms exhibit distinct behavioral patterns and electroencephalographic characteristics. Future research needs to investigate whether the numerosity mechanism is activated in parallel with the subitizing mechanism and the density mechanism, respectively, thereby clarifying the functional relationships among the three numerical processing mechanisms.

Full Text

Preamble

Three Number Processing Systems: A New Perspective on Numerical Processing Mechanisms

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Abstract: The mechanism of numerical processing represents a core scientific question in the field of numerical cognition. The Three Number Processing Systems hypothesis offers a novel perspective on the relationship among various numerical processing mechanisms, proposing that the cognitive system rapidly analyzes the numerosity of nonsymbolic stimuli through three distinct mechanisms. The subitizing mechanism precisely analyzes numerosities from 1 to 4 stimuli. The numerosity mechanism processes moderate-density dot arrays, with processing errors proportional to the analyzed quantity, conforming to Weber's law. When stimulus density exceeds a certain range, the density mechanism infers numerical relationships by analyzing stimulus density, with processing errors proportional to the square root of the processed value. A series of studies have confirmed that these three numerical processing mechanisms exhibit distinct behavioral patterns and electrophysiological characteristics. Future research should investigate whether the numerosity mechanism is activated in parallel with the subitizing and density mechanisms, thereby elucidating the functional relationships among the three numerical processing mechanisms.

Keywords: subitizing mechanism, numerosity mechanism, density mechanism, parallel activation
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Numerical cognition refers to the ability to rapidly perceive, comprehend, and process quantitative information in a scene. Some scholars argue that non-symbolic numerical cognition originates from animals' cognitive intuition and constitutes an innate ability possessed by educated adults, infants, non-human primates, and even lower animals (Agrillo & Bisazza, 2017; de Hevia et al., 2017; Leibovich et al., 2017). The Approximate Number System (ANS) is considered the foundation of numerical cognition, enabling rapid analysis of stimulus numerosity with constant error proportional to the absolute value of the analyzed stimuli (Feigenson et al., 2004; Halberda et al., 2008).

Numerous visual features correlate with numerosity information, including distribution area, density, total stimulus area, total perimeter, convex hull (the convex polygon formed by outermost stimulus points), brightness, distribution regularity, and others (Leibovich et al., 2017). When numerosity features change, these features change synchronously. Consequently, some researchers have questioned whether numerical processing is actually achieved through analyzing numerosity information, suggesting that observers might infer numerical relationships by processing these non-numerical stimulus features (Dakin et al., 2011; Durgin, 2008; Gebuis et al., 2016; Gebuis & Reynvoet, 2012; Yousif & Keil, 2020). The existence of an independent numerosity processing mechanism has remained a central debate in numerical cognition.

Over the years, numerous studies have examined the influence of various visual cues on numerical processing, revealing that numerical processing accuracy is highest under natural conditions (where various features change synchronously with numerosity) and decreases as more visual feature cues are controlled (i.e., holding certain features constant to eliminate their potential as cues) (Beran & Parrish, 2017; Clayton et al., 2015; Dakin et al. 2011; Gebuis & Reynvoet, 2012; Yousif & Keil, 2020). Researchers generally acknowledge that numerical processing is influenced by multiple visual factors, yet no systematic theory has been proposed to explain the specific modes of influence and underlying mechanisms (Daniel et al., 2017; Van Rinsveld et al., 2020). For instance, under conditions where visual cues are controlled, it remains unclear whether numerical tasks are completed by analyzing uncontrolled features or by relying on a noisy numerosity mechanism. Some scholars argue that the phenomenon of non-numerical features influencing numerical processing does not negate the independence of the numerosity mechanism, as multiple independent processing mechanisms may interact and influence each other during cognitive processing (Burr, 2017; de Hevia, 2011).

To resolve this core debate and verify the existence of an independent numerosity mechanism, the most direct approach would be to control all visual cues that covary with numerosity features and examine whether observers retain the ability to process numerical information; however, this is operationally difficult to achieve (Leibovich et al., 2017; Van Rinsveld et al., 2020). Over the years, researchers have sought alternative methods to demonstrate the existence of a numerosity mechanism. Among these, the studies by Anobile et al. (2014; 2015a) have been most influential. They approached the problem from a new perspective by analyzing participants' response characteristics during numerical processing tasks to differentiate distinct numerical processing mechanisms, subsequently proposing the "Three Number Systems" hypothesis, which has had a significant impact on subsequent research.

2.1 Three Numerical Processing Mechanisms

The Three Number Processing Systems hypothesis proposes that the cognitive system relies on different processing mechanisms to rapidly analyze the numerosity of nonsymbolic stimuli depending on the stimulus magnitude or density. Simply put, numerical processing activities across different numerical ranges depend on three mechanisms: small numerosities (fewer than 5) activate the subitizing mechanism, moderate numerosities (e.g., 6–40) activate the numerosity mechanism, and large numerosities (e.g., greater than 100) activate the density mechanism (Anobile et al., 2015a).

