CET: A New Complex Evidence Theory

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Abstract

Dempster-Shafer evidence theory, as an extension of probability theory, is widely used in the field of information fusion because it satisfies weaker conditions than probability theory when dealing with uncertain information. Nevertheless, the description space of current evidence theory is limited to real space only, and it cannot effectively describe and process uncertain information when confronted with multidimensional characteristic data and periodic data with phase angle changes. To address this gap, this paper extends Dempster-Shafer evidence theory to complex Dempster-Shafer evidence theory. In complex Dempster-Shafer evidence theory, the mass function used to describe uncertain information is extended from real space to complex space, termed as complex mass function, and the modulus of the mass function indicates the degree of support for a proposition. On this basis, other basic concepts used to describe uncertain information are also defined and discussed, such as complex belief function, complex plausibility function, etc. To perfect complex Dempster-Shafer evidence theory, the complex Dempster combination rule (CDCR) is supplemented. CDCR is an extension of Dempster combination rule (CDR), which satisfies the commutative and associative laws just as CDR does, and it can degenerate into CDR under certain conditions. In addition, we propose a method to generate complex mass function and apply it to target recognition. The recognition results show that compared with the mass function in real space, the target recognition rate can be higher when using complex mass function to describe uncertain information.

Full Text

Preamble

CET: A New Complex Evidence Theory

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Abstract

Dempster-Shafer evidence theory, as an extension of probability theory, is widely used in the field of information fusion due to its ability to satisfy weaker conditions than probability theory when dealing with uncertain information. Nevertheless, the description space of current evidence theory is limited to real space, and it cannot effectively describe and process uncertain information when faced with multidimensional characteristic data and periodic data with phase-angle changes.

To address this gap, this paper extends Dempster-Shafer evidence theory to complex Dempster-Shafer evidence theory. In complex Dempster-Shafer evidence theory, the mass function used to describe uncertain information extends from real space to complex space, termed the complex mass function, where the modulus of the mass function indicates the degree of support for a proposition. On this basis, other fundamental concepts used to describe uncertain information are also defined and discussed, such as complex belief function and complex plausibility function. To complete the complex Dempster-Shafer evidence theory framework, the complex Dempster combination rule (CDCR) is introduced. CDCR is an extension of the Dempster combination rule (DCR) that satisfies the commutative and associative laws just as DCR does, and it can degenerate into DCR under certain conditions. In addition, we propose a method to generate complex mass functions and apply it to target recognition. The recognition results demonstrate that compared with mass functions in the real plane, using complex mass functions to describe uncertain information can achieve higher target recognition rates.

Keywords: Dempster-Shafer evidence theory, complex Dempster-Shafer evidence theory, complex mass function, complex Dempster combination rule

1. Introduction

In nature, human cognition of things is often expressed through uncertain or imprecise information. To make such cognitive expression more reasonable, scholars have developed numerous theories to describe and handle uncertain or imprecise information, such as probability theory [?], Dempster-Shafer evidence theory [?, ?], fuzzy set theory [?], Atanassov' s intuitionistic fuzzy sets [?], entropy-based methods [?, ?], and others [?]. On the one hand, scholars have extended these theories to related frameworks, such as evidential reasoning [?], fuzzy soft sets [?], information volume [?], interval-valued Atanassov's fuzzy sets [?], Pythagorean fuzzy sets [?], quantum mass functions [?], and other hybrid methods [?, ?, ?, ?]. On the other hand, since these theories each have their own advantages, researchers have applied them to various fields including target classification [?, ?, ?, ?, ?], cluster analysis [?, ?, ?, ?, ?], medical diagnosis [?, ?, ?, ?, ?], reliability analysis [?, ?, ?, ?, ?], multi-criteria decision making [?, ?, ?, ?, ?], and other cross-disciplinary applications [?, ?, ?, ?, ?].

