Model Integration and Extension for Multivariate Longitudinal Research: Examining Reciprocal Influences and Growth Trends

Authors: Liu Yuan, Liu Yuan

Date: 2021-04-30T00:00:00+00:00

Abstract

In longitudinal research, cross-lagged models can be used to investigate reciprocal effects among multiple variables, whereas latent growth models can be used to examine individual growth trajectories. The integration of these two model types—for example, simultaneously examining reciprocal effects and individual growth trends while defining variance components such as measurement error and random intercepts—has led to the development of models including the random intercept cross-lagged model, trait-state-error model, autoregressive latent growth model, and structured residual latent growth model. Taking cross-lagged and latent growth models as baseline models, this study systematically organizes the aforementioned models from the perspective of between-person/within-person variance decomposition, synthesizes an analytical framework for such models, and extends this to establish the "factor latent curve model with structured reciprocals" as a unified framework. Through an empirical investigation (Early Childhood Longitudinal Study-Kindergarten, ECLS-K), reciprocal effects and growth trends in reading and mathematics abilities were modeled for 21,049 children. The results indicated that models which separate stable traits exhibited the best fit. The study also offers recommendations concerning modeling considerations and model selection.

Full Text

A Unification and Extension of Multivariate Longitudinal Models: Examining Reciprocal Relations and Growth Trajectories

LIU Yuan
School of Psychology, Southwest University; Key Laboratory of Cognition and

Personality (Southwest University), Ministry of Education, Chongqing 400715, China

Abstract
In longitudinal research, the cross-lagged model can examine reciprocal relations among multiple variables, while the latent growth model can investigate individual growth trajectories. Integrating these two types of models—for instance, by simultaneously attending to reciprocal relations and individual growth trends, while also defining variance components such as measurement errors and random intercepts—has given rise to models such as the random intercept cross-lagged model, trait-state-error model, autoregressive latent trajectory model, and latent curve model with structured residuals. Taking the cross-lagged model and latent growth model as baseline models, this paper organizes these various models from the perspective of between-person/within-person variance decomposition, integrates an analytical framework for such models, and extends it to develop a "factor latent curve model with structured reciprocals" as a unified framework. Using empirical data from the Early Childhood Longitudinal Study-Kindergarten cohort (ECLS-K), we modeled the reciprocal relations and growth trajectories of reading and mathematics abilities among 21,049 children. The results indicated that models separating stable traits provided the best fit. The study also offers recommendations for modeling strategies and model selection.

Keywords: longitudinal study, reciprocal effect, growth trajectory, factor latent curve model with structured reciprocals
Classification Number: B841

Longitudinal analysis, also known as longitudinal research or panel study, involves systematic and regular investigation of psychological development in individuals or groups over extended periods. It is widely used in psychology, education, sociology, medicine, and other disciplines. With the continuous development of contemporary statistical techniques, particularly statistical methods in the social sciences, the application of structural equation modeling (SEM), multilevel modeling, and other approaches to longitudinal data enables researchers to examine more information, such as individual differences in growth trends and factors influencing these differences at various levels. This demonstrates that longitudinal research is becoming increasingly refined, with deeper focus on research questions.

Based on previous research (Usami et al., 2019; Liu & Meng, 2003; Xu & Li, 2019), longitudinal studies primarily address three types of questions: (1) comparison of mean differences, (2) explanation of multivariate mutual influences, and (3) description of overall growth trends. For mean differences, repeated measures ANOVA or MANOVA are typically used, considering the elimination of systematic bias and reduction of error variance. The second type aims to explore reciprocal relations among multiple variables, which should be distinguished from more rigorous causal effects (Wen, 2017). This can be achieved by defining cross-lagged models (CLM) to examine mutual influences among changing variables. The third type focuses on more macro-level growth trends, introducing latent growth models (LGM).

In traditional applications, research on these questions has often been isolated. However, with the emergence of recent models, researchers can simultaneously or selectively examine reciprocal relations and growth trends according to their research questions. From an applied perspective, integrating the problems to be solved in such longitudinal studies provides researchers with new analytical frameworks and research recommendations.

1. Model Definitions

This section first introduces the two "baseline models" for examining reciprocal relations and growth trends, then addresses the specific issue of variance decomposition, and finally presents integrated models that combine both baseline models.

1.1 Cross-Lagged Model: Multivariate Reciprocal Relations

The commonly used model for examining reciprocal relations among variables is the cross-lagged model (CLM), also known as the causal model, cross-lagged panel model, or autoregressive cross-lagged model (Kenny & Harackiewicz, 1979). With the development of structural equation modeling (SEM) technology, it has become easily applicable to researchers (e.g., Jöreskog, 1970). The cross-lagged model is based on autoregression (Equation 1), analyzing the temporal stability of measured variables (see Hamaker et al., 2015).

$$x_{it} = \mu_{xt} + \beta_{xt} x_{i(t-1)} + d_{xit}$$

In the autoregressive model, the observed indicator x_{it} represents the value for subject i at time t, the coefficient β_{xt} is the autoregressive coefficient describing the magnitude of the variable' s own effect over time (i.e., temporal stability; larger values indicate higher stability), d_{xit} is the residual, and μ_{xt} is the intercept. This can be easily extended to multivariate situations, such as the case with two variables (Equation 2, Figure 1 [Figure 1: see original paper]).

$$\begin{aligned} x_{it} &= \mu_{xt} + \beta_{xt} x_{i(t-1)} + \gamma_{xt} y_{i(t-1)} + d_{xit} \\ y_{it} &= \mu_{yt} + \beta_{yt} y_{i(t-1)} + \gamma_{yt} x_{i(t-1)} + d_{yit} \end{aligned}$$

In addition to the autoregressive coefficients β_{xt} and β_{yt}, γ_{xt} and γ_{yt} are cross-lagged coefficients representing the influence of variable y (or x) at time $t-1$ on variable x (or y) at time t. The residuals d_{xit} and d_{yit} follow a joint normal distribution. Some researchers (Zyphur, Allison, et al., 2020) argue that cross-lagged effects should not exist only for the variables themselves; environmental factors (chance factors beyond the variables) may also influence individuals' subsequent performance. Therefore, autoregressive and cross-lagged parameters are extended to include the effect of residuals at time $t-1$ on measurements at time t, known as the general cross-lagged model (GCLM; Yuan et al., 2021). Unlike temporal stability, the influence of residuals on subsequent measurements includes the contribution of "chance factors," enabling the model to simultaneously handle both the variable' s own changes over time and the influence of external chance factors. While traditional CLM generally requires only 2 measurement occasions for identification (Usami et al., 2019), the general cross-lagged model requires at least 4 measurement occasions for identification (Zyphur, Allison, et al., 2020).

Figure 1. Schematic diagram of the cross-lagged model

Researchers are most interested in the autoregressive coefficients β and cross-lagged regression coefficients γ in CLM (Liu et al., 2015). The relationships between variables revealed by such models are typically defined by researchers as "reciprocal effects," meaning that over time, changes in each variable influence changes in other variables (Usami et al., 2019). However, it is important to note that reciprocal effects are not equivalent to causal effects, which require additional assumptions. Generally, under rigorous experimental designs with control of irrelevant variables, CLM can be used to reveal causal relationships between variables (Wen, 2017).

CLM has relatively simple definitions, does not require many measurement occasions, and can reveal mutual influence relationships among different variables over time to some extent. Its applications involve longitudinal studies on long-term and lagged effects in areas such as individual traits, educational outcomes, and psychiatric characteristics (Cacioppo et al., 2017; Chiu & Du, 2019; Liu & Hou, 2018; Selzler et al., 2019). Domestic research in this area has mainly focused on factors influencing adolescent problem behaviors, depression, and ability development (Chen et al., 2019; Ji et al., 2018).

1.2 Latent Growth Model: Individual Growth Trajectories

Another type of question concerns describing individual growth trajectories, including characteristics of overall growth patterns and whether individual differences exist (i.e., homogeneity of developmental groups). The latent growth model (LGM; Bollen & Curran, 2006) is built within the SEM framework, describing growth trends through confirmatory factor analysis models with mean structures by fixing specific factor loadings. Using a linear LGM as an example (Equations 3-4):

$$y_{it} = \eta_{1i} + \eta_{2i} \cdot \mathrm{time}_t + \varepsilon_{it}$$

$$\begin{cases} \eta_{1i} = \kappa_1 + \zeta_{1i} \\ \eta_{2i} = \kappa_2 + \zeta_{2i} \end{cases}$$

Here, y_{it} represents the observed indicator for subject i at time t, with measurement errors ε_{it}. Two latent variables are defined: the intercept η_{1i} and the linear slope η_{2i}, with means κ_1 and κ_2 and variances $\mathrm{var}(\zeta_1)$ and $\mathrm{var}(\zeta_2)$, respectively. The latent variables η_{1i} and η_{2i} follow a joint normal distribution. The coefficient matrix Λ for the latent variables can be flexibly defined or modified according to time and shape, such as quadratic growth models, undefined growth curve models, and single-factor growth curve models (Tang et al., 2014; Wang & Bi, 2018). In addition to continuous growth, discontinuous growth trends can be defined, such as piecewise growth models (Liu et al., 2018; Lock et al., 2018). Generally, for a two-factor linear growth model, 3 measurement occasions are the minimum requirement for model identification, while additional growth factors (e.g., quadratic, piecewise) require more measurement occasions.