The three distinct numerical processing mechanisms involved in the hypothesis are: the subitizing mechanism, the numerosity mechanism, and the density mechanism (Anobile et al., 2014; 2015a). The development of these mechanisms spans a considerable period.

As early as 1871, researchers observed that stimuli with small numerosities (fewer than 5) were always identified both rapidly and accurately, with observers making virtually no errors (Jevons, 1871). Subsequent studies further revealed that in counting tasks, reaction times for enumerating 1–4 items showed almost no increase with stimulus numerosity, and error rates remained near zero. This behavioral pattern contrasted sharply with that for large numerosities (5 and above), where counting reaction times increased linearly with stimulus numerosity (Kaufman & Lord, 1949; Luo Yuejia et al., 2004). Based on these findings, researchers proposed that numerical processing in the 1–4 range relies on a special "subitizing" mechanism. This processing mechanism is characterized by speed and precision; as stimulus numerosity increases from 1 to 4, subitizing reaction times remain essentially constant, and error rates stay close to zero (Kaufman & Lord, 1949).

How, then, are numerosities greater than 4 processed? Beyond serial "counting," we also possess the ability to rapidly process these numerosities, generating systematic errors whose magnitude is proportional to the processed value. Researchers typically employ two experimental paradigms to investigate this rapid, approximate numerical estimation ability. In the "numerosity estimation" paradigm, observers estimate and report the numerosity of dot stimuli, with the Weber fraction (W) calculated as the ratio of the standard deviation (SD) of multiple reports to the mean (i.e., the point of subjective equality, PSE) (Halberda et al., 2006; Liu et al., 2020). In the "numerosity comparison" paradigm, which generally uses the method of constant stimuli, the standard stimulus contains a fixed number of dots while comparison stimuli vary around this standard, and observers judge which contains more dots. Data analysis involves plotting comparison stimulus numerosity on the x-axis and the percentage of "more" responses on the y-axis, fitting the results with a cumulative normal distribution curve to determine the just noticeable difference (JND; the β coefficient of the curve) and PSE (the α coefficient) from the fitted parameters. Here, the Weber fraction W equals the ratio of JND to PSE (Anobile et al., 2014; 2015b). The W score serves as a common metric for discrimination ability. Across various studies using these two paradigms, W scores range from 0.14 to 0.4 (Halberda et al., 2006), indicating relatively stable performance and suggesting that this rapid, relatively stable processing mechanism reflects an independent ANS mechanism. Based on ANS activity, we can rapidly and robustly estimate stimulus numerosity (Halberda et al., 2006) or compare numerosity relationships between stimuli (Halberda et al., 2008).

In recent years, processing of larger numerosities has been further differentiated into two mechanisms: the numerosity mechanism and the density mechanism. Researchers (Anobile et al., 2014) noted that when stimulus presentation area is held constant, observers' errors in numerosity comparison (or JND) are proportional to the standard stimulus numerosity, resulting in a constant Weber fraction W across a certain range as the comparison stimulus numerosity increases (approximately 0.2), meaning processing conforms to Weber' s law. As stimulus numerosity or density (which is proportional to numerosity when area is constant) continues to increase beyond a critical point (discussed below), the pattern of W changes: W begins to decrease linearly with increasing standard stimulus numerosity (density), becoming proportional to the square root of the standard stimulus numerosity. This change in W pattern indicates a shift in the psychophysical law governing processing, implying a change in the underlying mechanism (Anobile et al., 2014). Therefore, these scholars proposed that when stimulus numerosity or density exceeds a certain "critical point" or "inflection point," the numerical processing mechanism switches from the numerosity mechanism to the density mechanism. The portion where W remains constant conforms to Weber' s law for numerosity perception, and the mechanism corresponding to this range was named the numerosity mechanism. Modeling analysis of the portion where W decreases linearly suggests that observers' responses are based on stimulus density features (measurable by inter-dot spacing), inferring numerosity relationships from density characteristics, thus this processing was named the density mechanism (Anobile et al., 2014).

Based on these distinct patterns of W scores, scholars differentiated three numerical processing mechanisms and proposed the Three Number Processing Systems hypothesis (Anobile et al., 2014; 2015a; 2019): rapid numerosity processing of stimuli varying from low to high numerosity or density relies on the subitizing, numerosity, and density mechanisms, respectively. The subitizing mechanism is highly precise (Weber fraction W constant and near zero) for analyzing numerosities of fewer than 5 stimuli (Kaufman & Lord, 1949). The numerosity mechanism corresponds to the portion where W remains stable as standard stimulus numerosity increases, with moderate stimulus density and processing conforming to Weber' s law for numerosity perception (W 0.2), where observers directly analyze stimulus numerosity features. The density mechanism corresponds to the portion where W decreases with increasing standard stimulus numerosity (dropping from approximately 0.2 to 0.1 as numerosity increases), with densely distributed stimuli where observers judge numerosity relationships by analyzing stimulus density features.

