Measurement Method for Polarization Characteristics of Protective Aluminum Coatings on Astronomical Mirrors (Postprint)

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Abstract

The instrumental polarization effects of telescopes are primarily caused by the polarization effects of reflection from metallic mirrors, constituting a major technical challenge currently faced by high-precision astronomical polarimetry. In order to eliminate the influence of instrumental polarization on polarimetric measurement accuracy, precise and comprehensive measurement and investigation of the polarization characteristics of metal-coated materials in the laboratory becomes essential. This paper proposes a method for measuring the polarization characteristics of protected aluminum coatings, which employs two photoelectric modulation methods to directly measure the key elements of the Mueller matrix of protected aluminum coatings. Based on a bilayer structural model, it retrieves optical parameters using measurement data at multiple incident angles and wavelengths without dependence on dispersion models or parameter assumptions. The retrieval results of the proposed method are consistent with those obtained by traditional methods, while the proposed method can provide better characterization of polarization characteristics.

Full Text

Measurement Method for Polarization Properties of Protected Aluminum Coatings on Astronomical Mirrors

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Abstract

The instrumental polarization effect of telescopes, primarily caused by the polarization-dependent reflection from metallic mirror coatings, represents the principal technical challenge facing modern high-precision astronomical polarimetry. To eliminate the impact of instrumental polarization on measurement accuracy, precise and comprehensive laboratory characterization of the polarization properties of metal-coated surfaces is essential. This paper proposes a measurement method for the polarization properties of protected aluminum coatings that directly measures key elements of the Mueller matrix using two photoelectric modulation schemes. Based on a double-layer structural model, the method retrieves optical parameters using multi-angle, multi-wavelength measurement data without relying on dispersion models or parameter assumptions. The retrieval results obtained by our method are consistent with those from conventional approaches, while providing superior characterization of the polarization properties.

Keywords: polarimetry; instrumental polarization; polarization properties of protected aluminum coating; Mueller matrix; retrieval of optical parameters

Polarimetry serves as a powerful observational tool in astrophysical and solar physics research, enabling direct acquisition of crucial physical and geometric information about astronomical targets that cannot be obtained through other observational means. For example, solar telescopes diagnose magnetic field structures in the solar atmosphere through spectropolarimetric observations [?], while the polarized radiation from stars contains even richer physical information [?]. Due to the intrinsically low polarization of most astronomical targets, astronomical polarimetry always operates in a photon-starved regime. Consequently, utilizing large-aperture telescopes to collect more photons and improve the signal-to-noise ratio of polarization signals has become an important development direction for astronomical polarimetry.

The instrumental polarization effect of telescopes—defined as the modification of the incident polarization state by the telescope's optical system—constitutes the primary technical challenge for current high-precision astronomical polarimetry. For large-aperture reflecting telescopes, instrumental polarization is mainly induced by the polarization-dependent reflection from metallic mirror surfaces. According to Fresnel equations, the amplitude attenuation and phase retardance of metal mirrors are functions of the incident angle and the refractive index of the coating material. Microscopically, metal films consist of randomly stacked metal crystallites with linear dimensions of several tens of nanometers, and their optical properties depend on the coating material, deposition process, operating environment, and service lifetime [?]. Therefore, to accurately understand the polarization characteristics of metal thin films, enable proper polarimetric optical design or modeling of telescopes, and ultimately eliminate the influence of instrumental polarization on measurement accuracy, we must conduct precise and comprehensive laboratory measurements of the polarization properties of metal coating materials.

Aluminum-coated mirrors are commonly employed in large-aperture telescopes, and their polarization properties have been extensively studied. Shortly after deposition, a native aluminum oxide film forms on the surface of pure aluminum coatings [**?**, **?**]. While this oxide layer provides protection for the aluminum, it also alters the polarization characteristics of the mirror, and its presence must be considered when characterizing the mirror's polarization properties [**?**]. However, the native oxide film cannot prevent contamination from moisture and other agents, so an additional protective silicon dioxide layer is typically deposited on top of the aluminum coating [**?**, **?**], which further significantly changes the polarization properties.

Conventional ellipsometric measurement methods fit spectroscopic ellipsometry data acquired at fixed incident angles. These approaches employ dispersion models to constrain the refractive index at each wavelength \cite{8–10} and sometimes utilize known literature values to restrict parameter variations. Consequently, the accuracy of parameters retrieved by traditional methods heavily depends on the choice of dispersion model. Moreover, due to reduced degrees of freedom and parameter constraints, complex samples require extremely sophisticated structural models to achieve adequate characterization. This paper proposes a polarization property measurement method for protected aluminum coatings that first directly measures the key elements of the Mueller matrix and then retrieves optical parameters using the measured Mueller matrix data based on a double-layer structural model, without dependence on dispersion models or parameter assumptions.