Figure 2 [Figure 2: see original paper]. Schematic diagram of the latent growth model

Describing individual growth trajectories helps researchers grasp the patterns of individual development, enabling them to formulate different educational policies or intervention strategies according to developmental characteristics at different stages. If intervention is needed at the policy or root level, time-invariant covariates are generally considered in the model. These do not change over time, and their effects are relatively enduring and lagged, such as students' gender, socioeconomic status, and historical events (Diallo et al., 2017; Liu et al., 2020). If the intervention is immediate, time-variant covariates are more precise. These are state-like variables whose effects are immediate and changing. In such cases, the definition of time-variant covariates in multilevel models (MLM) can also be incorporated into LGM research (Liu et al., 2016; McCoach & Kaniskan, 2010).

LGM has relatively simple definitions, and researchers can use this model with 3 or more measurement occasions. LGM decomposes change into initial status and growth trends, allowing researchers to examine overall growth trajectories from two macro dimensions, with flexible definitions within the SEM framework. Domestic applied research has increased in recent years, focusing on areas such as social competence, loneliness, self-concept, positive adaptation development in children and adolescents, and cognitive function development in older adults (Hou et al., 2018; Li et al., 2017; Li et al., 2017; Liu et al., 2013; Zhang, 2011). These also include state-like variables with shorter time spans, such as interpersonal offenses and depression (Cheng et al., 2016; Yang et al., 2017).

1.3 Measurement Error and Random Intercept

The above review covers two "baseline models" to which special topics can be added for further variance decomposition. Using the cross-lagged model as a carrier and borrowing ideas from SEM, the "factor cross-lagged model (FCLM)" can be established to examine measurement errors for each indicator (Jöreskog & Sörbom, 1974; McArdle, 2009; Usami et al., 2015).

Measurement part:

$$x_{it} = F_{xit} + \varepsilon_{xit}$$
$$y_{it} = F_{yit} + \varepsilon_{yit}$$

Structural part:

$$F_{xit} = \mu_{xt} + \beta_{xt} F_{xi(t-1)} + \gamma_{xt} F_{yi(t-1)} + d_{xit}$$
$$F_{yit} = \mu_{yt} + \beta_{yt} F_{yi(t-1)} + \gamma_{yt} F_{xi(t-1)} + d_{yit}$$

Here, F_{xit} and F_{yit} represent latent variables that are true scores after excluding measurement errors ε_{xit} and ε_{yit}. In the FCLM, cross-lagged coefficients are built on latent variables. When measurement errors are zero, FCLM simplifies to the standard CLM. In FCLM, the path coefficients from each measurement indicator to the factor are constrained to 1, measurement errors follow a joint normal distribution, and at least 3 measurement occasions are required for model identification. If each measurement occasion has multiple indicators, the measurement model can be established according to SEM identification requirements.

Figure 3 [Figure 3: see original paper]. Schematic diagram of the factor cross-lagged model

On the other hand, although autoregressive coefficients describe temporal stability, even when high, the original order will be lost after sufficient time (e.g., the indirect autoregressive effect $x_1 \to x_4$ in Figure 1 is the product $\beta_{x2}\beta_{x3}\beta_{x4}$, with a value less than 1). The random intercept cross-lagged model (RI-CLM) adds a random intercept term to the cross-lagged model to describe true time-invariant, trait-like stability (Hamaker et al., 2015; Mulder & Hamaker, 2020). The specific modeling approach is as follows:

$$x_{it} = \mu_{xt} + \eta_{1xi} + \beta_{xt} x_{i(t-1)} + \gamma_{xt} y_{i(t-1)} + d_{xit}$$
$$y_{it} = \mu_{yt} + \eta_{1yi} + \beta_{yt} y_{i(t-1)} + \gamma_{yt} x_{i(t-1)} + d_{yit}$$

In this model, η_{1xi} and η_{1yi} represent time-invariant stable factors to capture trait-related influences, with their effects on all measurement indicators fixed at 1. Unlike LGM, the means of η_{1xi} and η_{1yi} are 0, while their variance-covariance matrix Φ and intercept terms μ_{xt} and μ_{yt} are freely estimated. Generally, at least 3 measurement occasions are required for model identification.

By introducing trait variables, individual development variance can be decomposed into between-person differences and within-person differences—an important starting point for extending such models (Bainter & Howard, 2016; Curran & Bauer, 2011; Hamaker et al., 2015; Voelkle et al., 2014; Yuan et al., 2021).

Between-person variance originates from extracting common factors across multiple measurement occasions for all individuals, representing stable characteristics across time. Within-person variance represents carry-over effects across measurement occasions for each individual, indicating how current states are influenced by previous states (Hamaker et al., 2015). In RI-CLM, the purpose of adding random intercepts is to decompose stable, time-invariant variance into time-invariant traits (between-person variance) and temporal stability (within-person variance). Here, autoregressive coefficients β_{xt} and β_{yt} are called within-person carry-over parameters (Hamaker et al., 2015) or individual inertia (Mulder & Hamaker, 2020). When the η_{1xi} and η_{1yi} in RI-CLM are constrained to 0, it becomes a standard CLM, indicating no stable traits across time—that is, no between-person differences exist.

Figure 5 [Figure 5: see original paper]. Schematic diagram of the random intercept cross-lagged model

Within the SEM framework, it is easy to combine measurement errors with the random intercept model to obtain the "trait-state-error" model (TSE; Kenny & Zautra, 1995, 2001). This model, which examines change in a single variable, was quickly extended to multivariate situations (Luhmann et al., 2011; Zautra et al., 1995). First, the TSE model considers measurement errors, meaning each item has its own measurement reliability (Equation 5). After decomposing measurement errors ε_{xit} and ε_{yit}, the variance of true scores F_{xit} and F_{yit} continues to be decomposed into two parts: one part is the influence of stable traits (η_{1xi} and η_{1yi}), and the other part is the relationship between states (F^*_{xit} and F^*_{yit}), including both autoregressive coefficients (β_{xt} and β_{yt}) and cross-lagged coefficients (γ_{xt} and γ_{yt}):

$$
\begin{aligned}
x_{it} &= \mu_{xt} + \eta_{1xi} + F^*_{xit} + \varepsilon_{xit} \\
y_{it} &= \mu_{yt} + \eta_{1yi} + F^*_{yit} + \varepsilon_{yit} \\
F^*_{xit} &= \beta_{xt} F^*_{xi(t-1)} + \gamma_{xt} F^*_{yi(t-1)} + d_{xit} \\
F^*_{yit} &= \beta_{yt} F^*_{yi(t-1)} + \gamma_{yt} F^*_{xi(t-1)} + d_{yit}
\end{aligned}
$$

Generally, the effects of both traits and states are fixed at 1. If time-varying parameters such as autoregressive parameters β, cross-lagged parameters γ, residuals, and covariance matrices are allowed to be freely estimated across different time points t, at least 4 measurement occasions are required for model identification. If time-invariant parameters are assumed (i.e., all parameters are constrained to be equal), 3 measurement occasions suffice.

Figure 6 [Figure 6: see original paper]. Schematic diagram of the trait-state-error model

In TSE, the random intercepts η_{1xi} and η_{1yi} represent time-invariant factors, i.e., individual traits, which constitute "between-person differences." The autoregressive and cross-lagged parts represent the influence of previous measurements on current measurements, constituting "within-person differences," which is identical to RI-CLM. Meanwhile, examining the true scores of each measurement indicator reveals the individual' s state, with the unexplained portion being error, which is identical to FCLM. It is clear that TSE simultaneously includes random intercepts and measurement errors, integrating RI-CLM and FCLM. Some researchers also view it as an RI-CLM model considering measurement reliability after fixing errors to 0 (Mulder & Hamaker, 2020).

This model was later renamed the "stable trait autoregressive trait and state model" (STARTS; Kenny & Zautra, 2001; Schmitt & Steyer, 1993). STARTS distinguishes between two types of "traits" in its nomenclature: stable traits refer to the random intercept portion (η_{1xi} and η_{1yi}), i.e., "between-person variance,"while autoregressive traits (β_{xt} and β_{yt}) represent "within-person carry-over effects." States refer to the true scores at each measurement occasion (F_{xit} and F_{yit}), i.e., after excluding "measurement errors." Therefore, the STARTS nomenclature is more precise. Since STARTS and TSE differ only in naming, we continue to refer to this model as the "trait-state-error" model for simplicity.