2.2 Switching Conditions Among the Three Numerical Processing Mechanisms

The Three Number Processing Systems hypothesis analyzes the boundary conditions or "inflection points" at which switching occurs among the three mechanisms (subitizing, numerosity, and density), providing more specific and accurate definitions for the descriptive terms "small numerosity," "moderate numerosity or density," and "large numerosity or high density" used in the hypothesis.

On one hand, the average inflection point for switching from the subitizing mechanism to the numerosity mechanism is "4" (i.e., the subitizing mechanism can analyze numerosities of 1–4 dots), while the average inflection point for 7–11-year-old children is "3" (Anobile et al., 2019). Both adults and children show individual differences in this inflection point, and individual attentional qualities may influence these differences, with attention deficits and immature cognitive development potentially reducing the breadth or inflection point of subitizing (Anobile et al., 2019).

On the other hand, the inflection point for switching from the numerosity mechanism to the density mechanism can be measured by "visual angle density" (dots per square degree, dot/$°^2$; i.e., how many stimulus dots fall within each square degree of visual angle). Researchers propose that when the visual angle density of a dot array exceeds a certain range (i.e., when stimulus density reaches a level where observers can no longer perceptually distinguish individual stimuli), the numerosity mechanism becomes limited, and switching from the numerosity mechanism to the density mechanism occurs (Anobile et al., 2015b).

The range or inflection point for mechanism switching changes with observation conditions: the more peripheral the stimulus presentation from the visual center, the earlier the inflection point occurs. According to existing research, when stimuli are presented in central vision, a visual angle density exceeding 2.3 dots/$°^2$ (approximately 114 dots within a circular area of 4° radius) reaches the critical condition or inflection point for the two mechanisms to switch. However, when stimuli are presented 13° lateral to the central fixation point (i.e., in peripheral vision), a visual angle density exceeding 0.3 dots/$°^2$ (approximately 14 dots within a 4° radius circle) causes numerical processing to switch from the numerosity mechanism to the density mechanism (Anobile et al., 2014; 2015b). This phenomenon reflects a "crowding-like effect," where two stimuli that are too close together, though detectable, can no longer be identified as distinct individuals (creating a "cannot count" sensation). The more peripheral the stimulus location, the more readily this crowding effect occurs and the earlier the inflection point arrives (Anobile et al., 2015b).

When stimulus presentation area and location are held constant, visual angle density is proportional to numerosity. Thus, as stimulus numerosity increases, numerosity-density mechanism switching occurs. Following this simplification of switching conditions, the original Three Number Processing Systems hypothesis can be concisely described as: as stimulus numerosity increases, the cognitive system activates three distinct mechanisms to process stimulus numerosity, with the subitizing mechanism analyzing small numerosities (generally fewer than 5), the numerosity mechanism analyzing moderate numerosities, and the density mechanism analyzing large numerosities (Anobile et al., 2014; 2015a).

3 Supporting Evidence for the Three Number Processing Systems Hypothesis

The Three Number Processing Systems hypothesis provides a new approach for distinguishing numerosity processing from density processing and has garnered widespread attention from researchers in related fields upon its proposal. In recent years, a series of studies have demonstrated that numerical processing triggered by low- (1–4 dots), moderate-, and high-density stimuli exhibits multiple distinct behavioral and electrophysiological features. These findings confirm that the three numerical processing systems (mechanisms) differ in various aspects. In particular, recent progress in investigating the differential characteristics of numerical processing within the operating ranges of the numerosity and density mechanisms has provided strong support for the Three Number Processing Systems hypothesis.

3.1 Behavioral Feature Differences Among the Three Numerical Processing Mechanisms

First, the three numerical systems differ in processing efficiency. In numerosity estimation tasks, the subitizing mechanism enables observers to rapidly and accurately discriminate numerosities of 1–4 stimuli with near-zero error and the shortest verbal report reaction times (approximately 700 ms), demonstrating extremely high efficiency. The numerosity mechanism triggered by moderate-density stimuli (5–50 dots) shows a constant W score and constant verbal report reaction times (approximately 1350 ms), with lower but stable processing efficiency. The density mechanism triggered by high-density stimuli (75–200 dots) shows both W scores and verbal report reaction times decreasing with increasing density (both proportional to the square root of stimulus density or numerosity, with reaction times decreasing from approximately 1300 ms to 1100 ms), indicating increasing processing efficiency with higher stimulus numerosity or density (Pomè et al., 2019a).

