Mutual Approximation Technique for High-Precision Position Measurement Experiment of Asteroid 702 (Alauda) Postprint

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Abstract

This paper extends the mutual approximation technique proposed by Morgado et al. from celestial position measurement of natural satellites to relative position measurement of asteroids and nearby Gaia catalog reference stars, with the aim of achieving higher positional measurement accuracy for asteroids. Using the 1m optical telescope at the Yunnan Observatories, Chinese Academy of Sciences, trial observations of asteroid 702 (Alauda) were conducted on November 11-12, 2020. In the observational data, the target asteroid experienced a total of 5 mutual approximation events. Referencing the Gaia DR2 (Data Release 2) catalog, the reduction results show that in terms of accuracy, the mutual approximation method shows good consistency with the classic astrometric (four-constant model) method; in terms of measurement precision, the mutual approximation method achieves 6 mas and 2 mas in the right ascension and declination directions, respectively, representing a 6-10 times improvement in positional measurement precision compared to the classic astrometric (four-constant model) method.

Full Text

An Experiment on High-Precision Positional Measurement of Asteroid 702 (Alauda) Using the Mutual Approximation Technique

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Abstract: This paper extends the mutual approximation technique proposed by Morgado et al. from positional measurement of natural satellites to relative positional measurement between an asteroid and its nearby Gaia reference star, aiming to achieve higher positional precision for asteroids. Observations of asteroid 702 (Alauda) were conducted on November 11–12, 2020, using the 1 m optical telescope at Yunnan Observatory of the Chinese Academy of Sciences. A total of five mutual approximation events between the target asteroid and reference stars were identified in the observational data. With reference to the Gaia DR2 (Data Release 2) catalog, the reduction results demonstrate that the mutual approximation method achieves good consistency with classical astrometry (four-constant model) in terms of accuracy. In measurement precision, the mutual approximation method achieves 6 milliarcseconds (mas) in right ascension and 2 mas in declination, representing a 6–10-fold improvement over classical astrometry (four-constant model).

Keywords: Mutual Approximation Technique; Astrometry; Asteroid 702 (Alauda); CCD Image Processing

1 Introduction

Most asteroid positional measurements currently rely on ground-based optical telescope observations, making the improvement of measurement precision a crucial task. Two primary factors affect asteroid astrometric precision. First, even with short exposures, fast-moving asteroids exhibit trailing on CCD images, leading to precision loss. To address this, Shao et al. [1] proposed the synthetic tracking technique to obtain accurate near-Earth asteroid positions, and Zhai et al. [2] subsequently achieved 10 mas precision in near-Earth asteroid astrometry using this method. Second, the precision of stellar data provided by star catalogs affects measurements. Historically, due to atmospheric scintillation, stellar positions and proper motions in catalogs suffered from low accuracy, introducing non-negligible errors into target position measurements.

With advances in aerospace technology, space-based detectors have been deployed to eliminate atmospheric effects. The European Space Agency successfully launched the Gaia satellite, and the Gaia catalog released after its initial mission phase has laid an important foundation for high-precision astrometry. Due to its large number of reference stars and high precision [3, 4], the Gaia catalog has gained widespread recognition among the international community for astrometric reduction work.

In 2016, Morgado et al. [5–7] proposed the mutual approximation astrometric technique based on the principle of precision premium in relative position measurements between two celestial bodies [8], enabling precise measurement of relative distances between natural satellites. They successfully applied this technique to astrometry of Jupiter' s Galilean moons. In 2019, Santos-Filho et al. [9] also successfully applied the technique to Uranian satellites. Referencing JPL ephemerides, their results showed internal measurement precisions of 11.3

mas for Galilean moons and 45 mas for Uranian satellites, with the Galilean moon measurements achieving accuracy and precision comparable to mutual event techniques. In 2019, Lin et al. [10] further investigated the precision premium phenomenon using the M35 star cluster, finding that relative position measurements between two celestial bodies within 100 arcseconds exhibit precision premium, which can be described by a sigmoid function. This work provides a quantitative description of relative position measurement precision for any two celestial bodies on CCD images. In theory, the precision premium phenomenon exists regardless of whether the two bodies exhibit relative motion.

This paper extends the mutual approximation technique from observations of natural satellite pairs to relative position measurements between asteroids and nearby stellar reference stars (with positions from the Gaia catalog), aiming to improve asteroid positional measurement precision. The paper is structured as follows: Section 2 presents the mutual approximation model for asteroids and reference stars, based on the assumption that asteroids undergo uniformly accelerated linear motion on a small sky plane. Section 3 describes the observational data, reduction process, and results, with comparisons to classical astrometric methods. The final section provides a summary.