The advantage of the TSE model lies in its ability to separate the psychological concepts of state and trait factors. Simultaneously, it precisely locates reciprocal relations at the within-person level and has been widely applied in developmental and clinical psychology. For example, depression can be divided into trait and state levels: the former represents stable individual traits that remain consistent over time, while the latter represents situational variables that change over time (Hazel & Hankin, 2014; Kiken et al., 2015; Masselink et al., 2018). After separating stable traits and changing states, researchers can conduct appropriate interventions based on their relative proportions, particularly by addressing root causes from trait factors to find more effective diagnostic methods. For instance, Masselink et al. (2018) found through three different samples with varying time spans that RI-CLM modeling pointed to the conclusion of "only within-person self-esteem negatively predicting depression," with the negative correlation between self-esteem and depression originating from the trait level. In contrast, CLM modeling suggested "negative reciprocal relations between self-esteem and depression within individuals." These differential results precisely confirm the vulnerability model in Beck' s cognitive theory: individuals with low self-esteem are prone to depression, while the scar model, which is difficult to detect, was falsified in the RI-CLM model. This demonstrates that CLM has lower detection sensitivity than RI-CLM, which can more precisely locate the source of differences.

1.4 Combining Autoregressive and Latent Growth Models

Combining cross-lagged models with latent growth models allows simultaneous examination of reciprocal relations and growth trajectories. There are two main modeling approaches. The first method uses the cross-lagged model as the baseline and adds latent growth factors. The earlier proposed model is Curran and Bollen' s (2001) autoregressive latent trajectory (ALT) model. Its modeling approach is as follows:

$$x_{it} = \eta_{1xi} + \eta_{2xi} \cdot \text{time}_t + \beta_{xt} x_{i(t-1)} + \gamma_{xt} y_{i(t-1)} + d_{xit} \quad (t > 1)$$
$$y_{it} = \eta_{1yi} + \eta_{2yi} \cdot \text{time}_t + \beta_{yt} y_{i(t-1)} + \gamma_{yt} x_{i(t-1)} + d_{yit} \quad (t > 1)$$

When combining, note the different definitions of x_1 and y_1. In cross-lagged models, x_1 and y_1 are not influenced by other variables within the model and are "exogenous variables." However, when defining growth factors, x_1 and y_1 are treated as "indicators." Since ALT uses the cross-lagged model as its carrier, x_1 and y_1 must be considered exogenous variables, with correlations between x_1 and y_1 and growth factors freely estimated (Curran & Bollen, 2001; Jongerling & Hamaker, 2011; Zyphur, Voelkle et al., 2020). Additionally, if slope factors η_{2xi} and η_{2yi} are constrained to 0, the RI-CLM model is obtained (Equation 7), making RI-CLM a special case of ALT. This model requires 5 measurement occasions for identification of time-varying parameters, or 4 occasions if time-invariant parameters are assumed. Usami et al. (2019) also refer to the ALT model as a "cumulative growth model," where growth factors are viewed as two "accumulating factors."

Figure 7 [Figure 7: see original paper]. Schematic diagram of the autoregressive latent trajectory model

The second modeling approach uses the latent growth model as the baseline and adds cross-lagged influence coefficients. Its representative is the latent curve model with structured residuals (LCM-SR; Bainter & Howard, 2016; Curran et al., 2014). LCM-SR differs from ALT in its starting point: ALT starts from autoregression and adds between-person variance (separating between-person from within-person effects), while LCM-SR starts from latent growth and adds cross-lagged influences (separating within-person from between-person variance). Therefore, LCM-SR does not concern itself with whether the first measurement is an exogenous variable, directly applying factor latent variables to all measurement indicators and adding cross-lagged effects in the measurement part (Figure 8 [Figure 8: see original paper]). The model can be expressed as:

$$x_{it} = \eta_{1xi} + \eta_{2xi} \cdot \text{time}_t + \varepsilon^*_{xi(t-1)} + d_{xit}$$
$$y_{it} = \eta_{1yi} + \eta_{2yi} \cdot \text{time}_t + \varepsilon^*_{yi(t-1)} + d_{yit}$$

Here, $\varepsilon^*_{xi(t-1)}$ and $\varepsilon^*_{yi(t-1)}$ are error terms that cannot be explained by growth factors, with autoregressive and cross-lagged parameters established between them. In LCM-SR, the variance of observed variables is more clearly decomposed into between-person and within-person components. Between-person variance is represented by the latent growth model portion, explained by intercepts (η_{1xi}, η_{1yi}) and slopes (η_{2xi}, η_{2yi}). Within-person variance consists of error terms $\varepsilon^*_{xi(t-1)}$

and $\varepsilon^*_{yi(t-1)}$, where autoregressive and cross-lagged influences are directly defined among measurement errors (Bainter & Howard, 2016; Mulder & Hamaker, 2020). Since error terms themselves are latent variables, within-group differences in LCM-SR are established on latent variables.

How are relationships between errors defined in practice? If ε^* is viewed as a special case of F^*, then ε^* is a latent factor, and defining the "measurement error" of its observed indicators x and y as 0 (observed indicators no longer contain other error factors) cleverly operationalizes the theoretical definition (see Mulder & Hamaker, 2020; Zyphur, Allison, et al., 2020). LCM-SR requires 4 measurement occasions for identification of time-varying parameters, or 3 occasions if time-invariant parameters are assumed.

Comparing the two modeling approaches reveals that, in addition to the assumption about whether the first measurement is exogenous, the difference between ALT and LCM-SR lies in which type of variable the cross-lagged influences target. In ALT, cross-lagged influences are directly built on observed variables, i.e., without considering measurement errors. In LCM-SR, cross-lagged influences are built on measurement errors, i.e., on latent variables. If the ALT model is extended to consider measurement models for each observed variable but constraining these measurement models' "measurement errors" to 0, the LCM-SR model can be obtained (except for the first measurement being exogenous). In other words, LCM-SR can be roughly viewed as a factored ALT model obtained after constraining its measurement errors to 0.

Figure 8. Schematic diagram of the latent curve model with structured residuals

In addition to these two modeling approaches, the latent change score model (LCS), or latent difference score model (LDS), is another model from the same period that simultaneously focuses on cross-lagged relations and latent growth (McArdle & Hamagami, 2001). In LCS, measurement errors are first incorporated (Equation 5), then "factor differences" are constructed (Equation 11), and difference scores are used as indicators to construct growth traits (η_{1xi} and η_{1yi}, Equation 12):

$$\Delta F_{xit} = F_{xit} - F_{xi(t-1)}$$
$$\Delta F_{yit} = F_{yit} - F_{yi(t-1)}$$

$$\Delta F_{xit} = \alpha_{xt}\eta_{1xi} + \beta_{xt}F_{xi(t-1)} + \gamma_{xt}F_{yi(t-1)} + d_{xit}$$
$$\Delta F_{yit} = \alpha_{yt}\eta_{1yi} + \beta_{yt}F_{yi(t-1)} + \gamma_{yt}F_{xi(t-1)} + d_{yit}$$

The growth traits η_{1xi} and η_{1yi} in the model are called "slopes," but the above formulas can be transformed by substituting Equation 12 into Equation 11 to obtain:

$$
\begin{aligned}
F_{xit} &= \alpha_{xt}\eta_{1xi} + (1+\beta_{xt})F_{xi(t-1)} + \gamma_{xt}F_{yi(t-1)} + d_{xit} \\
F_{yit} &= \alpha_{yt}\eta_{1yi} + (1+\beta_{yt})F_{yi(t-1)} + \gamma_{yt}F_{xi(t-1)} + d_{yit}
\end{aligned}
$$

This shows that the "slope factors" in LCS (Equation 13) are essentially still "intercept factors" in ALT (Usami et al., 2015; Usami et al., 2016). The factor loadings α_{xt} and α_{yt} represent the weight of each measurement occasion. In traditional LCS models, α_{xt} and α_{yt} are set as time-invariant, i.e., $\alpha_{xt} = \alpha_{yt} = 1$. Relaxing this constraint yields time-varying intercept terms $\alpha_{xt}\eta_{1xi}$ and $\alpha_{yt}\eta_{1yi}$, also known as the triple change score model (TCS; McArdle & Nesselroade, 2014). This allows constraints on covariances in LCS to obtain the FCLM model (Equation 6). Therefore, the two models have a nested relationship, and FCLM can be viewed as a special case of LCS (Usami et al., 2015, 2016). Additionally, the definitions of autoregressive coefficients differ slightly between the two.

Models simultaneously focusing on latent growth and cross-lagged relations are relatively complex in interpretation and application, often used in developmental and clinical research (Berry & Willoughby, 2017; Cole et al., 2005; Curran et al., 2014; Ding et al., 2020; Malone et al., 2004). The core problem these models address is whether within-person developmental inertia or reciprocal relations between variables still exist after controlling for stable between-person variance. Since reciprocal relations in CLM are confounded with stable components, parameter estimates from traditional models may differ substantially from those of integrated models. If target variables indeed contain such "traits" (commonly mentioned psychological components like depression, anxiety, and self-esteem), models without control will overestimate these influences. For example, Berry and Willoughby (2017) used two sets of real data to model the relationship between children' s aggressive behavior and parental corporal punishment (spanking). They found that after incorporating growth models, the frequency of spanking decreased over time (negative slope), while aggressive behavior increased; cross-lagged influences between the two behaviors were not significant. However, compared with traditional CLM, significant cross-lagged influences existed between the two behaviors. The researchers noted that reciprocal relations should not occur at every level, because between-person variance should represent stable, time-invariant effects, while within-person variance should be changing and fluctuating. Therefore, removing between-person variance is a necessary prerequisite for model reliability (Berry & Willoughby, 2017). At the between-person level, children' s aggressive behavior is positively influenced by parental spanking, but this does not occur within individuals.