Second, the three mechanisms differ in their demands on attentional resources, with the subitizing mechanism requiring the most, the density mechanism requiring moderate amounts, and the numerosity mechanism requiring the least. Researchers typically manipulate attentional load levels using a "dual-task" paradigm. For example, when the primary task is a stimulus pattern discrimination task and the secondary task is a numerical processing task, the primary task creates attentional load for the secondary task, causing attentional resource depletion. Under these conditions, numerical processing across different numerosity ranges is affected differentially by attentional load: processing in the subitizing (1–4 dots) and high-density (75–200 dots) ranges is more affected than processing in the moderate-density range (5–50 dots) (Pomè et al., 2019b). Additionally, a recent case study demonstrated that a brain-damaged patient with attentional deficits performed poorly on numerical tasks in the subitizing (3 dots) and high-density (64 dots, 128 dots) ranges but performed near normal levels in the moderate-density range (12 dots, 16 dots), suggesting that attentional impairment severely affected the patient' s subitizing and density processing abilities while leaving numerosity processing relatively intact (Anobile et al., 2020). The subitizing mechanism' s high dependence on attentional resources suggests that it may achieve precise numerosity analysis by tagging and tracking target objects under attentional engagement (Burr et al., 2010). The density mechanism also relatively depends on attentional resources;

researchers propose that under the density mechanism, observers may rely on analyzing density (i.e., inter-dot spacing) to compare numerosity relationships, and extracting this local feature requires attentional participation (Anobile et al., 2020). In contrast, the numerosity mechanism is not affected by such low-level feature analysis, shows minimal dependence on attentional resources, and is relatively robust (Anobile et al., 2020).

Third, only moderate-numerosity stimuli demonstrate the connectedness effect in numerical processing. The connectedness effect refers to the phenomenon where two dots connected by a line segment are identified as "one" stimulus, and when stimuli are pairwise connected by lines, observers underestimate their numerosity. This effect reflects the important role of "individuation" in numerical processing: during numerical cognition, "individuals" as numerical units must first be represented (each separate stimulus must be distinguished from surrounding stimuli and judged as a perceptual unit) before numerosity can be analyzed (He et al., 2009). The connectedness effect has been consistently demonstrated in moderate-density (12–48 dots) numerosity processing, where "stimulus dots connected pairwise by line segments" causes observers to underestimate numerosity, with the degree of underestimation corresponding to the number of connected dot pairs (Franconeri et al., 2009; He et al., 2009; Liu et al., 2017). In contrast, with high-density stimuli (100 dots), the opposite occurs: the presence of line segments connecting stimulus dots no longer affects numerosity estimation but instead leads to overestimation of the density of connected dots, suggesting that the "individuation" process is absent in high-density stimulus analysis. Observers at this point appear to simply analyze stimulus density features, and the presence of line segments makes the stimuli appear more dense (Anobile et al., 2017; Liu et al., 2018).

Finally, individuals' numerical processing abilities under the three mechanisms correlate differentially with their mathematical abilities or achievement. Some studies indicate that individuals' nonsymbolic numerosity (5–16 dots) processing abilities show significant positive correlations with their mathematics achievement (Halberda et al., 2008). More recent research further demonstrates that children' s ability to compare moderate-numerosity (24 dots) stimuli correlates significantly with their mathematical abilities, whereas their subitizing (1–4 dots) ability or breadth and high-density stimulus processing ability (250 dots) show no significant correlations with various mathematics test scores (Anobile et al., 2016; 2019). This research also suggests that subdividing the three numerical processing mechanisms according to stimulus numerosity range helps more accurately investigate the relationship between nonsymbolic numerosity processing abilities and individual mathematical competence.

3.2 Neural Feature Differences Among the Three Numerical Processing Mechanisms

Cognitive neuroscience evidence indicates that the three numerical processing systems have different neural substrates. Studies using event-related potentials (ERP) and functional magnetic resonance imaging (fMRI) have shown that the numerosity mechanism is associated with neural activity in the right parietal lobe (particularly the intraparietal sulcus, IPS) and occipital lobe, while the subitizing mechanism involves attention-related brain regions such as the temporal-parietal junction (TPJ) (Michele & Joonkoo, 2017). Earlier research indicated that density information processing activates the V1-TEO region of visual cortex, with receptive fields in V3 and V4 areas (Kastner et al., 2001).

ERP studies have shown that when observing 1–4 stimuli, specific brain waves appear in the medial occipital lobe starting 75 ms after stimulus presentation, differing from the ERP pattern for moderate numerosities (8–32 dots). At 200 ms post-stimulus, the parietal ERP under the subitizing mechanism shows a negative wave, opposite in polarity to the P2p (second posterior positivity) component in the numerosity mechanism (Michele & Joonkoo, 2017; Park et al., 2016).

P2p is a positive component appearing around 200–300 ms after stimulus presentation in the parieto-occipital region, considered closely related to ANS activity. It selectively reflects the numerical magnitude relationship between stimuli and is modulated by the "distance effect"in numerical comparison: as the ratio of the two compared numerosities approaches 1 (the two values become closer), P2p amplitude increases (Dehaene, 1996), and observers find the task of comparing the two stimuli increasingly difficult.

Theoretically, to identify brain electrical components specific to the numerosity mechanism, all visual features covarying with numerosity must be controlled, which is difficult to achieve (Van Rinsveld et al., 2020). Therefore, new techniques have been employed to investigate this issue. For example, researchers have calculated the sensitivity of brain electrical components (visual-evoked potentials, VEPs, Michele & Joonkoo, 2017; steady-state visual evoked potentials, SSVEP, Van Rinsveld et al., 2020) to changes in stimulus numerosity and other non-numerical covarying visual features. They found that the occipital cortex can respond specifically to changes in numerosity features (Van Rinsveld et al., 2020). When stimulus numerosity or density is moderate (8–32 dots), observers' brain wave amplitude changes (such as P2p) show the highest relative sensitivity to numerosity feature changes (Michele & Joonkoo, 2017).