1. Measurement Principle and Method

The polarization state of astronomical targets can be represented by the Stokes vector:

$$\mathbf{S}_{\mathrm{in}} = (I_{\mathrm{in}}, Q_{\mathrm{in}}, U_{\mathrm{in}}, V_{\mathrm{in}})^T$$

where I_{in} represents the total intensity of the incident light, Q_{in} and U_{in} are intensity parameters for orthogonal linear polarization components, V_{in} is the intensity parameter for circular polarization, and the superscript T denotes matrix transposition. After reflection from a mirror surface, the Stokes vector becomes:

$$\mathbf{S}_{\mathrm{out}} = \mathbf{M}_{\mathrm{mirror}} \cdot \mathbf{S}_{\mathrm{in}}$$

where $\mathbf{M}_{\mathrm{mirror}}$ is the Mueller matrix of the mirror, which can be expanded as:

$$\mathbf{M}_{\mathrm{mirror}} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}$$

The Mueller matrix elements can be classified into three categories based on their subscripts [?]: m_{12}, m_{13}, m_{14} represent instrumental polarization (diattenuation); m_{21}, m_{31}, m_{41} represent instrumental crosstalk; and $m_{22}, m_{23}, m_{24}, m_{32}, m_{33}, m_{34}, m_{42}, m_{43}, m_{44}$ represent instrumental depolarization.

When the incident and emergent Stokes vectors are defined in the intrinsic coordinate system of the mirror reflection transformation as shown in [Figure 1: see original paper], where the coordinate axes correspond to the s- and p-polarization components of light, the Mueller matrix for metallic mirror reflection can be simplified to [?]:

$$\mathbf{M}_{\text{mirror}} = \begin{pmatrix} 1 & -\cos 2\psi & 0 & 0 \\ -\cos 2\psi & 1 & 0 & 0 \\ 0 & 0 & \sin 2\psi \cos \Delta & \sin 2\psi \sin \Delta \\ 0 & 0 & -\sin 2\psi \sin \Delta & \sin 2\psi \cos \Delta \end{pmatrix}$$

where ψ and Δ are ellipsometric parameters. Considering the symmetry of the metallic reflection Mueller matrix, its polarization properties are primarily characterized by three terms: $-\cos 2\psi$, $\sin 2\psi \cos \Delta$, and $\sin 2\psi \sin \Delta$. Therefore, in subsequent discussions, the Mueller matrix refers specifically to these three non-zero elements.

Experimental data acquisition was performed using a commercial phase-modulation ellipsometer (UVISEL-Plus, Horiba), whose optical configuration is shown in [Figure 2: see original paper]. Light from a xenon lamp source passes through a polarizer (Rochon prism) and impinges obliquely on the sample surface. Due to the sample's polarization properties, the polarization state of the reflected light changes. After modulation by a photoelastic modulator (PEM) and an analyzer (Rochon prism), the light is detected by a spectrometer, yielding three measured quantities I_s, I_c, and I_n. These quantities are functions of the polarizer azimuth P, modulator azimuth M, and analyzer azimuth A, expressed as [?]:

$$\begin{aligned} I_s &= \sin 2(P-M) \sin 2(A-M) \sin \Delta \sin \delta \\ I_c &= \sin 2(P-M) \sin 2(A-M) \cos \Delta \sin \delta \\ I_n &= 1 - \cos 2(P-M) \cos 2(A-M) - \sin 2(P-M) \sin 2(A-M) \cos \delta \end{aligned}$$

where δ is the modulation amplitude of the PEM.

Traditional methods require converting the measured I_s, I_c, and I_n data into ellipsometric parameters, which are then used to determine sample properties. However, solving for ellipsometric parameters involves inverse cosine or inverse sine operations with a domain restricted to $[-1, 1]$. Due to measurement errors, the calculations can exceed this domain and produce errors. For computational accuracy, we can directly study the polarization properties of aluminum mirrors by fitting the I_s, I_c, and I_n data. Equation (3) shows that by adjusting the azimuths of the polarizer, modulator, and analyzer, we can obtain different measurement datasets. Comparing with Equation (2), the non-trivial elements of the aluminum mirror's Mueller matrix can be obtained through two modulation configurations:

Configuration 1: $P = 45^\circ$, $M = 0^\circ$, $A = 45^\circ$, yielding measured data:

$$I_{s1} = \sin 2\psi \sin \Delta, \quad I_{c1} = \sin 2\psi \cos \Delta, \quad I_{n1} = 1 - \cos 2\psi$$