Dempster-Shafer evidence theory serves as a mathematical tool for dealing with uncertain information and represents a generalization of probability theory. Compared with probability theory, evidence theory satisfies weaker uncertainty conditions, and the multi-element expression of propositions demonstrates its sufficient fault-tolerant capability. The Dempster combination rule (DCR) can gradually increase the degree of support for the objective proposition while decreasing support for non-objective propositions. From an algebraic operations perspective, DCR satisfies the commutative and associative laws of multiplication, which guarantees the stability and symmetry of algebraic operations. Due to these advantages, many decision theories have been extended and applied to various fields based on evidence theory. However, it has been found that the description of uncertain information in existing evidence theories is confined to real number space, leading to a lack of effective mathematical tools for describing and processing the multidimensional characteristics of uncertain information. Furthermore, when uncertain information is presented as periodic data with phase-angle changes, existing description methods cannot capture these variations [?]. Consequently, current research methods exhibit a non-negligible gap when describing and processing uncertain information.

Hence, in this paper, we extend evidence theory to complex evidence theory. Specifically, this includes expanding the description space of uncertain information from real space to complex space. Then, a new mass function is defined on the complex number plane, termed the complex mass function, where the support degree of the complex mass function for a proposition is non-negative and normalized. On this basis, some fundamental concepts are also mapped from real space to complex space, such as complex belief function, complex plausibility function, etc. Moreover, a new combination rule is proposed on the complex plane, called the complex Dempster combination rule (CDCR). CDCR inherits the advantages of DCR; it can not only gradually increase the support degree for the target proposition but also satisfy the commutative and associative laws of multiplication, ensuring the stability and symmetry of CDCR on the complex plane. When the complex mass function degenerates to the classical mass function, CDCR degenerates to DCR. In addition, we propose a method to generate complex mass functions. Within the framework of complex evidence theory, complex mass functions are applied to target recognition to improve recognition accuracy.

The remainder of this paper is structured as follows. Section 2 introduces relevant background on evidence theory. Section 3 proposes complex Dempster-Shafer evidence theory and provides numerical examples for illustration. Section 4 presents a method for generating complex mass functions and applies it to target recognition. Section 5 summarizes this work.

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2. Preliminaries

This section provides a brief review of Dempster-Shafer evidence theory [**?**, **?**].

Definition 1. (Frame of discernment) Suppose there exists a positive definite non-empty set whose elements satisfy mutual exclusion, $\Theta = \{\ _1,\ _2, \cdots,\ \}$. Then this set is a frame of discernment (FOD), and its power set is described as follows:

$$2^{\Theta} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \cdots, \{\theta_n\}, \{\theta_1, \theta_2\}, \cdots, \Theta\}$$

where   is called the empty set.

Definition 2. (Mass function) The mass function is used to indicate the degree of support for a proposition. It is defined as $m : 2^{\Theta} \rightarrow [0, 1]$ which satisfies the following conditions:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \subseteq \Theta} m(A) = 1$$

When $m(A) > 0$, A is called a focal element of the mass function, and $m(A)$ indicates the degree of support for proposition A. The mass function $m(A)$ is also called the basic probability assignment (BPA).

Definition 3. (Dempster combination rule) Given two BPAs m_1 and m_2 in FOD Θ, the Dempster combination rule $m_1 \oplus m_2$ is formulated as follows:

$$m(A) = \frac{\sum_{i,j: A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{\sum_{i,j: A_i \cap B_j \neq \emptyset} m_1(A_i) m_2(B_j)} \quad \text{for } A \neq \emptyset$$

where $K = 1 - \sum_{i,j: A_i \cap B_j \neq \emptyset} m_1(A_i) m_2(B_j)$ is a normalization constant called the conflict coefficient. $K \in [0, 1]$, where $K = 0$ indicates no conflict between m_1 and m_2, and $K = 1$ indicates complete conflict between m_1 and m_2. Under normal circumstances, $0 \leq K < 1$.

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3. Complex Dempster-Shafer Evidence Theory

In this section, we introduce complex Dempster-Shafer evidence theory.