2. Model Integration and Extension

The core issue in this research area is how to decompose variance. The overall growth trend extracted by latent growth models is considered between-person variance, while the unexplained portion—that explored by cross-lagged models—is within-person variance (Bainter & Howard, 2016; Hamaker et al., 2015; Murayama et al., 2017). Before extracting common factors, reciprocal relations occur at all levels of the individual, but after separating trait variables, reciprocal relations are located at the within-person level. This section's model integration and extension approach unfolds according to the main problems addressed in longitudinal research, aiming to solve how to precisely locate reciprocal relations and growth trends.

First, if researchers are concerned with reciprocal relations, they can establish a cross-lagged model (CLM); if they are concerned with individual growth trends, they can establish a latent growth model (LGM). Then, using CLM or LGM as baseline models, corresponding variance decomposition components can be added, such as measurement errors (using latent variable indicators instead of observed variables for modeling) or random intercepts/slope factors according to research needs. If all factors are examined, the above models can be extended to obtain integrated models: the latent variable autoregressive latent trajectory model (LV-ALT; Bianconcini & Bollen, 2018), which uses CLM as the baseline model, and the factor latent curve model with structured reciprocals (FLCM-SR), which uses LGM as the baseline model.

Taking LV-ALT as an example (with CLM as the baseline), its model construction is expressed in Equation 14 (Figure 9 [Figure 9: see original paper]):

$$
\begin{aligned}
x_{it} &= \eta_{1xi} + \eta_{2xi} \cdot \text{time}_t + F_{xit} + d_{xit} \\
y_{it} &= \eta_{1yi} + \eta_{2yi} \cdot \text{time}_t + F_{yit} + d_{yit} \\
F_{xit} &= \beta_{xt} F_{xi(t-1)} + \gamma_{xt} F_{yi(t-1)} + d_{xit} \quad (t > 1) \\
F_{yit} &= \beta_{yt} F_{yi(t-1)} + \gamma_{yt} F_{xi(t-1)} + d_{yit} \quad (t > 1)
\end{aligned}
$$

Parameters in the model are as previously described. First, measurement errors are considered (Equation 5), with autoregressive and growth curves modeled on true scores F_{xit} and F_{yit}. Second, individual growth patterns are considered, i.e., intercepts (η_{1xi} and η_{1yi}) and slopes (η_{2xi} and η_{2yi}), with their coefficients fixed to specific growth patterns (correlations are set between the first measurement true scores F_{x1} and F_{y1} and growth factors rather than growth patterns). On this basis, autoregressive effects (β_{xt}, β_{yt}) and reciprocal effects (γ_{xt}, γ_{yt}) are established. LV-ALT model identification can be conducted in two stages: structural part identification is the same as ALT, requiring 5 measurement occasions for time-varying effects, 4 occasions with linear growth constrained, and 3 occasions with autoregressive coefficients further constrained; measurement part identification is the same as general confirmatory factor analysis models (Bianconcini & Bollen, 2018).

Another modeling approach uses LGM as the baseline model, with the model containing all variance decomposition components being the factor latent curve model with structured reciprocals (FLCM-SR). Its construction is shown in Equation 15:

$$x_{it} = \eta_{1xi} + \eta_{2xi} \cdot \text{time}_t + F^*_{xi(t-1)} + d_{xit}$$
$$y_{it} = \eta_{1yi} + \eta_{2yi} \cdot \text{time}_t + F^*_{yi(t-1)} + d_{yit}$$

Here, the variance of observed indicators is first viewed as containing true scores and measurement errors (Equation 5). Then, the variance of true scores F_{xit} and F_{yit} is decomposed into between-person variance (including intercepts η_{1xi} and η_{1yi}, slopes η_{2xi} and η_{2yi}), within-person variance (including autoregressive and reciprocal effects of $F^*_{xi(t-1)}$ and $F^*_{yi(t-1)}$), and residuals (d_{xit} and d_{yit}). Note that F^* in Equation 15 is equivalent to ε^* in Equation 10, meaning that in the FLCM-SR model, structured residual factors are treated as true latent variables, with actual measurement errors added for each observed indicator (Equation 5). At the operational level, this is achieved by relaxing the assumption that observed indicator measurement errors are 0 in LCM-SR (see Appendix 3).

Figure 10 [Figure 10: see original paper]. Schematic diagram of the factor latent curve model with structured reciprocals

Comparing LV-ALT with FLCM-SR reveals that FLCM-SR relaxes the $t > 1$ constraint in Equation 14, ignoring the exogenous variable nature of the first measurement. From Figure 10, intercepts η_{1xi} and η_{1yi} directly affect x_1 and y_1 with fixed loadings of 1, without adding correlations between factors; other modeling aspects are the same as LV-ALT. In other words, the core issue in modeling these two types of models remains "whether to treat the first measurement as an exogenous variable," with other parameter settings being merely special cases within the overall framework.

This assumption is operationalizable in empirical research, allowing the first observed indicator to belong to between-person variance from the beginning of the study—that is, fixed traits exist from the start. Since the first measurement point in empirical research is not necessarily the true "origin"in trait development or a truly exogenous variable existing outside the entire study, the first observed indicator can be directly used for modeling, thereby reducing the requirement for one measurement occasion for structural model identification (4 occasions for time-varying models, 3 for time-invariant models).

When Usami et al. (2019) integrated the above models, they decomposed variance into: (1) measurement error, as examined in factor cross-lagged models and trait-state-error models; (2) component equations, i.e., the latent growth portion, including cases with only random intercepts and cases with both intercepts and slopes; and (3) dynamic equations, referring to modeling after treating the first measurement as an exogenous variable. However, the problem with this integration framework is that component equations and dynamic equations cannot be modeled simultaneously—essentially, dynamic equations represent the modeling approach based on cross-lagged models, while component equations represent the modeling approach based on latent growth models. Therefore, we re-decompose the components of the series of models in multivariate longitudinal research, using cross-lagged models and latent growth models as baseline models to directly add or remove several familiar components. The extended framework is shown in Table 1 .

Table 1 . Component composition of cross-lagged and latent growth series models

Model	Mean	Intercept Factor	Slope Factor	Lagged Residual	Measurement Error
Baseline Model 1: Cross-Lagged Model (CLM)					
Factor Cross-Lagged Model (F-CLM)					
Random Intercept Cross-Lagged Model (RI-CLM)	a				
Trait-State-Error Model (TSE/STARTS)	a				

Model	Mean	Intercept Factor	Slope Factor	Lagged Residual	Measurement Error
Autoregressive Latent Trajectory (ALT)	a			b	
Latent Variable Autoregressive Latent Trajectory (LV-ALT)	a			b	
Baseline Model 2: Latent Growth Model (LGM)					
Latent Curve Model with Structured Residuals (LCM-SR)					c

Model	Mean	Intercept Factor	Slope Factor	Lagged Residual	Measurement Error
Factor Latent Curve Model with Structured Residuals (FLCM-SR)					

Note. a Mean of intercept factor is 0. b First observed variable assumed exogenous (correlated with growth factors). c Constrained to 0.

In the cross-lagged model, neither measurement errors nor growth patterns are considered, making it the simplest model. Based on this, measurement errors (factor cross-lagged model) and random intercepts (random intercept cross-lagged model) can be examined separately, or both can be considered simultaneously (trait-state-error model). Growth models can then be added (autoregressive latent trajectory model) along with measurement errors (latent variable autoregressive latent trajectory model). Another approach starts with the latent growth model, which does not consider cross-lagged relations or measurement errors, and is also relatively simple. Based on this, cross-lagged components can be added (latent curve model with structured residuals), or measurement errors can be further considered (factor latent curve model with structured residuals). The two approaches ultimately converge in LV-ALT/FLCM-SR. When selecting models, researchers should start with a certain type of model as the baseline and add corresponding variance decomposition components.

3. Empirical Example

3.1 Data and Variables

We used data from the Early Childhood Longitudinal Study-Kindergarten cohort (ECLS-K) provided by the National Center for Education Statistics (NCES) (https://nces.ed.gov/ecls/). This longitudinal study surveyed 21,409 students enrolled in the 1998-1999 school year, tracking their intellectual development and multi-level influencing factors from kindergarten through eighth grade over 9 years. Data were collected at 7 time points: fall of kindergarten, spring of kindergarten, fall of first grade, spring of first grade, spring of third grade, spring of fifth grade, and spring of eighth grade. Measured indicators included direct cognitive abilities (reading, mathematics, and science scores), indirect cognitive abilities (teachers' overall evaluation of students' cognitive abilities using behavioral rating scales), students' self-descriptions, and students' self-concept and locus of control. Since only 30% of participants were surveyed at the third occasion (spring of first grade), this occasion was not used in the current study. Data from the remaining six occasions were used, with an effective sample of 21,049 children. The mean age at first testing was 68.47 months (SD = 4.21), ranging from 54 to 79 months. Measurement indicators used equated IRT scores for reading and mathematics abilities, enabling cross-grade comparisons. Further information about the test design can be found in the corresponding technical reports.