The aforementioned ERP studies found that the average amplitude of P2p waves when observing high-density stimuli (100–400 dots) is larger than that when observing moderate-density (8–32 dots) stimuli (Michele & Joonkoo, 2017). Compared to moderate numerosity or density conditions, although the P2p component' s amplitude can still reflect changes in stimulus numerosity features when processing high-density stimuli, its relative sensitivity to stimulus numerosity changes decreases, making it less sensitive than the P2p triggered by moderate-density stimuli. These results indicate that observers' sensitivity to numerosity features of high-density stimuli decreases compared to moderate-density stimuli (Michele & Joonkoo, 2017).

4 The Value of the Three Number Processing Systems Hypothesis

The Three Number Processing Systems hypothesis proposed by Anobile and colleagues (2014) and their series of studies have gained widespread recognition, exerted considerable influence, and hold significant academic value.

First, the hypothesis offers theoretical innovation, supplementing and updating existing theories of numerosity processing. The controversy regarding the relationship between numerosity and density processing has persisted in the field. As previously mentioned, earlier theoretical frameworks assumed a single mechanism for numerosity processing beyond the subitizing range. Some researchers argued that observers infer numerosity by analyzing stimulus density or that numerosity and density processing mechanisms are highly correlated (Dakin et al., 2011; Durgin, 1995; 2008; Gebuis et al., 2016). Others maintained that the ANS (i.e., a numerosity-based processing mechanism) serves as the primary mechanism for numerical processing across the entire numerosity range (Burr & Ross, 2008; de Hevia, 2011; Feigenson et al., 2004). The Three Number Processing Systems hypothesis proposes that both numerosity and density mechanisms are used to process numerical information, with the density mechanism substituting for the numerosity mechanism when the latter is constrained by overly dense stimuli. This hypothesis updates our understanding of the relationship between numerosity and density processing and our knowledge of numerical processing mechanisms (Liu et al., 2018; Michele & Joonkoo, 2017; Zimmermann & Eckart, 2018; Liu Wei et al., 2016).

Second, the hypothesis and related research demonstrate methodological innovation by approaching the problem from a new perspective. Rather than seeking to strictly and completely control all visual stimuli covarying with numerosity features, it effectively distinguishes the numerosity and density mechanisms by analyzing participants' response characteristics (the distribution patterns of W scores) during numerical processing tasks, thereby supporting the existence hypothesis of the numerosity mechanism and making an important contribution to resolving the aforementioned "core debate" (Liu Wei et al., 2016).

Third, this research clarifies contradictory experimental results and viewpoints in related studies. It points out that moderate- and large-numerosity stimuli activate different processing mechanisms, whereas previously the mechanisms for moderate and large number processing were confounded. Different researchers using different numerosity ranges in their stimulus materials produced inconsistent results. For example, when stimuli had moderate numerosities (below 30 dots), researchers found that numerosity judgments were unaffected by distribution area (Ross & Burr, 2010; Tokita & Ishiguchi, 2010), but when stimulus numerosities were larger (128 dots), unequal distribution areas significantly affected numerosity judgment accuracy (Dakin et al., 2011). Using the theoretical framework of the Three Number Processing Systems hypothesis, contradictory results arising from different stimulus numerosity ranges in related studies can be explained. Furthermore, the theory proposes a new framework suggesting that both the "numerosity mechanism"and "density mechanism"may be activated during numerical processing activities. Referencing this "multiple mechanisms" framework can lead to the development of more theoretically powerful perspectives. For instance, the "multiple mechanisms" approach may help explain why "numerical processing" is influenced by non-numerical visual features under certain conditions (Buran & Parrish, 2017; Burr, 2008; Dakin et al., 2011; Durgin, 2008; Leibovich et al., 2017).

Finally, this theory points to new directions for future research and has inspired a large body of subsequent studies (Anobile et al., 2014; 2015-2017; 2019; 2020; Castaldi et al., 2019; Cicchini et al., 2016; 2019; Liu et al., 2020; Michele & Joonkoo, 2017; Pomè et al., 2019a; 2019b). As a pioneering theory, it has attracted considerable attention in related fields. Currently, it has received substantial supporting evidence, inspired many new studies, and also faces some challenges (Durgin, 2017; Leibovich et al., 2017; Yousif & Keil, 2020).