Configuration 2: $P = 0^\circ$, $M = 45^\circ$, $A = 45^\circ$, yielding measured data:

$$I_{s2} = -\cos 2\psi, \quad I_{c2} = 0, \quad I_{n2} = 1$$

This approach enables direct measurement of Mueller matrix elements. Compared to traditional ellipsometers that measure ellipsometric parameters, this method provides more comprehensive information about the mirror. The measured data from the aluminum mirror can be expressed in Mueller matrix form:

$$\mathbf{M}_{\text{meas}} = \begin{pmatrix} 1 & I_{s2} & 0 & 0 \\ I_{s2} & 1 & 0 & 0 \\ 0 & 0 & I_{c1} & I_{s1} \\ 0 & 0 & -I_{s1} & I_{c1} \end{pmatrix}$$

To thoroughly investigate the polarization properties of aluminum mirrors, we require sufficient information. Given the instrument's measurement capabilities, we collected spectroscopic data from an aluminum mirror sample (PF05-03-G01, Thorlabs) at multiple incident angles (range: 50–80°, interval: 2°) across a wavelength range of 400–1200 nm (interval: 25 nm). [Figure 3: see original paper] displays the measured key elements of the Mueller matrix, with the vertical axis representing the magnitude of each element. These three Mueller matrix elements exhibit significant spectral variations with changing incident angle.

2. Data Analysis

The polarization properties of aluminum mirrors with protective silicon dioxide coatings are more complex than those of bare aluminum films. [Figure 4: see original paper] illustrates the ideal layered structure of a protected aluminum coating, where n_0 is the refractive index of air, $\tilde{n}_1$ is the complex refractive index of aluminum (defined as $\tilde{n}_1 = n_1 - ik_1$), n_2 is the refractive index of silicon dioxide, and d is the thickness of the oxide layer. The incident angle in air is θ_0, and while θ_1 is complex and no longer represents a simple refraction angle, it still obeys Snell's law. The refraction angle within the oxide layer is θ_2.

The Fresnel reflection coefficients for this structure are:

$$r_{01p} = \frac{n_1 \cos\theta_0 - n_0 \cos\theta_1}{n_1 \cos\theta_0 + n_0 \cos\theta_1}, \quad r_{01s} = \frac{n_0 \cos\theta_0 - n_1 \cos\theta_1}{n_0 \cos\theta_0 + n_1 \cos\theta_1}$$
$$r_{12p} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}, \quad r_{12s} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

The total reflection coefficient is:

$$r_{p,s} = \frac{r_{01p,s} + r_{12p,s} e^{-2i\beta}}{1 + r_{01p,s} r_{12p,s} e^{-2i\beta}}$$

where $\beta = 2\pi n_1 d \cos\theta_1 / \lambda$ is the phase thickness of the oxide film, and the subscripts p and s denote p- and s-polarization components, respectively.

The effective refractive indices for the three materials (air, oxide layer, aluminum film) are defined as:

$$\text{s-component:} \quad \tilde{N}_s = n \cos\theta$$
$$\text{p-component:} \quad \tilde{N}_p = \frac{\cos\theta}{n}$$

Based on Equations (2) and (6), and combining the definition of ellipsometric parameters $\tan\psi = |r_p/r_s|$ and $\Delta = \arg(r_p) - \arg(r_s)$, we can establish the relationship between polarization properties and physical parameters for the structure shown in [Figure 4: see original paper]. Assuming $n_0 = 1$, for a specific incident angle and wavelength, the relationship between the key Mueller matrix terms and the optical parameters is:

$$-\cos 2\psi = f_1(n_2, k_2, d, \theta_0, \lambda)$$
$$\sin 2\psi \cos\Delta = f_2(n_2, k_2, d, \theta_0, \lambda)$$
$$\sin 2\psi \sin\Delta = f_3(n_2, k_2, d, \theta_0, \lambda)$$

If we attempt to solve for the optical parameters using the measured Mueller matrix elements and the above relationships, only three equations can be established. Our method must solve for four parameters without using any literature values, resulting in an underdetermined system (fewer equations than unknowns). While multiple incident angles cannot provide additional information for single-layer aluminum films, the multi-layer structure with an oxide layer introduces multiple internal reflections, allowing different incident angles to supply extra information [**?**]. Therefore, we incorporate measurement data from additional incident angles to increase the number of equations—each new angle adds three equations, and theoretically, only two angles are needed to achieve an overdetermined system. In practice, we use as many angles as possible to improve parameter retrieval accuracy.

