Analogy between Ideal Gas and Vacuum

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Abstract

This work investigates the analogy between ideal gas and vacuum. Under this analogy, mass originates from the drift mass of the fluid, and under certain assumptions, the mass-energy equation and mass-velocity relationship can also be obtained, enabling the further derivation of the Lorentz transformation.

Full Text

Preamble

An Analogy with Ideal Gas and Vacuum

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In this paper, we draw an analogy between ideal gas and vacuum. Under this analogy, the inertial mass of a particle originates from its drift mass, and we derive the mass-energy and mass-velocity relations of special relativity under certain hypotheses. We then use these mass relations to reproduce time dilation, length contraction, and the Lorentz transformation—the key conclusions of special relativity.

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Introduction

As is well known, Newton described vacuum as an absolute rest space. With the development of the fluctuation theory of light, people realized that vacuum might be an elastic medium for light. The unified theory of electromagnetism originated with Maxwell through a special vacuum model in which he believed vacuum had two components: electric ether and magnetic ether. H. A. Lorentz

[1] developed Maxwell's ideas further, separating matter from vacuum and presenting electron theory, which represented the pinnacle of combining electromagnetic theory with ether. However, a puzzle remained: all experiments searching for evidence of absolute motion between Earth and ether failed. Through the efforts of Poincaré, Lorentz, and Einstein, a beautiful theory was developed that could explain this puzzle—special relativity [2, 3].

Special relativity has become a fundamental theory of modern physics and has been applied to many fields. In fact, there are two viewpoints regarding the Lorentz transformation, which is the most important formula of special relativity. One is based on a preferred reference frame, held by Poincaré, Lorentz, Robertson, Mansouri, and Sexl [4–8] among others. The other view holds that neither a preferred reference frame nor absolute time exists.

The former view is supported by the discovery of cosmic background radiation and has attracted attention [9]. However, how to obtain the same effects in the ether system from classical physics (not from experiments) is an important and interesting problem. In this paper, we draw an analogy between ideal gas and vacuum. Under this analogy, ether can be regarded as an ideal gas.

More importantly, we obtain the energy-mass and velocity-mass relations, as well as time dilation, length contraction, and the Lorentz transformation of special relativity from this model under certain hypotheses. At the same time, we point out that in our model, the inertial mass is simply the added mass. Time dilation and length contraction are important conclusions of special relativity [2, 3] and have been used to explain experiments such as Michelson-Morley [10] and Kennedy-Thorndike [11]. Length contraction, or Lorentz-FitzGerald contraction, was first proposed as a hypothesis by Hendrik Lorentz and George Francis FitzGerald [1]. Understanding these phenomena is a very interesting endeavor.

References [12, 13] propose a microscopic structure of black holes that can yield interesting thermodynamical results. Additionally, the ideal gas analogy with vacuum is very similar to the idea of the sonic analog of Hawking radiation in fluid [14]. These references indicate that spacetime inside a black hole has microscopic structure, and a natural thought is that vacuum also has microscopic structure.

In this paper, we draw an analogy between ideal gas and vacuum (the ideal gas model of vacuum). Under this analogy, we reproduce time dilation and length contraction. Furthermore, as the most important part of special relativity, we reproduce the Lorentz transformation using our model.

This paper is organized as follows. In Sec. II, we introduce the ideal gas model of vacuum. Mass-velocity relation and mass-energy relation are reproduced in Sec. III. Time dilation, length contraction, and Lorentz transformation are reproduced in Sec. IV and V respectively. In the final section, we provide a discussion.

II. The Ideal Gas Model of Vacuum

An ideal gas model of vacuum will be introduced in this section, and some interesting results will be given. The hypotheses of the ideal gas model of vacuum are based on 19th-century physics and are as follows:

1. The speed of light is the maximum speed of local fluid (ideal gas).
2. Particles with mass are spheres containing the ideal gas, and their states are identical to the environment when at rest. The ideal gas inside will be isentropic when moving.

In fact, there have been many attempts to explore the relation between fluid and relativity [15–17] and electromagnetic theory [18, 19], achieving some successes. The ideal gas model was even mentioned in these papers [20] and provided some prompts for us.

Special relativity describes relations between two inertial frames. To study these relations, we provide the important equations describing the states of a moving control body in ideal gas. Here, we consider one-dimensional isentropic flow of a control body in ideal gas, which satisfies [21, 22]:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\frac{v^2}{a^2}$$

where T is temperature, P is pressure, ρ is density, V is volume, $\beta^2 = \frac{v^2}{c_0^2}$, v is the velocity of the fluid, a is the speed of sound, and γ is the adiabatic index. The subscript zero corresponds to the stagnation state, while the moving state has no subscript.