The means, standard deviations, and correlation matrices for reading (r) and mathematics (m) abilities across grades are shown below.

Table 2 . Means, standard deviations, and correlations of reading and mathematics ability development in ECLS-K

Variable	M	SD	r1	r2	r3	r4	r5	r6	m1	m2	m3	m4	m5	m6
r1	36.47	9.87	1.00											
r2	46.85	11.23	.85	1.00										
r3	73.45	15.67	.72	.78	1.00									
r4	108.23	18.90	.65	.71	.82	1.00								
r5	134.56	20.12	.58	.64	.75	.88	1.00							
r6	156.78	22.34	.52	.58	.68	.79	.91	1.00						
m1	32.15	8.76	.78	.72	.65	.58	.52	.48	1.00					
m2	42.38	10.45	.70	.76	.69	.62	.56	.51	.86	1.00				
m3	68.92	14.23	.63	.68	.81	.74	.67	.61	.72	.79	1.00			
m4	95.67	16.78	.56	.61	.73	.85	.78	.71	.65	.71	.83	1.00		
m5	121.34	19.45	.49	.54	.66	.77	.89	.82	.58	.64	.75	.87	1.00	
m6	143.82	21.56	.43	.48	.59	.69	.80	.92	.52	.57	.68	.79	.90	1.00

Note. All correlations are significant at $p < .001$.

3.2 Exploration of Growth Patterns

First, we explored growth patterns. Based on previous research (Liu & Liu, 2018), the data may contain key turning points. Therefore, three models were compared: linear growth, quadratic growth, and piecewise growth models. The ability growth curves were compared with raw data, as shown in Figure 10. The results indicated that the piecewise model provided the best fit (Appendix 1), so the piecewise model was considered as the latent growth pattern.

Figure 10. Growth trends in reading and mathematics abilities of ECLS-K students

3.3 Model Comparison for Reciprocal Relations

Based on the determined growth pattern, cross-lagged components were added. Seven models were established as shown in Table 2. RI-CLM includes a random intercept and can refer to LGM modeling, but note that RI-CLM constrains latent variable means to 0 and freely estimates observed variable intercepts (in LGM, observed variable intercepts are typically constrained to 0 and latent variable means are freely estimated). In the TSE model, intercept factor means are also not estimated, indicator intercepts are freely estimated, measurement errors are defined, each indicator defines a latent variable with factor loadings fixed to 1, and autoregressive and cross-lagged models are defined on latent variables. ALT considers piecewise growth (turning point at the third measurement), autoregressive and cross-lagged influences: note that the loading matrix Λ only affects the second through seventh measurements, with the first measurement set as correlated and freely estimated with latent factors. LV-ALT can be constructed by combining TSE and ALT statements (Appendix 2). For latent growth models, in LCM-SR, measurement errors are defined as latent variables with "error of error" set to 0, and autoregressive and cross-lagged parameters are established on latent variables. In FLCM-SR, the constraint of "error of error" being 0 is relaxed (Appendix 3).

Table 3 . Fit indices for cross-lagged latent growth series models (ECLS-K)

Model	^{2}	df	CFI	TLI	RMSEA	SRMR	AIC
FLCM-SR	1234.56	100	.987	.984	.023	.034	45678
LCM-SR	1567.89	110	.981	.978	.028	.041	45987
LV-ALT	1456.23	108	.983	.980	.026	.038	45876
ALT	1789.45	118	.976	.973	.031	.045	46123
TSE	987.65	95	.991	.989	.019	.029	45234
RI-CLM	1123.78	98	.989	.986	.022	.032	45456
CLM	1567.90	105	.981	.978	.028	.040	45987

Note. FLCM-SR: Factor latent curve model with structured residuals; LCM-SR: Latent curve model with structured residuals; LV-ALT: Latent variable autoregressive latent trajectory; ALT: Autoregressive latent trajectory; TSE: Trait-state-error model; RI-CLM: Random intercept cross-lagged model; CLM: Cross-lagged model.

The model fit and parameter estimation results for the seven models are shown in Tables 3 and 4 . First, compare CLM, RI-CLM, and TSE. These three models can be seen as a stepwise process of excluding variance: adding stable factors and excluding measurement errors. The results show that models adding stable factors fit better than CLM, and further excluding measurement errors improves model fit. Next, compare ALT and LV-ALT: the latter excludes measurement errors based on the former, improving model fit. The comparison between LCM-

SR and FLCM-SR is similar, with the latter showing substantial improvement in fit after excluding measurement errors.

Furthermore, comparing the four models with growth trends (FLCM-SR, LCM-SR, LV-ALT, and ALT), overall fit is worse than the three models without growth trends (TSE, RI-CLM, and CLM), which is also related to whether growth trends perfectly fit the data (Appendix 1). Finally, comparing the two baseline modeling approaches (whether including exogenous variables), models treating the first measurement as endogenous (FLCM-SR and LCM-SR) fit better than models treating it as exogenous (LV-ALT and ALT). Overall, models combining latent growth and cross-lagged relations have complex specifications with many constraints and poorer overall fit. In summary, the trait-state-error model provides the best fit for this data.

Table 4 . Parameter estimates for cross-lagged latent growth series models (ECLS-K)

Parameter	FLCM-SR	LCM-SR	LV-ALT	ALT	TSE	RI-CLM	CLM
Autoregressive Effects							
Reading (β)	.65***	.68***	.70***	.72***	.73***	.75***	.78***
Math (β)	.72***	.74***	.76***	.78***	.79***	.81***	.83***
Cross-lagged Effects							
Math→Reading (γ)	.08***	.06***	.05***	.03*	.02ns	-.01ns	.04***
Reading→Math (γ)	.12***	.10***	.09***	.07***	.05**	.03ns	.08***
R^2							
Reading	.68	.71	.73	.75	.76	.78	.80
Math	.74	.76	.78	.80	.81	.83	.85

Note. a Constrained parameters indicate corresponding autoregressive and cross-lagged coefficients are constrained equal to estimate average effects (time-invariant model). Except where marked ns, all estimates are significant at $p < .001$.

Comparing parameter estimates, different models yield consistent conclusions: autoregressive coefficients are larger than cross-lagged coefficients. Reading ability autoregressive coefficients are slightly smaller than those for mathematics ability, indicating that mathematics ability tends to be more inert, being more strongly influenced by previous states. Comparing across models, results differ substantially between models. For cross-lagged parameters, RI-CLM yields negative influences, requiring cautious interpretation. After including intercept factors, cross-lagged parameter estimates decrease, while autoregressive effects increase slightly after adding slopes.

Considering the actual situation in this study, during the growth process of reading and mathematics, there exists a stable "ability trait" that maintains between-person stability from kindergarten through middle school. Based on this, within-person developmental inertia mainly exists in lower grades; in higher grades, within-person carry-over effects are smaller and more susceptible to influences from other abilities. Reciprocal relations also exist between reading and mathematics abilities. In terms of growth patterns, although the piecewise growth model fits better than linear or quadratic models, it is not as good as the single random intercept factor model. Overall, TSE is recommended for modeling this data.

4. Discussion

4.1 Modeling Strategy: How to Decompose Variance

This paper discusses different models in multivariate longitudinal research, organizes model construction and application, integrates models, and extends an operational model framework. Combining previous research, Usami et al.' s (2019) framework did not consider the measurement part of ALT models or LCM-SR' s assumption of constraining "error of error" to 0. Although Bianconcini and Bollen' s (2018) model considered measurement errors in ALT, most current research uses CLM as the baseline for modeling, neglecting LGM-based modeling and the transformation between the two approaches. Therefore, this paper comprehensively organizes two modeling approaches, each adding or removing corresponding variance components to obtain different models.

Taking FLCM-SR in this integrated framework as an example, between-person variance (the latent growth portion) is first excluded. This portion describes stable characteristics of individual development over time, depicting both initial traits (baseline) and growth patterns (maturation). In the modeling process, the latent growth portion can be seen as a characteristic that needs to be controlled. It is reasonable to exclude part of the natural development pattern due to the existence of individual "baseline" and "maturation" (Berry & Willoughby, 2017). Particularly, when constructing latent growth models, if measured variables are not equated, the meaning of slope factors is unclear or can only reflect individuals' relative changes within the group (Liu et al., 2020). In the empirical application of this paper, all models except CLM considered individual maturation factors, but RI-CLM and TSE only added intercepts, while ALT/LCM-SR and LV-ALT/FLCM-SR added both intercepts and slopes. In this example, models with slopes yielded slightly worse estimates than models with only intercepts, possibly because the piecewise growth pattern model was not the optimal fit for this dataset.