Optical parameters exhibit correlations (e.g., n_2 and d are positively correlated [?]). To constrain these correlations, we simultaneously fit data across multiple wavelengths, where each wavelength shares the same d but has different n_2 and k_2 values. When fitting spectroscopic data, conventional methods typically use dispersion models to constrain the wavelength-dependent refractive index and rely on single-angle data. The fitting quality and parameter accuracy are highly dependent on model selection, and complex samples require elaborate structural models due to reduced degrees of freedom and parameter constraints.

Our method, based on the double-layer model shown in [Figure 4: see original paper], uses spectroscopic data from multiple incident angles to simultaneously fit multiple wavelength points for n_2, k_2, and d without employing dispersion models. This approach offers higher degrees of freedom and enables better characterization of spectroscopic data across multiple angles. We employ the Levenberg-Marquardt algorithm [?] to solve this nonlinear least-squares optimization problem:

$$\chi^2 = \sum_{i=1}^{N_{\text{angle}}} \sum_{j=1}^{N_{\text{wave}}} \sum_{k=1}^{3} \left(\frac{M_k^{\text{meas}}(\theta_i, \lambda_j) - M_k^{\text{calc}}(\theta_i, \lambda_j)}{\sigma_k(\theta_i, \lambda_j)} \right)^2$$

where N_{angle} and N_{wave} are the numbers of incident angles and wavelengths, respectively; M_k^{meas} represents the measured key terms ($-\cos 2\psi$, $\sin 2\psi \cos \Delta$, $\sin 2\psi \sin \Delta$); and M_k^{calc} are the corresponding calculated terms.

More incident angle data provide additional information and improve the precision of retrieved unknown parameters. Therefore, we simultaneously fit spectroscopic data across incident angles from 50° to 80° using the method described above, with results compared against those from conventional methods. In the traditional approach, the complex refractive index of aluminum is modeled using the Drude-Lorentz dispersion model, while silicon dioxide uses the Lorentz dispersion model, with data at 60° and 70° fitted separately (angles near the pseudo-Brewster angle of the sample provide high measurement sensitivity).

Fitting quality is evaluated using relative residuals, defined as the ratio of the difference between calculated and original data to the original data, expressed as a percentage. Figure 5: see original paper shows the relative residuals at 60° and 70° for both methods, where solid lines represent our method and dashed lines represent the conventional method. For all three Mueller matrix elements, our method demonstrates better fitting performance than the traditional approach, particularly for $\sin 2\psi \sin \Delta$, where traditional method residuals reach approximately 50%, while our method's residuals are nearly zero. Since our method fits data from multiple incident angles simultaneously, the results conform to polarization properties across all angles, achieving comparable residual levels for other angles as well.

Both residual curves exhibit sudden increases at certain wavelengths where the sample's polarization properties are extremely weak (e.g., at 725 nm for 70° incidence, the diattenuation is only 7×10^{-4}), causing the relative error to become large when used as a denominator. Excluding these anomalous points, our method still shows superior fitting performance. presents the oxide layer thickness retrieved by both methods, which differ by only ~0.5 nm, indicating consistent thickness measurements.

Table 1. Retrieved oxide layer thickness from our method and traditional methods

Method	Thickness (nm)
Our method	28.3
Traditional (60°)	27.8
Traditional (70°)	28.1

The retrieved optical parameters from both methods are compared with literature reference values in Figure 5: see original paper-(d). Solid lines connect results from multiple wavelengths, while dashed curves represent literature reference values for aluminum's complex refractive index (from McPeak [**?**]) and silicon dioxide's refractive index (from Gao [**?**]). For n_2 and k_2, all three curves show consistent trends, with minor numerical differences between our method and traditional methods arising from two factors: (1) conventional dispersion models cannot fully characterize the mirror's polarization properties as they ignore many influencing factors, and (2) our method accounts for variations in d. For k_2 shown in Figure 5: see original paper, differences between our results and the reference curve increase significantly beyond 1000 nm. Since the reference curve was also obtained using traditional methods on an aluminum sample at 70° incidence, it closely matches the traditional method results for our sample at 70°, though differences remain beyond 1000 nm. The discrepancy between our method and the reference curve can thus be attributed to sample differences, while differences between our method and traditional methods represent results balanced across multiple incident angles without assuming sample isotropy.

The curves from our method are not perfectly smooth because we do not use dispersion models to constrain the wavelength dependence of optical parameters. Instead, only a common oxide thickness connects the wavelength points, allowing the results to accurately reflect the true polarization properties at each wavelength. Our method fits data from multiple incident angles simultaneously, yielding retrieval results that conform to polarization properties across all angles. Based on a simple double-layer structure, this approach facilitates future prediction and verification of polarization properties at unknown incident angles.

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Note: Figure translations are in progress. See original paper for figures.