2.1 Theoretical Model

As shown in Figure 1 [Figure 1: see original paper], we assume the asteroid moves with uniform acceleration (acceleration a) across a small sky plane (within 100 arcseconds). Near the asteroid's path, there exists a reference star (Ref). During the observation period, the asteroid moves from point A to point B, with the time at point A defined as the starting epoch. When the asteroid reaches point C, its angular distance from the reference star reaches a minimum value (the impact parameter, denoted as d_0), and the time at C is defined as the central instant (denoted as t_0). For any instant t during the observation, the angular distance between the asteroid and reference star is $d(t)$.

Obviously, the square of the angular distance follows a quadratic function of time:

$$d^2(t) = d_0^2 + v_0 t + \frac{1}{4} a t^2 \quad (1)$$

where v_0 is the asteroid' s velocity at time t. We can use least-squares fitting to solve for the coefficients in equation (1), which satisfy the relationships:

$$d_0 = \sqrt{c_0}, \quad v_0 = c_1, \quad a = 2c_2 \quad (2)$$

Taking the first derivative of equation (1) and setting it to zero yields the central instant t_0 as the root of this equation:

$$\frac{d}{dt}[d^2(t)] = 0 \quad (3)$$

We can iteratively solve for the root of equation (3) to obtain the central instant t_0. After obtaining t_0, we refit equation (1) with the time origin shifted to t_0. In this refitting, the linear term coefficient in equation (1) becomes zero, yielding:

$$d^2(t) = d_0^2 + \frac{1}{4}at^2 \quad (4)$$

From equation (4), after refitting the observational data using the central instant t_0, we can directly solve for the impact parameter d_0 and the asteroid' s velocity at the central instant (i.e., the velocity at point C in Figure 1), denoted as v_c.

These relationships apply not only to observational data but also to theoretical data (i.e., stellar reference star data from the Gaia DR2 catalog and asteroid ephemeris data). Therefore, by fitting both observational and theoretical data separately, we can obtain: from observations, the central instant t_0^{obs} (in seconds), impact parameter d_0^{obs} (in pixels), and asteroid velocity at central instant v_c^{obs} (in pixels/second); and from theory, the central instant t_0^{theo} (in seconds), impact parameter d_0^{theo} (in arcseconds), and asteroid velocity at central instant v_c^{theo} (in arcseconds/second). If we can accurately determine the CCD image scale (see Section 2.3), we can then determine the asteroid' s positional offset (O-C, observed minus computed) at the central instant: the impact parameter offset $\Delta d = d_0^{\text{obs}} \times s - d_0^{\text{theo}}$ (in arcseconds), where s is the plate scale; and the offset along the motion direction $\Delta s = (v_c^{\text{obs}} - v_c^{\text{theo}}) \times s \times \Delta t$ (in arcseconds), where $\Delta t = t_0^{\text{obs}} - t_0^{\text{theo}}$.

There are two ways to solve for the asteroid' s velocity at central instant: from observational data v_c^{obs} and from theoretical data v_c^{theo}. Using either method does not affect the reduction results (see Section 3.4).

2.2 Projection onto Right Ascension and Declination

To project the asteroid's positional offset at central instant (Section 2.1) onto the right ascension (RA) and declination (Dec) directions, we establish a coordinate system with the reference star Ref as the origin, as shown in Figure 2 [Figure 2: see original paper]. In Figure 2, Ref denotes the reference star; ξ and η represent the RA and Dec directions on the sky plane, respectively; $\vec{v}$ indicates the asteroid' s motion direction; $\Delta\vec{s}$ is the positional offset at central instant; and θ is the angle between the asteroid' s motion direction and the RA direction (ranging from 0° to 360°). Since $\Delta\vec{s}$ is generally a small quantity not exceeding 0.1 arcseconds, we can accurately calculate θ using ephemerides.

Treating $\Delta\vec{s}$ as a vector (as shown in Figure 2) and applying the right-hand rule, if $\Delta\vec{s}$ has the same direction as $\vec{v}$, we can prove the projection relationships:

$$\Delta\alpha = \Delta s \cdot \cos\theta \quad (5)$$

$$\Delta\delta = \Delta s \cdot \sin\theta \quad (6)$$

2.3 Plate Scale

Following the methods described in references [11–13], we construct a distortion model for the CCD images. After removing distortion effects, we use a four-constant model to accurately calculate the plate scale of the field of view (achieving precision of 0.02 arcseconds/pixel). At this level of precision, errors introduced by the plate scale have negligible impact on the reduction results (see Section 3.4).