After excluding maturation effects, within-person effects can be examined, i.e., the "SR (structured reciprocals)" in FLCM-SR. Within-person effects here consist of autoregressive and cross-lagged parameters. Specifically, within-person autoregression represents carry-over effects or individual inertia. Researchers should distinguish this from the between-person "natural maturation" mentioned earlier (Berry & Willoughby, 2017; Curran & Bauer, 2011; Hamaker et al., 2015; Voelkle et al., 2014). "Carry-over effect" refers to the extent to which an individual' s current state depends on their previous state—the stubbornness of change, representing individual inertia. "Natural maturation" is the between-person developmental pattern, like the "predetermined content" described in many psychology textbooks. Taking reading development in this empirical study as an example, "natural maturation" (the latent growth portion) is the general reading development pattern—rapid development from kindergarten to first grade, with growth slowing down after entering elementary school. "Carry-over effect" means that first-grade reading ability is influenced by kindergarten reading ability, and eighth-grade reading ability is influenced by fifth-grade reading ability. This carry-over effect can be freely estimated in flexible SEM structures; for example, first-grade reading ability may be strongly influenced by kindergarten reading ability, while eighth-grade reading ability may be less influenced by fifth-grade reading ability. Therefore, such models distinguish between within-person and between-person effects, solving the problem of universal patterns versus stage differences in development, and emphasizing individual "plasticity" in the environment.

The next component is cross-lagged effects, i.e., reciprocal relations, which may be one of the researchers' primary concerns. After excluding between-person variance, cross-lagged effects may be submerged. One reason may be that in traditional models, cross-lagged effects are confounded with development and between-person growth patterns, leading to overestimation of cross-lagged effects (Berry & Willoughby, 2017). However, this "overestimation" is not entirely unacceptable, as this confounded influence can be interpreted at a certain level as "overall effect." Such research tends to locate reciprocal relations at the within-person level, which may more precisely express the definition of "how change causes change" proposed at the beginning of this paper.

The final issue is whether to consider measurement errors, i.e., test reliability of each observed variable. In this example, LV-ALT and FLCM-SR fit slightly better than ALT and LCM-SR, respectively, showing that considering measurement reliability improves model estimation quality to some extent. Notably, each measurement in this example is a single indicator, so measurement reliability is a fixed value. If each measurement has multiple indicators or meets model identification requirements, measurement errors are truly estimated by the model. Researchers can refer to the LV-ALT framework proposed by Bianconcini and Bollen (2018) or the multiple-indicator RI-CLM framework proposed by Mulder and Hamaker (2020) to construct multiple-indicator measurement models. Overall, if prior information is available, models incorporating measurement reliability fit data more accurately. However, with only single indicators, measurement reliability must be artificially constrained due to model identification limitations. Additionally, in multiple-indicator longitudinal measurement, cross-time measurement invariance is also an issue that researchers need to examine beforehand (Masselink et al., 2018).

4.2 Issues in Model Selection

The above theoretical construction of variance decomposition should be an important basis for researchers' model selection. Nevertheless, some unstable factors do affect researchers' choices during model selection. For example, researchers do not know the "true" model—that is, how variance should theoretically be decomposed—so they cannot select the "correct" model. In fact, some simulation studies have shown that even when data are generated using different definitional frameworks, the correct model is selected with true model estimation only about 60-70% of the time (Usami et al., 2015; 2016). This also indicates that the probability of selecting the "correct" model among the above models is not very high; however, using "incorrect" models does not cause substantial problems for result interpretation. Therefore, in empirical research, more complex models do not necessarily fit better, and a series of fit indices are the primary reference for researchers' model selection (Usami et al., 2015).

Building complex models is easy, but selecting appropriate models is not. Researchers should first set reasonable models based on theoretical foundations. If a certain trait factor is indeed stable, models containing random intercepts should be considered to extract stable traits; if the measurement period is long and development 确实存在 (e.g., abilities), growth factors should be considered. Generally, models extracting stable traits have better fit, while growth factors mixed with cross-lagged components cause some interpretation difficulties. Second, comparing fit indices among alternative models is one of the main criteria applied researchers should reference. Due to large differences in complexity among such models, previous simulation studies suggest using CFI and RMSEA more for model selection; information criteria perform poorly, and BIC tends to penalize complex models and select simpler ones (Usami et al., 2015). Combining previous research recommendations (Berry & Willoughby, 2017; Usami et al., 2019) and results from this empirical data, applied researchers should use comprehensive judgments based on indices such as CFI, TLI, RMSEA, SRMR, and AIC.

4.2 Other Issues and Future Directions

For model selection, recent scholars have proposed "structural equation model trees (SEM trees)" and "structural equation model forests (SEM forests)," borrowing decision tree concepts from management psychology to provide evidence for model selection (Brandmaier et al., 2016; Brandmaier, Von et al., 2013). Future simulation studies could provide recommendations for model selection, such as how to select the true model, the cost of model misspecification, and which models are recommended when between-person/within-person differences reach certain levels.

On the other hand, constructing time-varying models can make model estimation more precise. For example, time-invariant effects in autoregressive models can be relaxed to some extent within the SEM framework; carry-over effects β_t can be freely estimated across different measurement occasions, with fewer model constraints that can effectively improve overall model fit. Bringmann et al. (2018) proposed a time-variant autoregressive model, assuming that autoregressive influences differ across time intervals (Bringmann et al., 2018). If the theoretical assumption of time-invariant carry-over effects is uncertain, this assumption can be relaxed for better overall fit (Ding et al., 2020). However, parameter identification issues need to be noted.

Additionally, researchers have different precision requirements for time variables depending on measurement intervals in longitudinal studies. In the SEM framework, the loading matrix Λ mostly fixes measurement times, which increases estimation error and reduces model sensitivity (Liu et al., 2015; Sterba, 2014). Based on this, borrowing from the multilevel modeling framework, the assumption of fixed measurement times can be relaxed, and SEM has gradually begun using continuous time models (CTM), defining participants' involvement time as a random number rather than fixed measurement occasions (Deboeck & Preacher, 2016; Hamaker & Wichers, 2017; Schuurman et al., 2016). In newer versions of SEM software (e.g., Mplus 8.0), such issues have already received technical support for flexible use by researchers.

References

Bainter, S. A., & Howard, A. L. (2016). Comparing within-person effects from multivariate longitudinal models. *Developmental Psychology, 52*(12), 1955–1968.

Berry, D., & Willoughby, M. T. (2017). On the practical interpretability of cross-lagged panel models: Rethinking a developmental workhorse. *Child Development, 88*(4), 1186–1206.

Bianconcini, S., & Bollen, K. A. (2018). The latent variable-autoregressive latent trajectory model: A general framework for longitudinal data analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 25*(5), 791–808.

Bollen, K. A., & Curran, P. J. (2006). *Latent curve models: A structural equation approach.* NJ: Wiley.

Brandmaier, A. M., Prindle, J. J., McArdle, J. J., & Lindenberger, U. (2016). Theory-guided exploration with structural equation model forests. *Psychological Methods, 21*(4), 566–582.

Brandmaier, A. M., Von Oertzen, T., McArdle, J. J., & Lindenberger, U. (2013). Structural equation model trees. *Psychological Methods, 18*(1), 71–86.

Bringmann, L. F., Ferrer, E., Hamaker, E. L., Borsboom, D., & Tuerlinckx, F. (2018). Modeling nonstationary emotion dynamics in dyads using a time-varying vector-autoregressive model. *Multivariate Behavioral Research, 53*(3), 293–314.

Cacioppo, J. T., Chen, H. Y., & Cacioppo, S. (2017). Reciprocal influences between loneliness and self-centeredness: A cross-lagged panel analysis in a population-based sample of African American, Hispanic, and Caucasian adults. *Personality and Social Psychology Bulletin, 43*(8), 1125–1135.

Chiu, C.-J., & Du, Y.-F. (2019). Longitudinal investigation of the reciprocal relationship between depressive symptoms and glycemic control: The moderation effects of sex and perceived support. *Journal of Diabetes Investigation, 10*(3), 801–808.

Cole, D. A., Martin, N. C., & Steiger, J. H. (2005). Empirical and conceptual problems with longitudinal trait-state models: Introducing a trait-state-occasion model. *Psychological Methods, 10*(1), 3–20.

Curran, P. J., & Bauer, D. J. (2011). The disaggregation of within-person and between-person effects in longitudinal models of change. *Annual Review of Psychology, 62*(1), 583–619.

Curran, P. J., & Bollen, K. A. (2001). The best of both worlds: Combining autoregressive and latent curve models. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 105–136). Washington, DC: American Psychological Association.

Curran, P. J., Howard, A. L., Bainter, S. A., Lane, S. T., & McGinley, J. S. (2014). The separation of between-person and within-person components of individual change over time: A latent curve model with structured residuals. *Journal of Consulting and Clinical Psychology, 82*(5), 879–894.

Deboeck, P. R., & Preacher, K. J. (2016). No need to be discrete: A method for continuous time mediation analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 23*(1), 61–75.

Diallo, T. M. O., Morin, A. J. S., & Lu, H. (2017). The impact of total and partial inclusion or exclusion of active and inactive time invariant covariates in growth mixture models. *Psychological Methods, 22*(1), 166–190.

Ding, X., Ansari, A., Li, X., Liu, Y., & Yan, N. (2020). Transactional effects between parental sensitivity and child social adjustment: Specifying trait-state aspects of parenting. *Developmental Psychology, 56*(7), 1331–1345.

Hamaker, E. L., Kuiper, R. M., & Grasman, R. P. P. P. (2015). A critique of the cross-lagged panel model. *Psychological Methods, 20*(1), 102–116.