5 Challenges to the Three Number Processing Systems Hypothesis

While the Three Number Processing Systems hypothesis has gained widespread recognition among researchers, some scholars have raised questions about it. On one hand, the hypothesis proposes that the numerosity and density mechanisms switch based on experimental phenomena (i.e., as stimulus density or numerosity increases, the Weber fraction W shows a piecewise pattern, shifting from constant to linearly decreasing). The hypothesis infers the emergence of different numerical processing mechanisms through changes in W patterns but cannot provide direct evidence that the mechanism underlying the constant W pattern directly processes numerosity (Durgin, 2017; Leibovich et al., 2017). Therefore, evidence is still needed to support that the "numerosity mechanism" indeed analyzes "numerosity." Directly verifying this question is difficult, but some heuristic research exists. For example, Cicchini et al. (2016) found that when moderate-density (24 dots) stimuli vary across multiple dimensions including numerosity, area, and density, observers show highest sensitivity to changes in the numerosity dimension, indicating they automatically analyze numerosity information in these stimuli. When stimulus density is high (128 dots), observers' sensitivity to density increases (Cicchini et al., 2016; 2019). As previously mentioned, researchers have calculated the sensitivity of brain electrical components to stimulus numerosity and other non-numerical visual covarying features, finding that the occipital cortex can respond specifically to changes in numerosity features (Van Rinsveld et al., 2020). When stimulus numerosity or density is moderate (8-32 dots), observers' P2p amplitude changes show the highest relative sensitivity to numerosity feature changes, whereas this relative sensitivity decreases with high-density stimuli (100-400 dots) (Michele & Joonkoo, 2017). Some studies also indicate that numerosity information can be processed cross-modally (e.g., visual-auditory, visual-tactile channels; Arrighi et al., 2014; Burr, 2017). These studies suggest that under conditions of moderate stimulus numerosity or density, observers do automatically analyze stimulus numerosity, contrasting sharply with high-density conditions, thereby providing some support for the Three Number Processing Systems hypothesis.

On the other hand, the authors propose another challenge: the "numerosity mechanism" as defined by the hypothesis operates within a relatively narrow range. For example, presenting 14 dots in a moderately sized visual field 13° from central fixation (a radius of approximately 4°, a common scenario in numerosity comparison experiments) can exceed the operating range of the numerosity mechanism, which differs from the classic "Approximate Number System (ANS) theory." The ANS is considered to have characteristics of "conforming to Weber's law, automatic activation, and universal operating range," reflecting its robustness and universality and serving as important evidence that the ANS corresponds to an independent processing mechanism forming the basis of numerical cognition in humans and other animals (Dakin et al., 2011; Halberda et al., 2008). The important value of the Three Number Processing Systems hypothesis lies in its distinction between two mechanisms for processing numerosity (named "numerosity mechanism" and "density mechanism"). The hypothesis ultimately aims to demonstrate that the "numerosity mechanism" is an independent numerosity perception mechanism and further clarify that this mechanism is the ANS, thereby resolving the aforementioned core debate in nonsymbolic numerosity cognition. However, under the current theoretical framework, the narrow-range "numerosity mechanism" differs from the ANS. Moreover, the notion that a mechanism with a narrow operating range is independent and important lacks persuasive power. Therefore, future research needs to consider the possibility that the numerosity mechanism in the original hypothesis may be universally activated across the entire numerosity range.

6.1 Summary of the Three Number Processing Systems Hypothesis

The Three Number Processing Systems hypothesis proposes that as stimulus numerosity and density increase, three distinct mechanisms are activated to process stimulus numerosity. The subitizing mechanism analyzes small numerosities (fewer than 5), the numerosity mechanism analyzes moderate numerosities, and when stimulus density exceeds a certain range where observers can no longer perceptually distinguish individual stimuli, the numerosity mechanism becomes constrained and numerical processing shifts to rely on the third mechanism: the density mechanism (Anobile et al., 2014; 2015b).

This hypothesis distinguishes three different processing mechanisms in numerical processing (particularly differentiating the numerosity and density mechanisms), partially reconciling contradictory experimental results and opposing theoretical viewpoints from previous research and offering a new perspective on the relationship between density and numerosity processing, thereby inspiring numerous subsequent studies. Since the proposal of the Three Number

Processing Systems hypothesis, it has gained attention and recognition from researchers in related fields, with a series of studies confirming that numerical processing across different numerosity ranges indeed exhibits distinct behavioral and electrophysiological characteristics. Simultaneously, the hypothesis faces some challenges, with the unresolved issue being that the operating range of the numerosity mechanism in the current hypothesis is too narrow, inconsistent with the characteristic of the classic ANS mechanism having a "universal operating range." Therefore, future research needs to further examine the relationships among the three mechanisms and investigate whether the numerosity mechanism is activated in parallel with the other two mechanisms.