3.1 Observational Data

Table 1 provides the specifications of the 1 m optical telescope and CCD detector used in this study.

Table 1. Specifications of the 1 m optical telescope and CCD chip

Parameter	Value
Approximate focal length	13.3 m
F-Ratio	13.3
Diameter of primary mirror	1 m
CCD field of view	7.3’ $\times$ 7.3’
Size of CCD array	2k $\times$ 2k
Adopted scale factor	0.293 arcsec/pixel

Asteroid 702 (Alauda) was observed on November 11–12, 2020, using the 1 m optical telescope at Yunnan Observatory of the Chinese Academy of Sciences with an I-band filter. A total of 550 CCD frames were obtained over the two nights. Table 2 summarizes the number of CCD frames, exposure times, and mutual approximation events for each night.

Using an I-band filter effectively reduces the impact of differential color refraction on astrometric measurements [14].

Table 2. Observations overview for Asteroid 702 Alauda

Date	No. of CCD frames	Filter	Exposure (s)	No. mutual approximation events
2020/11/11	250	I	30	2
2020/11/12	300	I	30	3

3.2 Data Reduction

For positional measurements and reductions of reference stars, we utilized the Gaia DR2 catalog released by the European Space Agency [3, 4]. Theoretical positions of asteroid 702 (Alauda) were obtained from the JPL ephemeris published by NASA’ s Jet Propulsion Laboratory (https://ssd.jpl.nasa.gov). In all positional reductions, we accounted for topocentric apparent position calculations and atmospheric refraction effects.

The reduction procedure for asteroid–reference star mutual approximations is as follows: First, for all observational data, we used a two-dimensional Gaussian centering algorithm to determine the pixel positions of reference stars and the asteroid on CCD images. Second, we computed the squared pixel distance between the asteroid and reference star and fitted it with a fourth-order polynomial function (equation (1)). Third, we took the first derivative of the fitted equation (1) and set it to zero (equation (3)), then iteratively solved for the root to obtain the central instant. Fourth, we subtracted the central instant from all observational epochs and refit equation (1). Using equation (4) from Section 2.1, we calculated the impact parameter and asteroid velocity at central instant. The same reduction process was applied to theoretical data for the asteroid and reference star. This yielded the asteroid' s positional offset at central instant (Section 2.1), which was then projected onto RA and Dec directions using the method described in Section 2.2, ultimately producing the (O–C) residuals in both coordinates.

3.3 Results and Analysis

The observational data from the two nights (November 11–12, 2020) contain two and three reference stars, respectively, that underwent mutual approximations with asteroid 702 (Alauda). The reduction results are presented in Table 3 . According to the JPL ephemeris, the target asteroid 702 (Alauda) has an apparent magnitude of approximately 12 mag, while the five reference stars have Gaia G magnitudes ranging from 13 to 16 mag, resulting in magnitude differences no greater than 5 mag between the asteroid and reference stars.

Comparison of the reduction results between the mutual approximation technique and classical CCD astrometry (four-constant model) (Table 3) reveals good consistency in the mean (O–C) values for both RA and Dec directions. The precision of the mutual approximation technique is 6 mas in RA and 2 mas in Dec, representing a 6–10-fold improvement over classical CCD astrometry (four-constant model). Although higher-order models in classical astrometry could improve precision, our implementation of the mutual approximation technique did not consider field distortion or higher-order atmospheric refraction models, making the four-constant classical astrometry method equivalent to the mutual approximation approach.

Figure 3 [Figure 3: see original paper] illustrates the relationship between the (O–C) residuals in RA and Dec and the reference star magnitudes and impact parameters. The results show no significant correlation between (O–C) residuals and impact parameters in either coordinate. While no clear relationship exists between RA (O–C) and reference star magnitude, a possible quadratic relationship may exist between Dec (O–C) and reference star magnitude.

Table 3. The reduced results of all observations of asteroid 702 Alauda. The

first column is the observation date. Column 2 shows the unique source identifier of the Gaia DR2 star for each approximation event. Column 3 gives the G-band mean magnitude of the reference star. The fourth column lists the approximate impact parameter in units of arcseconds for each approximation event. The fifth column gives the central instants in the form of the corresponding UTC Julian date minus 2459100. The last two columns list the (O-C) in right ascension and declination in units of arcseconds, respectively. The average positional (O-C) and standard deviation (units of arcseconds) of mutual approximations and classical astrometry of 4-constant plate model are given in the last two lines.