Hamaker, E. L., & Wichers, M. (2017). No time like the present: Discovering the hidden dynamics in intensive longitudinal data. *Current Directions in Psychological Science, 26*(1), 10–15.

Hazel, N. A., & Hankin, B. L. (2014). A trait-state-error model of adult hassles over two years: Magnitude, sources, and predictors of stress continuity. *Journal* *of Social and Clinical Psychology, 33*(2), 103–123.

Jongerling, J., & Hamaker, E. L. (2011). On the trajectories of the predetermined ALT model: What are we really modeling. *Structural Equation Modeling: A Multidisciplinary Journal, 18*(3), 370–382.

Jöreskog, K. G. (1970). A general method for analysis of covariance structures. *Biometrika*(2), 239–251.

Jöreskog, K. G., & Sörbom, D. (1974). *Advances in factor analysis and structural equation models*. Cambridge, MA: Abt Books.

Kenny, D. A., & Harackiewicz, J. M. (1979). Cross-lagged panel correlation: Practice and promise. *Journal of Applied Psychology, 64*(4), 372–379.

Kenny, D. A., & Zautra, A. (1995). The trait-state-error model for multiwave data. *Journal of Consulting & Clinical Psychology, 63*(1), 52–59.

Kenny, D. A., & Zautra, A. (2001). Trait-state models for longitudinal data. In *New methods for the analysis of change* (pp. 243–263). Washington, DC, US: American Psychological Association.

Kiken, L. G., Garland, E. L., Bluth, K., Palsson, O. S., & Gaylord, S. A. (2015). From a state to a trait: Trajectories of state mindfulness in meditation during intervention predict changes in trait mindfulness. *Personality and Individual Differences, 81*, 41–46.

Liu, Y., Hau, K.-T., Liu, H., Wu, J., Wang, X., & Zheng, X. (2020). Multiplicative effect of intrinsic and extrinsic motivation on academic performance: A longitudinal study of Chinese students. *Journal of Personality, 88*(3), 584–595.

Liu, Y., & Hou, S. (2018). Potential reciprocal relationship between motivation and achievement: A longitudinal study. *School Psychology International, 39*(1), 38–55.

Liu, Y., Liu, H., & Hau, K.-t. (2016). Reading ability development from kindergarten to junior secondary: Latent transition analyses with growth mixture modeling. *Frontiers in Psychology, 7*, 1659.

Liu, Y., Liu, H., Li, H., & Zhao, Q. (2015). The effects of individually varying times of observations on growth parameter estimations in piecewise growth model. *Journal of Applied Statistics, 42*(9), 1843–1860.

Liu, Y., Liu, H., & Zheng, X. (2018). Piecewise growth mixture model with more than one unknown knot: An application in reading development. *Nonlinear Dynamics Psychology and Life Sciences, 22*(4), 485–507.

Lock, E. F., Kohli, N., & Bose, M. (2018). Detecting multiple random changepoints in Bayesian piecewise growth mixture models. *Psychometrika, 83*(3), 733–750.

Luhmann, M., Schimmack, U., & Eid, M. (2011). Stability and variability in the relationship between subjective well-being and income. *Journal of Research in Personality, 45*(2), 186–197.

Malone, P. S., Lansford, J. E., Castellino, D. R., Berlin, L. J., Dodge, K. A., Bates, J. E., & Pettit, G. S. (2004). Divorce and child behavior problems: Applying latent change score models to life event data. *Structural Equation Modeling: A Multidisciplinary Journal, 11*(3), 401–423.

Masselink, M., Van Roekel, E., Hankin, B. L., Keijsers, L., Lodder, G. M. A., Vanhalst, J., . . . Oldehinkel, A. J. (2018). The longitudinal association between self-esteem and depressive symptoms in adolescents: Separating between-person effects from within-person effects. *European Journal of Personality, 32*(6), 653-671.

McArdle, J. J. (2009). Latent variable modeling of differences and changes with longitudinal data. *Annual Review of Psychology, 60*, 577–605.

McArdle, J. J., & Hamagami, F. (2001). Latent difference score structural models for linear dynamic analyses with incomplete longitudinal data. In *New methods for the analysis of change* (pp. 139–175). Washington, DC, US: American Psychological Association.

McArdle, J. J., & Nesselroade, J. R. (2014). *Longitudinal data analysis using structural equation models.* Washington, DC: American Psychological Association.

McCoach, D. B., & Kaniskan, B. (2010). Using time-varying covariates in multilevel growth models. *Frontiers in Psychology, 1*:17.

Mulder, J. D., & Hamaker, E. L. (2020). Three extensions of the random intercept cross-lagged panel model. *Structural Equation Modeling: A Multidisciplinary Journal, 1–11.*

Murayama, K., Goetz, T., Malmberg, L.-E., Pekrun, R., Tanaka, A., & Martin, A. J. (2017). Within-person analysis in educational psychology: Importance and illustrations. *British Journal of Educational Psychology Monograph Series II, 12*, 71–87.

Schmitt, M., & Steyer, R. (1993). A latent state-trait model (not only) for social desirability. *Personality and Individual Differences, 14*(4), 519–529.

Schuurman, N. K., Ferrer, E., De Boersonnenschein, M., & Hamaker, E. L. (2016). How to compare cross-lagged associations in a multilevel autoregressive model. *Psychological Methods, 21*(2), 206–221.

Selzler, A.-M., Rodgers, W. M., Berry, T. R., McFadden, K., Husband, C., & Hall, C. (2019). Reciprocal relationships between self-efficacy, outcome satisfaction, and attendance at an exercise programme. *British Journal of Health Psychology, 24*(1), 123-140.

Sterba, S. K. (2014). Fitting nonlinear latent growth curve models with individually varying time points. *Structural Equation Modeling: A Multidisciplinary Journal, 21*(4), 630-647.

Usami, S., Hayes, T., & McArdle, J. J. (2015). On the mathematical relationship between latent change score and autoregressive cross-lagged factor approaches: Cautions for inferring causal relationship between variables. *Multivariate Behavioral Research, 50*(6), 676-687.

Usami, S., Hayes, T., & McArdle, J. J. (2016). Inferring longitudinal relationships between variables: Model selection between the latent change score and autoregressive cross-lagged factor models. *Structural Equation Modeling: A Multidisciplinary Journal, 23*, 331-342.

Usami, S., Murayama, K., & Hamaker, E. (2019). A unified framework of longitudinal models to examine reciprocal relations. *Psychological Methods, 24*(5), 637-657.

Voelkle, M. C., Brose, A., Schmiedek, F., & Lindenberger, U. (2014). Toward a unified framework for the study of between-person and within-person structures: Building a bridge between two research paradigms. *Multivariate Behavioral Research, 49*(3), 193-213.

Zautra, A. J., Marbach, J. J., Raphael, K. G., Dohrenwend, B. P., & Kenny, D. A. (1995). The examination of myofascial face pain and its relationship to psychological distress among women. *Health Psychology, 14*(3), 223-231.

Zyphur, M. J., Allison, P. D., Tay, L., Voelkle, M. C., Preacher, K. J., Zhang, Z., . . . Diener, E. (2020). From data to causes I: Building a general cross-lagged panel model (GCLM). *Organizational Research Methods, 23*(4), 651-687.

Zyphur, M. J., Voelkle, M. C., Tay, L., Allison, P. D., Preacher, K. J., Zhang, Z., . . . Diener, E. (2020). From data to causes II: Comparing approaches to panel data analysis. *Organizational Research Methods, 23*(4), 688-716.

Appendix 1. Parameter Estimates for Linear, Quadratic, and Piecewise Growth Models

Parameter	Linear Growth Model (LGM)	Quadratic Growth Model (QGM)	Piecewise Growth Model (PGM)
Means			
Intercept	36.47***	36.47***	36.47***
Slope	15.23***	15.23***	18.45***
Slope^2	–	-0.89***	-1.12***
Variances			
Intercept	89.56***	89.56***	89.56***
Slope	12.34***	12.34***	15.67***
Slope^2	–	0.35***	0.73***

Parameter	Linear Growth Model (LGM)	Quadratic Growth Model (QGM)	Piecewise Growth Model (PGM)
Residuals	45.67***	28.91***	22.34***
Correlations			
Intercept with Slope	-0.55***	-0.55***	-0.60***
Intercept with Slope^2	–	0.02***	0.04***
Slope with Slope^2	–	-0.02***	-0.01***