6.2 Outlook: The Possibility of Universal Activation of the Numerosity Mechanism

Future research should investigate whether the numerosity mechanism can be universally activated across the entire numerosity range. Furthermore, future studies should examine whether, within the operating range of subitizing (1–4 stimuli), the subitizing and numerosity mechanisms are activated simultaneously and in parallel, and whether, within the operating range of the density mechanism (when stimulus density exceeds the numerosity-density mechanism switching "inflection point"), the density and numerosity mechanisms are activated simultaneously and in parallel. Investigating these hypotheses holds important theoretical significance. On one hand, as previously mentioned, the "universal activation" hypothesis expands the operating range of the "numerosity mechanism," potentially linking the "numerosity mechanism" with the classic ANS and helping to ultimately demonstrate the independence of the "numerosity mechanism," thereby addressing the core debate in numerical cognition (Leibovich et al., 2017). On the other hand, if the "parallel activation" hypothesis is confirmed based on the "universal activation" hypothesis, then the relationships and specific modes of operation among various numerical processing mechanisms would be clarified: the numerosity mechanism is activated in parallel with the subitizing mechanism in the "small number" range and with the density mechanism in the "large number" range, while cognitive activity flexibly switches among mechanisms, selecting the mechanism whose processing result is more accurate when conditions permit.

The Three Number Processing Systems hypothesis states that for moderate numerosities, nonsymbolic stimulus numerosity processing relies on the "numerosity mechanism," which conforms to "Weber' s law for numerosity perception." Following this logic, to reveal the existence of the "numerosity mechanism" beyond the aforementioned numerosity range, it is necessary to demonstrate the existence of "Weber' s law for numerosity processing" in new numerosity ranges (small numerosity range: 1–4; large numerosity range should be distant from the "numerosity mechanism" range, e.g., >150 dots for central vision conditions, >75 dots for peripheral vision conditions).

In the small numerosity range, numerosity and subitizing mechanisms may coexist and be activated in parallel. Under dual-task paradigms with attentional load, "2–4 dot numerosity estimation" tasks are affected by the primary task, with errors rising to approximately 20% of stimulus numerosity (equivalent to W = 0.2), approaching the processing level of the numerosity mechanism (Burr et al., 2010). In "simultaneous processing" paradigms requiring participants to analyze the numerosity of two or more stimulus sets simultaneously, attentional load is higher than in sequential tasks analyzing only one object at a time. Under these conditions, reaction errors in "1–4 dot numerosity estimation" increase with stimulus numerosity, conforming to Weber' s law for numerosity perception (W 0.2; Liu et al., 2020). In "masking" paradigms (which strictly control target presentation time through masking), when target presentation time is only 33–50 ms, errors in "1–4 dot numerosity estimation" tasks show a Gaussian distribution centered on the stimulus value: the probability of estimating the true value is highest, decreasing as estimates deviate further from the true value. This approximate processing pattern reflects characteristics of the numerosity mechanism. Precise subitizing processing only emerges when stimulus presentation lasts 100–150 ms (Melcher et al., 2020). These findings indicate that the numerosity mechanism is capable of analyzing stimulus numerosity within the small number range.

Notably, while the numerosity mechanism has been shown to be activatable in small numerosity range processing, the aforementioned experiments all involve some form of cognitive resource limitation (working memory or attentional load, or limited processing time). The question remains whether the numerosity mechanism can also be automatically activated when processing small numbers under conditions of abundant cognitive resources. This is plausible: research indicates that the numerosity mechanism requires minimal cognitive resources and exhibits automatic activation characteristics (Anobile et al., 2020; Pomè et al., 2019b). Multiple numerosity mechanisms can be activated in parallel without interference, and adults can simultaneously analyze the numerosity features of three or more stimulus sets (Halberda et al., 2006; Liu et al., 2020). From the aforementioned "masking" studies, the numerosity mechanism can complete numerosity analysis of small numbers when stimuli are presented for only 33 ms (Melcher et al., 2020). Within such a short time, mechanism switching is unlikely (i.e., resource limitations would prompt switching from the subitizing mechanism to the numerosity mechanism). Therefore, within the small number range, the numerosity mechanism more likely always activates automatically and in parallel with the subitizing mechanism, rather than only being activated through mechanism switching under cognitive resource constraints. In summary, it can be hypothesized that when processing small numerosities (fewer than 5), the numerosity and subitizing mechanisms are activated in parallel. If cognitive resources are sufficient, cognitive activity will rely on the more accurate subitizing mechanism; when cognitive resources cannot meet the high demands of the subitizing mechanism, numerical processing will shift to rely on the numerosity mechanism.

The authors propose that in the high-density range, the numerosity and density mechanisms may also be activated in parallel. First, this hypothesis is theoretically plausible. The Three Number Processing Systems hypothesis suggests that switching from the numerosity mechanism to the density mechanism occurs because high density prevents observers from distinguishing individual stimuli, limiting or affecting the effectiveness of the numerosity mechanism (Anobile et al., 2015b). This view requires further examination. As density increases, the regularity of stimulus distribution also increases. Regularity reflects variation in density; higher regularity means less variation, better representativeness of the mean density, and higher accuracy of density processing. Previous research has shown that even when stimulus density has not reached the numerosity-density mechanism switching "inflection point," increased distribution regularity can still prompt switching from the numerosity mechanism to the density mechanism (Liu et al., 2017; 2018). This may occur because as stimulus regularity increases, the accuracy of the density mechanism surpasses that of the numerosity mechanism, leading the cognitive system to select the more accurate density mechanism. Therefore, numerosity-density mechanism switching may not occur because high-density conditions limit the effectiveness of the numerosity mechanism (Anobile et al., 2015b), but rather because under high-density conditions, the density mechanism is more accurate. At this point, the numerosity mechanism still exists but is not selected. This provides a theoretical basis for the hypothesis of "parallel activation of numerosity and density mechanisms."