Consider a frame of discernment $K = \{\kappa_1, \kappa_2, \cdots, \kappa_n\}$. The power set of K is described as follows:

$$2^K = \{\emptyset, \{\kappa_1\}, \{\kappa_2\}, \cdots, \{\kappa_n\}, \{\kappa_1, \kappa_2\}, \cdots, K\}$$

The complex mass function on frame of discernment K is defined below:

Definition 4. (Complex mass function) Let $|CM|$ denote the modulus of complex number CM, which satisfies $|CM| : 2^K \to [0, 1]$ with:

$$CM(\emptyset) = 0, \quad CM(A) = M(A)e^{i\theta_A} = x_A + iy_A$$

where $M(A) = \sqrt{x_A^2 + y_A^2}$ represents how strongly this evidence supports proposition A (not x_A). The phase angle $\theta_A = \arctan(y_A/x_A)$, and $\theta_A \in [-\pi, \pi]$. When $\theta_A = 0$, the complex mass function degenerates to the traditional mass function, and $CM(A)$ becomes a real number. The complex mass function is also known as the complex basic probability assignment (CBPA). The following normalization condition holds:

$$\sum_{A \subseteq K} |CM(A)| = \sum_{A \subseteq K} M(A) = 1$$

A numerical example illustrates the difference between CBPA and BPA.

Example 3.1 Suppose there is a CBPA CM on $K = \{\kappa_1, \kappa_2, \kappa_3\}$:

$$\begin{aligned} CM(\kappa_2) &= 0.4832 + 0.1242i = 0.4989e^{i \cdot \arctan(0.1242/0.4832)}, \\ CM(\kappa_2, \kappa_3) &= 0.2403 + 0.1529i = 0.2848e^{i \cdot \arctan(0.1529/0.2403)}, \\ CM(\kappa_1, \kappa_2, \kappa_3) &= 0.0825 + 0.2000i = 0.2163e^{i \cdot \arctan(0.2000/0.0825)}. \end{aligned}$$

Obviously, the supporting degrees of evidence for propositions $\{\kappa_2\}$, $\{\kappa_2, \kappa_3\}$, and $\{\kappa_1, \kappa_2, \kappa_3\}$ are 0.4989, 0.2848, and 0.2163, respectively. It is worth noting that when $\theta = 0$, the above three CBPAs degenerate into BPAs, but the support degree of the proposition remains the original value. For a BPA, the degree of support for a proposition corresponds to a single expression, whereas for a CBPA, the degree of support corresponds to a set of expressions, with the BPA being just one element of this set. This relationship is illustrated more clearly in Figure 1 [Figure 1: see original paper].

In complex evidence theory, several important functions are defined as follows:

Definition 5. (Complex belief function)

$$CBel(A) = \sum_{B \subseteq A} CM(B)$$

Definition 6. (Complex plausibility function)

$$CPl(A) = \sum_{B \cap A \neq \emptyset} CM(B)$$

Here, $|CBel(A)|$ represents the minimum support degree of evidence for proposition A, and $|CPl(A)|$ represents the maximum support degree. Obviously, $|CBel(A)| \leq |CPl(A)|$, forming the upper and lower bounds of support degree $[|CBel(A)|, |CPl(A)|]$.

Definition 7. (Complex Pignistic probability transformation) Pignistic probability transformation was proposed by Smets to transform mass functions into probability distributions in real number space [?]. Therefore, we also transform complex mass functions into probability distributions through the following equation:

$$BetP(A) = \sum_{B \subseteq K, B \neq \emptyset} \frac{|A \cap B|}{|B|} \cdot \frac{|CM(B)|}{1 - |CM(\emptyset)|}, \quad \forall A \subseteq K$$

Let CM_1 and CM_2 be two CBPAs on K, with A_i and B_j being subsets of K. The complex Dempster combination rule (CDCR) of CM_1 and CM_2 is denoted $CM_1 \circledast CM_2$, defined as follows:

Definition 8. (Complex Dempster combination rule)

$$CM(A) = \frac{\sum_{i,j: A_i \cap B_j = A} CM_1(A_i) CM_2(B_j)}{\sum_{i,j: A_i \cap B_j \neq \emptyset} |CM_1(A_i) CM_2(B_j)|} \quad \text{for } A \neq \emptyset$$

In CDCR, the conflict coefficient is:

$$K(CM_1, CM_2) = 1 - \sum_{i,j: A_i \cap B_j \neq \emptyset} |CM_1(A_i) CM_2(B_j)|$$

where $K \in [0, 1]$ denotes the degree of conflict between evidences. The larger the K value, the greater the conflict and the lower the credibility of the fusion results. In particular, when $K = 1$, there is an irreconcilable conflict in the evidence. Additionally, for any CM_1 and CM_2, $K(CM_1, CM_2)$ satisfies the following properties:

- **P(1): Nonnegativity.** $K(CM_1, CM_2) \geq 0$
- **P(2): Symmetry.** $K(CM_1, CM_2) = K(CM_2, CM_1)$
- **P(3): Boundedness.** $0 \leq K(CM_1, CM_2) \leq 1$

Example 3.2 Suppose there are two CBPAs CM_1 and CM_2 on $K = \{\kappa_1, \kappa_2\}$, expressed as follows:

$$\begin{aligned} CM_1(\kappa_1) &= \rho e^{i\theta}, & CM_1(\kappa_2) &= (1-\rho) e^{i(\pi/2 - \theta)}, \\ CM_2(\kappa_1) &= (1-\rho) e^{i(\pi/2 - \theta)}, & CM_2(\kappa_2) &= \rho e^{i\theta}, \end{aligned}$$

where $\rho \in [0, 1]$ and $\theta \in [-\pi/2, \pi/2]$. The calculated conflict coefficient K is shown in Figure 2 [Figure 2: see original paper]. As can be seen from Figure 2, when $\rho = 0$ or $\rho = 1$, K is always 1 for any θ. When $\rho = 0.5$ and $\theta = 0.1492$, then $K = 0.5$.

Example 3.3 Two CBPAs CM_1 and CM_2 on $K = \{\kappa_1, \kappa_2, \kappa_3\}$ are described as follows:

$$\begin{aligned} CM_1: \quad & CM_1(\kappa_2) = 1; \\ CM_2: \quad & CM_2(\kappa_2) = 0.4921 + 0.0739i, \\ & CM_2(\kappa_2, \kappa_3) = 0.2937 + 0.0982i, \\ & CM_2(\kappa_1, \kappa_2, \kappa_3) = 0.1344 + 0.1381i. \end{aligned}$$

In this example, $K = 0.0289$. If we change CM_2 to $CM_2(\kappa_2) = 1$, then $K = 0$. If analyzed using classical Dempster-Shafer evidence theory, $K = 0$ in both cases. Obviously, the conflict coefficient K in complex evidence theory imposes stricter requirements on the expression form of CBPAs.

Example 3.4 Suppose there are four CBPAs on the frame of discernment $K = \{\kappa_1, \kappa_2, \kappa_3\}$, described as follows:

$$\begin{aligned} CM_1: \quad & CM_1(\kappa_1) = 0.7765 + 0.0009i, \\ & CM_1(\kappa_1, \kappa_2) = 0.0756 + 0.0674i, \\ & CM_1(\kappa_1, \kappa_2, \kappa_3) = 0.0464 + 0.1130i; \\ CM_2: \quad & CM_2(\kappa_2) = 0.5900 - 0.0068i, \\ & CM_2(\kappa_2, \kappa_3) = 0.4091 + 0.0263i; \\ CM_3: \quad & CM_3(\kappa_2) = 0.5430 - 0.0177i, \\ & CM_3(\kappa_2, \kappa_3) = 0.4564 + 0.0166i; \\ CM_4: \quad & CM_4(\kappa_2) = 0.5580 + 0.0287i, \\ & CM_4(\kappa_2, \kappa_3) = 0.3425 + 0.0649i, \\ & CM_4(\kappa_1, \kappa_2, \kappa_3) = 0.0910 + 0.0177i. \end{aligned}$$

Then, $CM_1 \circledast CM_2 \circledast CM_3 \circledast CM_4$ is calculated as follows:

$$CM(\kappa_2) = 0.4417 + 0.8452i, \quad CM(\kappa_2, \kappa_3) = 0.0047 + 0.0461i.$$

In the combined results, $|CM(\kappa_2)| + |CM(\kappa_2, \kappa_3)| = 1$. In addition, the support degree for κ_2 is the largest, with $|CM(\kappa_2)| = 0.9537$, which is a non-counterintuitive result.

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4. The Generation Method and Application of Complex Mass Function

In this section, we propose a generation method for complex mass functions and apply the generated complex mass functions to target recognition on the iris dataset.