****p < .001***

Appendix 2. Mplus Syntax and Diagram for Latent Variable Autoregressive Latent Trajectory (LV-ALT) Model

```
TITLE: this is an example of LV-ALT
DATA: FILE IS eclsk_{theta}+alt.dat;
VARIABLE:
  NAMES ARE C1R4RTHT_R C1R4MTHT_R C1RGTHT_R C2R4RTHT_R C2R4MTHT_R C2RGTHT_R
          C3R4RTHT_R C3R4MTHT_R C3RGTHT_R C4R4RTHT_R C4R4MTHT_R C4RGTHT_R
          C5R4RTHT_R C5R4MTHT_R C5R2STHT_R C6R4RTHT_R C6R4MTHT_R C6R2STHT_R
          C7R4RTHT_R C7R4MTHT_R C7R2STHT_R P1LEARN P2LEARN P4LEARN T1LEARN
          T2LEARN T4LEARN T5LEARN T6LEARN;
  USEVARIABLES ARE C1R4RTHT_R C2R4RTHT_R C4R4RTHT_R C5R4RTHT_R C6R4RTHT_R C7R4RTHT_R
                   C1R4MTHT_R C2R4MTHT_R C4R4MTHT_R C5R4MTHT_R C6R4MTHT_R C7R4MTHT_R;
  MISSING ARE ALL (-9);
  MISSING ARE ALL (-8);
  MISSING ARE ALL (-7);
  MISSING ARE ALL (-1);
ANALYSIS:
  MODEL = NOCOVARIANCES;
  estimator=mlr;
  miteration = 5000;
  convergence=0.01;
MODEL:
  ! Define growth (excluding first measurement as exogenous, define piecewise latent growth
  ! first three measurements on slope 1, after third measurement on slope 2)
  INTR by C2R4RTHT_R@1 C4R4RTHT_R@1 C5R4RTHT_R@1 C6R4RTHT_R@1 C7R4RTHT_R@1;
  SLPR by C2R4RTHT_R@0.5 C4R4RTHT_R@1.5 C5R4RTHT_R@3.5 C6R4RTHT_R@5.5 C7R4RTHT_R@8.5;
  SLPR2 by C2R4RTHT_R@0 C4R4RTHT_R@0 C5R4RTHT_R@2 C6R4RTHT_R@4 C7R4RTHT_R@7;
```

```
  INTM by C2R4MTHT_R@1 C4R4MTHT_R@1 C5R4MTHT_R@1 C6R4MTHT_R@1 C7R4MTHT_R@1;
  SLPM by C2R4MTHT_R@0.5 C4R4MTHT_R@1.5 C5R4MTHT_R@3.5 C6R4MTHT_R@5.5 C7R4MTHT_R@8.5;
  SLPM2 by C2R4MTHT_R@0 C4R4MTHT_R@0 C5R4MTHT_R@2 C6R4MTHT_R@4 C7R4MTHT_R@7;
  [C2R4RTHT_R-C7R4MTHT_R@0];
  INTR SLPR INTM SLPM slpr2 slpm2;
  [INTR SLPR INTM SLPM slpr2 slpm2];
  intr with slpr slpr2; slpr with slpr2;
  intm with slpm slpm2; slpm with slpm2;

  ! Define state factors (define state variables, i.e., reliability of each measurement)
  R1 BY C1R4RTHT_R@1; R2 BY C2R4RTHT_R@1; R4 BY C4R4RTHT_R@1;
  R5 BY C5R4RTHT_R@1; R6 BY C6R4RTHT_R@1; R7 BY C7R4RTHT_R@1;
  M1 BY C1R4MTHT_R@1; M2 BY C2R4MTHT_R@1; M4 BY C4R4MTHT_R@1;
  M5 BY C5R4MTHT_R@1; M6 BY C6R4MTHT_R@1; M7 BY C7R4MTHT_R@1;

  ! Define exogenous indicators (define first measurement factors as exogenous variables)
  R1 with intr slpr slpr2; M1 with intm slpm slpm2;

  ! Define measurement errors
  C1R4RTHT_R-C7R4RTHT_R(1); C1R4MTHT_R-C7R4MTHT_R(2);

  ! Define autoregressive and cross-lagged regression
  R2 ON R1; R4 ON R2; R5 ON R4; R6 ON R5; R7 ON R6;
  M2 ON M1; M4 ON M2; M5 ON M4; M6 ON M5; M7 ON M6;
  M2 ON R1; M4 ON R2; M5 ON R4; M6 ON R5; M7 ON R6;
  R2 ON M1; R4 ON M2; R5 ON M4; R6 ON M5; R7 ON M6;

  ! Define residuals (define exogenous variable variances, within-person residuals)
  R1; R2-R7(3); M1; M2-M7(4);

  ! Define correlation between residuals (define factor covariances/correlations)
  R1 WITH M1; R2 WITH M2; R4 WITH M4; R5 WITH M5; R6 WITH M6; R7 WITH M7;
OUTPUT: STDYX; tech4; tech1;
```

Appendix 3. Mplus Syntax and Diagram for Factor Latent Curve Model with Structured Residuals (FLCM-SR)

```
TITLE: this is an example of FLCM-SR
DATA: FILE IS eclsk_{theta}+alt.dat;
VARIABLE:
  NAMES ARE C1R4RTHT_R C1R4MTHT_R C1RGTHT_R C2R4RTHT_R C2R4MTHT_R C2RGTHT_R
          C3R4RTHT_R C3R4MTHT_R C3RGTHT_R C4R4RTHT_R C4R4MTHT_R C4RGTHT_R
          C5R4RTHT_R C5R4MTHT_R C5R2STHT_R C6R4RTHT_R C6R4MTHT_R C6R2STHT_R
          C7R4RTHT_R C7R4MTHT_R C7R2STHT_R P1LEARN P2LEARN P4LEARN T1LEARN
          T2LEARN T4LEARN T5LEARN T6LEARN;
  USEVARIABLES ARE C1R4RTHT_R C2R4RTHT_R C4R4RTHT_R C5R4RTHT_R C6R4RTHT_R C7R4RTHT_R
```

```
                 C1R4MTHT_R C2R4MTHT_R C4R4MTHT_R C5R4MTHT_R C6R4MTHT_R C7R4MTHT_R;
  MISSING ARE ALL (-9);
  MISSING ARE ALL (-8);
  MISSING ARE ALL (-7);
  MISSING ARE ALL (-1);
ANALYSIS:
  MODEL = NOCOVARIANCES;
  estimator=mlr;
  miteration = 5000;
  convergence=0.01;
MODEL:
  ! Define growth (define piecewise latent growth model: first three measurements on slope 1
  INTR by C1R4RTHT_R@1 C2R4RTHT_R@1 C4R4RTHT_R@1 C5R4RTHT_R@1 C6R4RTHT_R@1 C7R4RTHT_R@1;
  SLPR by C1R4RTHT_R@0 C2R4RTHT_R@0.5 C4R4RTHT_R@1.5 C5R4RTHT_R@3.5 C6R4RTHT_R@5.5 C7R4RTHT_
  SLPR2 by C1R4RTHT_R@0 C2R4RTHT_R@0 C4R4RTHT_R@0 C5R4RTHT_R@2 C6R4RTHT_R@4 C7R4RTHT_R@7;
  INTM by C1R4MTHT_R@1 C2R4MTHT_R@1 C4R4MTHT_R@1 C5R4MTHT_R@1 C6R4MTHT_R@1 C7R4MTHT_R@1;
  SLPM by C1R4MTHT_R@0 C2R4MTHT_R@0.5 C4R4MTHT_R@1.5 C5R4MTHT_R@3.5 C6R4MTHT_R@5.5 C7R4MTHT_
  SLPM2 by C1R4MTHT_R@0 C2R4MTHT_R@0 C4R4MTHT_R@0 C5R4MTHT_R@2 C6R4MTHT_R@4 C7R4MTHT_R@7;
  [C1R4RTHT_R-C7R4MTHT_R@0];
  INTR SLPR INTM SLPM slpr2 slpm2;
  [INTR SLPR INTM SLPM slpr2 slpm2];
  intr with slpr slpr2; slpr with slpr2;
  intm with slpm slpm2; slpm with slpm2;

  ! Define state factors (define state variables, i.e., reliability of each measurement)
  R1 BY C1R4RTHT_R@1; R2 BY C2R4RTHT_R@1; R4 BY C4R4RTHT_R@1;
  R5 BY C5R4RTHT_R@1; R6 BY C6R4RTHT_R@1; R7 BY C7R4RTHT_R@1;
  M1 BY C1R4MTHT_R@1; M2 BY C2R4MTHT_R@1; M4 BY C4R4MTHT_R@1;
  M5 BY C5R4MTHT_R@1; M6 BY C6R4MTHT_R@1; M7 BY C7R4MTHT_R@1;

  ! Define measurement errors
  C1R4RTHT_R-C7R4RTHT_R; C1R4MTHT_R-C7R4MTHT_R;

  ! Define autoregressive and cross-lagged regression
  R2 ON R1; R4 ON R2; R5 ON R4; R6 ON R5; R7 ON R6;
  M2 ON M1; M4 ON M2; M5 ON M4; M6 ON M5; M7 ON M6;
  M2 ON R1; M4 ON R2; M5 ON R4; M6 ON R5; M7 ON R6;
  R2 ON M1; R4 ON M2; R5 ON M4; R6 ON M5; R7 ON M6;

  ! Define residuals (define within-person residuals)
  R1; R2-R7(3); M1; M2-M7(4);

  ! Define correlation between residuals (define factor covariances/correlations)
  R1 WITH M1; R2 WITH M2; R4 WITH M4; R5 WITH M5; R6 WITH M6; R7 WITH M7;
OUTPUT: STDYX; tech4; tech1;
```