

Postprint: EEMD-Based Analysis of the Hallstatt Cycle in Solar Barycentric Motion and Solar Activity

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Abstract

Based on the kinematic equations of the planetary conjunction index, we derived a calculation formula for the variation in the distance from the Sun's barycenter to the Solar System's barycenter. Using this formula, we reconstructed a time series of the Sun's barycenter distance variation relative to the Solar System's barycenter as a proxy indicator for solar orbital motion. The Ensemble Empirical Mode Decomposition (EEMD) method was employed to analyze the time series of the distance from the Sun's barycenter to the Solar System's barycenter and the $\Delta 14C$ record from 0–25.0 ka BP. The results demonstrate that both contain Intrinsic Mode Function (IMF) components with a period of approximately 2300 years (Hallstatt cycle). Cross-correlation analysis confirms that the Hallstatt periodic components contained in both series are correlated, with correlation coefficients reaching 0.52 and 0.44 for the intervals 0–13.9 ka BP and 13.9–25.0 ka BP, respectively, and that the variation in the distance from the Sun's barycenter to the Solar System's barycenter leads the variation in the solar activity indicator $\Delta 14C$. This indicates that when the Sun's barycenter moves away from the Solar System's barycenter, atmospheric $14C$ content increases, corresponding to periods of weaker solar activity; when the Sun's barycenter approaches the Solar System's barycenter, atmospheric $14C$ content decreases, corresponding to periods of stronger solar activity. This understanding provides new evidence and insights for further investigating the formation mechanism of the Hallstatt cycle of solar activity.

Full Text

Hallstatt Cycle Analysis of Solar Inertial Motion and Solar Activity Based on EEMD

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Abstract

Based on the kinematic equation of the planetary juncture index, we derived a formula for calculating the distance between the solar centroid and the solar system's centroid. Using this formula, we reconstructed a time series of the solar centroid's distance variation relative to the solar system's centroid as a proxy for solar orbital motion. We then applied ensemble empirical mode decomposition (EEMD) to analyze this time series and the atmospheric radiocarbon ($\Delta^{14}\text{C}$) record. The results reveal that both sequences contain a ~2300-year Hallstatt cycle component, with cross-correlation analysis confirming their relationship. From 0 to 13.9 ka BP, the solar centroid's distance variation leads $\Delta^{14}\text{C}$ changes by approximately 320 years, while from 13.9 to 25.0 ka BP, the lead time is about 1870 years. This indicates that when the solar centroid moves away from the solar system's centroid, atmospheric $\Delta^{14}\text{C}$ content increases, corresponding to periods of weaker solar activity. Conversely, when the solar centroid approaches the solar system's centroid, $\Delta^{14}\text{C}$ content decreases, corresponding to periods of stronger solar activity. These findings provide new evidence and insights for understanding the physical mechanism underlying the Hallstatt cycle of solar activity.

Keywords: solar inertial motion; solar activity; Hallstatt cycle; ensemble empirical mode decomposition (EEMD)

1. Introduction

Solar activity directly or indirectly influences Earth's dynamic systems and profoundly affects numerous geophysical phenomena. Consequently, research on solar activity variations has long attracted scholarly attention. Solar activity exhibits periodic fluctuations, most notably the ~210-year de Vries cycle and the ~2300-year Hallstatt cycle. The Hallstatt cycle, detected in radiocarbon records, has been identified by multiple studies using various spectral analysis techniques. However, the physical origin of this prominent ~2300-year solar activity cycle remains debated, with no consensus explanation.

Current theories explaining solar activity variations fall into two main categories: the solar dynamo theory and the solar system planetary driving theory. The latter can be further divided into two mechanisms: (1) the gravitational tidal

effects of planetary systems on the Sun, and (2) the Sun's orbital motion around the solar system's centroid under planetary system influences. The latter has gradually become a research focus in recent years. Charvatova first proposed that the ~2100-2500 year oscillation might result from solar inertial motion, using a simplified model to simulate solar revolution with an average period equivalent to the alignment of Jupiter and Saturn. Subsequent studies have detected this cycle in various solar activity proxies and planetary motion indices.

When investigating relationships between solar orbital motion and $\Delta^{14}\text{C}$, previous research has relied on simple image comparisons or pre-defined function fitting, introducing artificial factors into the analysis. Moreover, most studies have focused on the Holocene period. This study addresses these limitations by deriving a formula for solar centroid distance variation based on the planetary juncture index, calculating the corresponding time series, and applying EEMD to directly decompose both the distance and $\Delta^{14}\text{C}$ sequences to extract their Hallstatt cycle components without imposing predetermined functional relationships.