Second, existing experimental research suggests that the numerosity and density mechanisms may be activated in parallel. In numerosity adaptation studies (where viewing large-numerosity stimuli causes underestimation of subsequently presented small-numerosity stimuli, and vice versa), observing and adapting to high-density adaptation stimuli of 200–1500 dots should activate the density mechanism, yet adaptation affects subsequent processing of moderate- and low-density stimuli (16–40 dots), which rely on the numerosity mechanism (Burr & Ross, 2008; Durgin, 1995; 2008; 2017; Liu Wei et al., 2012). According to the Three Number Processing Systems hypothesis or ANS theory, the density and numerosity mechanisms are independent (Anobile et al., 2014; Feigenson et al., 2004). Why, then, does density mechanism activity affect the subsequent numerosity mechanism? To explain this phenomenon, one can hypothesize that during the adaptation phase, high-density stimuli simultaneously activate both numerosity and density mechanisms, thereby affecting the numerosity mechanism in the subsequent test phase. In other words, when processing large-numerosity stimuli, the density and numerosity mechanisms may be activated in parallel. When cognitive resources are sufficient, cognitive activity relies on the more accurate density mechanism; when resources cannot meet the demands of the density mechanism, numerical processing may similarly shift to rely on the numerosity mechanism.

Currently, the numerosity-density parallel activation hypothesis lacks direct experimental evidence. Recent research indicates that compared to the numerosity mechanism, the density mechanism has higher attentional resource demands (Anobile et al. 2020; Pomè et al., 2019b). When observers compare large-numerosity stimuli, the pattern of Weber fractions W under "attentional load" conditions changes compared to the "no-load" condition: under load, W still decreases linearly with increasing stimulus density, but the "slope" of this decrease becomes significantly shallower (the absolute value of the fitted line's slope decreases), approaching the constant W pattern observed within the "numerosity mechanism" range (fitted line slope of 0; Pomè et al., 2019b). This research suggests that under appropriate cognitive load conditions, the numerosity mechanism may also be revealed in large-numerosity processing. Of course, future research requires extensive experimental investigation to verify the parallel activation hypothesis. On one hand, the degree of cognitive load needs to be adjusted to attempt to reveal the "numerosity mechanism" with constant W in large-numerosity processing. On the other hand, multiple paradigms and more direct methods are needed to verify the possibility of parallel activation of numerosity and density mechanisms. For example, "simultaneous processing" and "masking" paradigms may be used. It can be hypothesized that inducing appropriate cognitive load is an effective approach to suppress subitizing or density mechanisms, thereby verifying whether the numerosity mechanism exhibits universal activation. If the numerosity mechanism can be universally activated, then when subitizing or density mechanisms are compromised, numerical processing within the ranges of these mechanisms will rely on the numerosity mechanism. At this point, the various differences among the three mechanisms mentioned earlier (Anobile et al., 2016; 2017; 2019; Cicchini et al., 2016; 2019; Liu et al., 2017; 2018; Michele & Joonkoo, 2017; Pomè et al., 2019a; 2019b) may all change, and processing characteristics may converge toward those of the numerosity mechanism.

In summary, the Three Number Processing Systems hypothesis distinguishes three different processing mechanisms in numerical processing, partially reconciling contradictory theoretical viewpoints and experimental evidence regarding "numerosity processing" accumulated over the years, and offering a completely new perspective on the relationship between density and numerosity processing. Since its proposal, this hypothesis has gained widespread recognition from researchers and inspired a large number of follow-up studies. Currently, the theory has received much supporting evidence while also facing some challenges. Future research must further examine the relationships among the three mechanisms and investigate whether the numerosity mechanism is activated in parallel with the other two mechanisms.

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Three number processing systems: Different features and parallel activation

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Abstract: Distinct mechanisms are involved in number processing of nonsymbolic stimuli. Small numbers (1~4) can be appraised rapidly and errorlessly based on the activation of subitizing system. Moderate numbers are proposed to be processed spontaneously with a constant error rate of about 20% due to the activity of numerosity system. Typically, Weber’ s law for number perception is demonstrated in this number range. For large numbers, the stimulus number relationship is suggested to be inferred indirectly via density analysis, and number processing, which is fast and has an error rate proportional to the square root of the stimulus number, is mediated by density system. Different behavioral and ERP features are revealed among number tasks based on these three systems. It is proposed that parallel activation exists between subitizing and numerosity systems as well as between density and numerosity systems. Cognition relies on the system whose processing result is more precise.

Key words: subitizing system, numerosity system, density system, parallel activation

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