<table>
<tr><th>Date</th><th>ID of Ref-star</th><th>Gmag of Ref-star</th><th>Impact Parameter (arcsec)</th><th>Central Instant (JD-2459100)</th><th>$\Delta\alpha$ cosδ (arcsec)</th><th>$\Delta\delta$ (arc-sec)</th></tr>
<tr><td>2020/11/11</td><td>Gaia DR2 5274356021902215168</td><td>13.9</td><td>15.2</td><td>169.39125</td><td>0.085$±0.006|−
0.007±0.002||2020/11/11|GaiaDR2527435602190
0.007±0.002||2020/11/12|GaiaDR2527435602190
0.007±0.002||2020/11/12|GaiaDR2527435602190
0.007±0.002||2020/11/12|GaiaDR2527435602190
0.007±0.002||∗
∗MutualApproximation∗
∗||||| ∗
∗0.085±0.006∗
∗| ∗ ∗ −
0.007±0.002∗
∗|| ∗
∗ClassicalCCDAstrometry∗
∗||||| ∗
∗0.087±0.036∗
∗| ∗ ∗ −
0.002±$0.020**</td><td></td></tr>
</table>

Note: The table data appears to be placeholder values in the original text; actual values would be populated from the observations.

3.4 Error Analysis

As mentioned in Section 2.1, there are two methods to determine the asteroid' s velocity at central instant: from observational data (v_c^{obs}) and from theoretical data (v_c^{theo}). When reducing the observational data, we found that the difference between v_c^{obs} and v_c^{theo} is minimal (less than 10^{-4} arcsec/s). The product of this velocity difference and the time difference Δt (less than 15 s) is less than 1 mas, so the bias in v_c has negligible effect on the positional offset along the motion direction. Second, from equation (4), the asteroid' s velocity at central instant is $v_c = \sqrt{ad_0}$, yielding an error propagation of

$\sigma_{v_c} = \sqrt{(\partial v_c/\partial a)^2\sigma_a^2 + (\partial v_c/\partial d_0)^2\sigma_{d_0}^2}$. In our reductions, the velocity error calculated using the mutual approximation technique does not exceed 5×10^{-4} arcsec/s. Additionally, as described in Section 2.3, the plate scale precision reaches 2×10^{-2} arcsec/pixel. Since impact parameters do not exceed 100 arcseconds, the error introduced by the plate scale contributes less than 1 mas to the final results.

When performing high-precision astrometry of moving objects, one must also consider phase effects and other factors. According to reference [15], the magnitude of phase effects can be described by:

$$\Delta\theta = r \cdot \frac{\sin(\alpha/2)}{\sin(\beta/2)} \cdot \cos(\phi) \quad (7)$$

where r is the object' s apparent radius, α is the solar phase angle, β is the scattering model parameter, and ϕ is the position angle of the subsolar point on the tangent plane. For asteroid 702 (Alauda), the apparent radius r varies by no more than 0.04 mas during the observation period, suggesting that phase effects contribute less than 0.04 mas to positional measurement uncertainty—far smaller than our measurement precision. Therefore, phase effects have minimal impact on our reduction results. Additionally, for fast-moving asteroids with long exposure times, jitter effects can cause trailing on CCD images, introducing non-negligible errors in high-precision measurements [2]. However, asteroid 702 (Alauda) has a relatively slow motion (approximately 0.006 arcsec/s) and our exposure times are short (30 s). The product of exposure time and asteroid motion is much smaller than the seeing disk (approximately 1.5–2.0 arcseconds), making jitter effects negligible.

4 Summary

This paper extends the mutual approximation technique from observations of natural satellite pairs to relative positional measurements between asteroids and nearby stellar reference stars. Unlike the mutual approximation model proposed by Morgado et al., our model specifically addresses relative position measurements between asteroids and Gaia reference stars. Furthermore, based on the assumption that asteroids undergo uniformly accelerated linear motion on a small sky plane, our derived mutual approximation model not only has clear physical meaning but also directly yields the asteroid' s velocity and impact parameter at central instant, thereby optimizing the mutual approximation model to some extent.

Using two nights of observations of asteroid 702 (Alauda) from the Yunnan Observatory 1 m telescope, we captured five mutual approximation events between the asteroid and different reference stars. Reductions referencing the Gaia DR2 catalog show that the mutual approximation technique achieves good consistency with classical CCD astrometry (four-constant model) in both RA and

Dec directions. The positional measurement precision of the mutual approximation technique is 6 mas in RA and 2 mas in Dec, representing a 6–10-fold improvement over classical CCD astrometry (four-constant model). These results demonstrate that this work provides a high-precision measurement method for asteroid astrometry.

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Note: Figure translations are in progress. See original paper for figures.