2. Data and Methods

2.1 Calculation of Solar Centroid Distance Variation

2.1.1 Planetary Juncture Index Definition The invariable plane of the solar system passes through the solar system's centroid and is perpendicular to the total angular momentum vector. This plane has an average inclination of 1.84° relative to the orbital planes of the eight major planets. Since all outer planets and most inner planets (except Mercury with 6.35° inclination) have smaller inclinations, we can approximate all planetary orbital planes as lying in the invariable plane for coordinate system establishment.

Treating the eight major planets as point masses, let P_i ($i = 1-8$) represent the position vector endpoints of each planet weighted by mass, with m_i , ϕ_i , and r_i denoting each planet's mass, heliocentric longitude, and position vector radius relative to the solar centroid, respectively. The position vector r_P of the planetary system's centroid (P) relative to the solar centroid (S) is:

$$r_P = \overline{SP} = \frac{\sum_{i=1}^8 m_i r_i}{\sum_{i=1}^8 m_i}$$

This shows the planetary system centroid is the mass-weighted vector sum of each planet's distance from the solar centroid. Based on this principle, Liu and Wang defined the planetary juncture index K as the magnitude of the mass-weighted radius vector sum of planetary orbits:

$$K = \sqrt{\left(\sum_{i=1}^8 r_i \sin(\phi_i + \omega_i t)\right)^2 + \left(\sum_{i=1}^8 r_i \cos(\phi_i + \omega_i t)\right)^2}$$

where K is the planetary juncture index, r_i is the weighted orbital radius, ϕ_i is the initial heliocentric longitude, ω_i is the mean angular velocity, and t is time. This index represents both the degree of planetary conjunction/opposition and the distance between the planetary system centroid and solar centroid.

2.1.2 Distance Calculation Formula Treating the eight planets as a single system, the solar centroid and planetary system centroid revolve around the solar system' s centroid according to the two-body problem. Assuming masses M_S and M_P , and distances R_S and R_P from the solar system' s centroid, respectively, we have $M_S R_S = M_P R_P$. Since $K = R_S + R_P$, we derive:

$$R_S = \frac{K M_P}{M_S + M_P}$$

Using astronomical data for the eight planets (mean angular velocities, mass-weighted radii, heliocentric longitudes, and masses), we calculated the time series of solar centroid distance variation relative to the solar system' s centroid at 12:00 UTC resolution.

2.2 Radiocarbon Data Source

We used the INTCAL13 radiocarbon calibration curve spanning 0-50.0 ka BP. The data for 0-13.9 ka BP derive primarily from tree-ring chronologies, while older data (13.9-25.0 ka BP) come mainly from marine records (corals, foraminifera). This distinction is important because trees directly absorb atmospheric carbon, whereas marine organisms absorb dissolved carbon that may have been diluted over long periods. The long-term trend in $\Delta^{14}\text{C}$ data reflects changes in Earth' s magnetic dipole field, while short-term variations indicate solar activity changes.

Atmospheric ^{14}C is produced primarily through neutron interactions with atmospheric nitrogen. Solar activity modulates this production: during periods of strong solar activity (increased sunspots), enhanced solar wind—a plasma stream of protons and electrons—provides magnetic shielding that reduces cosmic ray flux and thus decreases ^{14}C production. Conversely, weak solar activity corresponds to higher ^{14}C production rates.

2.3 EEMD Analysis Method

Huang et al. proposed empirical mode decomposition (EMD) for analyzing non-linear, non-stationary signals, decomposing any signal into a series of intrinsic mode functions (IMFs) representing different oscillatory modes. However, EMD

can suffer from mode mixing, where a single IMF contains disparate scales or a scale appears in different IMFs, potentially yielding components without physical meaning.

To address this, Wu and Huang developed ensemble empirical mode decomposition (EEMD), which adds Gaussian white noise to the signal to avoid mode mixing. The added noise helps separate scales naturally, and through ensemble averaging, the noise is canceled out while the true signal components are retained. This adaptive method decomposes time series into physically meaningful IMFs and a residual trend term without requiring predetermined basis functions.

We applied EEMD to both the solar centroid distance and $\Delta^{14}\text{C}$ time series (0-25.0 ka BP) to directly extract their Hallstatt cycle components. The analysis used an ensemble size of 100 and noise standard deviation of 0.2 times the original data's standard deviation.