[Figure 3: see original paper] The generation method of complex mass function

Method Description: Suppose there are N targets, denoted as $\{Class_1, Class_2, \cdots, Class_j, \cdots, Class_N\}$, each having K attributes represented as $\{Attribute_1, Attribute_2, \cdots, Attribute_i, \cdots, Attribute_K\}$. We can use the process shown in Figure 3 to generate complex mass functions for the jth attribute.

- **Step 1:** Select the training set and test set from the original dataset.
- **Step 2:** The N classes correspond to $2^N - 1$ propositions, denoted as $\mathcal{N} = \{Class_1, Class_2, \cdots, Class_{12}, \cdots, Class_{1,\cdots,N}\}$, where $Class_{12}$ represents the union of $Class_1$ and $Class_2$. Calculate the mean μ_{j_p} and standard deviation δ_{j_p} of the pth proposition.
- **Step 3:** Select an unknown target from the test set, add it to the training set, and recalculate the mean μ'_{j_p} and standard deviation δ'_{j_p} of the pth proposition.
- **Step 4:** The complex mass function of the pth proposition is generated by the following equation:

$$CM_j(Class_p) = \left(\frac{\delta'_{j_p}}{\delta_{j_p}}\right) e^{i\left(\frac{\mu'_{j_p}}{\mu_{j_p}}\right)}$$

Experimental Verification: We selected the Iris dataset, Seeds dataset, Banknote dataset, Blood Transfusion Service Center (BTSC) dataset, and Fertility-Diagnosis dataset from the UCI database (http://archive.ics.uci.edu/ml/index.php) for experiments. The dataset details are shown in Table 1 . For each dataset, 60% of the data were randomly selected as the training set and the remainder as the test set. We conducted 100 experiments, with results shown in Figure 4 [Figure 4: see original paper]. As can be seen from Figure 4, compared with mass functions in the real number plane [**?**], describing uncertain information using complex mass functions can improve target recognition accuracy, indicating that complex mass functions provide a more reasonable description of uncertain information.

Algorithm 1: Generation of Complex Mass Functions

```
Input: Training set and test set /* They all have n classes, each class has k attributes. */
Output: Complex Mass Functions for the test set
for j = 1 $\rightarrow$ k do
    for p = 1 $\rightarrow$ 2^N - 1 do
        Calculate  _{j_p}, $\delta$_{j_p},  '_{j_p}, $\delta$'_{j_p}
        Δ$\delta$_{j_p} = $\delta$'_{j_p} / $\delta$_{j_p}
        Δ _{j_p} =  '_{j_p} /  _{j_p}
        CM_j(Class_p) = Δ$\delta$_{j_p} · e^{i · Δ _{j_p}}
    end for
    return CM_j
end for
```

Table 1: Details of the Datasets | Dataset | Instances | Classes | Attributes | Attribute Distribution | Missing Values | |——|——-|——|——-|—————|

———-| | Iris | | | | | | | Seeds | | | | | | | Banknote| | | | | | | BTSC | | | | | | | Fertility-Diagnosis | | | | | |

[Figure 4: see original paper] The target recognition accuracy based on mass functions in real plane and complex plane

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5. Conclusion

To describe uncertain information with multidimensional features, we extended Dempster-Shafer evidence theory from real number space to complex number space and proposed complex Dempster-Shafer evidence theory. Within this framework, we defined complex mass functions, complex belief functions, and related concepts. Meanwhile, we proposed the complex Dempster combination rule, which satisfies the commutative and associative laws of multiplication. When a complex mass function degenerates to a traditional mass function, the complex Dempster combination rule also degenerates to the Dempster combination rule in real space. Additionally, we proposed a method to generate complex mass functions based on changes in the mean and variance of sample data. The results demonstrate that compared with mass functions in real number space, complex mass functions can effectively improve target recognition rates within the complex evidence theory framework.

In subsequent work, we will explain complex Dempster-Shafer evidence theory from an algebraic perspective, which will help perfect its theoretical foundation, and we will also apply complex Dempster-Shafer evidence theory to target classification, cluster analysis, intelligent decision making, and other fields to address more practical problems. In addition, since complex probability is related to the description of quantum systems [**?**, **?**], we will use complex evidence theory to further solve some quantum system problems, such as entanglement effects, interference effects, etc.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Note: Figure translations are in progress. See original paper for figures.