3. Results

3.1 EEMD Decomposition Results

The solar centroid distance varies between 0-2.2 solar radii, meaning at minimum the solar centroid nearly coincides with the solar system's centroid, while at maximum it exceeds one solar diameter from the centroid. EEMD decomposition of the distance time series yielded multiple IMFs, including a significant ~2300-year component (IMF10) corresponding to the Hallstatt cycle [Figure 2: see original paper].

Similarly, EEMD decomposition of the $\Delta^{14}\text{C}$ time series revealed a corresponding ~2300-year Hallstatt component [Figure 3: see original paper]. The periods of decomposed components are listed in Table 2. The ~2300-year periods from both series align closely, with the solar centroid distance component showing a period of 2273 years and the $\Delta^{14}\text{C}$ component showing 2456 years, both consistent with the Hallstatt cycle documented in literature.

3.2 Cross-Correlation Analysis

To quantify the relationship between the two Hallstatt components, we performed cross-correlation analysis using IBM SPSS Statistics 23. Given the different data sources, we analyzed two time intervals separately: 0-13.9 ka BP (tree-ring data) and 13.9-25.0 ka BP (marine records).

For 0-13.9 ka BP, the cross-correlation coefficient reaches its maximum value of 0.52 at a relative shift of 320 years (significant at the 0.01 level), indicating the solar centroid distance variation leads $\Delta^{14}\text{C}$ changes by 320 years [Figure 4: see original paper]. This lead time is similar to the ~580-year phase difference reported by Scafetta between planetary centroid motion and $\Delta^{14}\text{C}$ variations.

For 13.9-25.0 ka BP, the maximum correlation coefficient is 0.44 at a relative

shift of 1870 years (significant at the 0.01 level). The different phase relationship in this interval likely reflects the marine record characteristics, where dissolved carbon has undergone long-term dilution before incorporation into corals and foraminifera.

The positive correlation indicates that when the solar centroid moves away from the solar system' s centroid (increasing distance), atmospheric $\Delta^{14}\text{C}$ content increases, corresponding to weaker solar activity. Conversely, when the solar centroid approaches the solar system' s centroid, $\Delta^{14}\text{C}$ content decreases, indicating stronger solar activity. This relationship extends the study of solar inertial motion and solar activity beyond the Holocene to 25.0 ka BP.

4. Discussion and Conclusions

This study demonstrates that solar inertial motion may be the physical cause of the Hallstatt cycle observed in solar activity proxies. The mechanism likely involves angular momentum conservation: as the solar centroid moves farther from the solar system' s centroid, its orbital radius increases, increasing orbital angular momentum. Since total angular momentum is conserved, solar rotational angular momentum decreases, reducing rotation speed and weakening solar activity. The opposite occurs when the solar centroid approaches the solar system' s centroid.

The specific physical mechanisms remain to be fully elucidated. Possible pathways include: (1) direct gravitational or electromagnetic effects of planetary motions on the Sun and heliosphere, modulating cosmic ray flux and interplanetary dust concentration; and (2) angular momentum transfer affecting solar differential rotation and dynamo processes. The correlation between solar centroid distance and solar activity suggests that solar orbital motion may also influence Earth' s climate system, as evidenced by the coincidence of rapid climate events (e.g., at 8.2 ka, 10.3 ka, 5.9 ka, and 2.8 ka) with periods when the solar centroid was distant from the solar system' s centroid and solar activity was weak.

In summary: (1) We derived a formula for solar centroid distance variation based on the planetary juncture index and reconstructed the corresponding time series; (2) Using EEMD, we adaptively decomposed both the distance and $\Delta^{14}\text{C}$ time series to extract their intrinsic Hallstatt cycle components without imposing predetermined functions; (3) Cross-correlation analysis confirms a positive relationship between the ~ 2300 -year components, with the solar centroid distance variation leading $\Delta^{14}\text{C}$ changes by 320 years in the 0-13.9 ka BP interval and 1870 years in the 13.9-25.0 ka BP interval; (4) The results indicate that solar centroid distance variation correlates negatively with solar activity strength—greater distance corresponds to weaker solar activity (higher $\Delta^{14}\text{C}$), while smaller distance corresponds to stronger solar activity (lower $\Delta^{14}\text{C}$). These findings provide new evidence and methodological approaches for understanding the planetary driving mechanism of solar activity Hallstatt cycles.

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