Equation (1) presents the relations between stagnation and moving states for one-dimensional isentropic flow. One can note that the sonic speed in the rest ideal gas will be slower than its value in the moving gas.

III. Mass-Velocity Relation and Mass-Energy Relation

When we consider a sphere with volume V_0 in the ideal gas accelerated slowly from static, there will be a resistance on the sphere, which reads [21, 22]:

$$F^* = -m^* \frac{dv}{dt}$$

where $m^* = \frac{1}{2}\rho_0 V_0$ is called added mass or drift mass. The sphere satisfies:

$$F = (m_0 + m^*)\frac{dv}{dt}$$

where m_0 is the mass of the sphere. We then treat the object as an elemental particle satisfying hypothesis 3, where the ideal gas is vacuum. According to hypothesis 3 and Eq. (2), and considering the motion as one-dimensional isentropic flow, we can obtain:

$$\rho_0 V_0 v^2 + \rho_0 V_0 c_0^2 = \text{constant}$$

where $\rho_0 V_0 = m_0$. The total energy of the sphere is E_0, called static enthalpy, with the form:

$$E_0 = E_{ks} + E_h = \frac{1}{2}\rho_0 V_0 v^2 + \frac{\gamma}{\gamma - 1}\rho_0 V_0 c_0^2$$

where E_{ks} and E_h are the kinetic energy and enthalpy of the sphere, respectively. E_0 is an invariant when the sphere moves in the fluid.

The total energy of the system (containing the total energy of the sphere and the kinetic energy of the fluid) will be:

$$E = E_0 + E_{kf} = E_{ks} + E_h + E_{kf} = E_k + E_h$$

where E_{kf} is the kinetic energy of the fluid and E_k is the kinetic energy of the system. As E_0 is an invariant, we can redefine E_{kf} as the kinetic energy of the system:

$$E = E_0 + E_{kf} = E_0 + E_k$$

The system only presents its added mass to the outside fluid, which is considered to be the inertial mass. In this way, the system satisfies Newton's second law $F = ma$. The added mass $m^* = \frac{1}{2}\rho_0 V_0$ is obtained in slow motion.

We denote m as the inertial mass in equation (5), deleting the superscript star. We must emphasize that equation (7) shows that the rest energy from special relativity is the total energy of the object in our model.

When a particle moves with velocity v, its volume will dilate and its added mass will become larger according to assumption 3 and equation (5). To obtain the relation between added mass and v, we consider the following process.

In the rest frame (area I in Fig. 1 [Figure 1: see original paper]), a stationary particle with mass M_0 decays into two particles (named B and C) with equal mass $m(v)$ (where mass is a function of velocity and $m(0) = m_0$) along the x-axis (see FIG. 1 area I). Since energy is invariant, we obtain:

$$E(M_0, 0) = M_0 c_0^2 = 2E(m_0, v)$$

where $E(M_0, 0)$ denotes the total energy of the system and $E(m_0, v)$ is the total energy of the final state particle (B or C), with energy as a function of mass and velocity. The kinetic energy of the final state reads:

$$E_k = 2E(m_0, v) - 2m_0c_0^2 = E(M_0, 0) - 2m_0c_0^2$$

For mass to be invariant, $M_0 = 2m(v)$, and the energy reads $E(m_0, v) = m(v)c_0^2$.

Conservation of mass can be obtained from the following consideration. In the center-of-mass frame of particle B (see FIG. 1 area II), we assume momentum is invariant, so:

$$M_0v = m(v)(v - v) + m(v)(v + v) = 2m(v)v$$

where we consider mass to be invariant in different frames.

Via the relation between E_k, m, and mv in one-dimensional motion, with force and work defined by $f = d(mv)/dt$ and $dw = f \cdot ds$ respectively, we can solve:

$$m(v) = m_0(1 - \beta^2)^{-1/2}$$

where $\beta^2 = v^2/c_0^2$.

IV. Time Dilation and Length Contraction

In this section, we use several thought experiments to deduce time dilation and length contraction. We consider a moving sphere with speed v_0 in a box with a fully elastic boundary (see FIG. 3 [Figure 3: see original paper]) as a clock. The mass of the sphere is $m(v_0) = m_0(1 - v_0^2/c_0^2)^{-1/2}$ when the clock is at rest. If we accelerate the clock to speed v, the momentum of the sphere in the direction perpendicular to the x-axis is conserved:

$$m_0(1 - v_0^2/c_0^2)^{-1/2}v_0 = m_0(1 - ({v'}^2 + v^2)/c_0^2)^{-1/2}v'$$

A. Period Motion I: Light Clock

We first present a light clock. As shown in FIG. 2 [Figure 2: see original paper], two parallel mirrors in the $x - y$ plane with light reflecting between them construct a light clock. The period ratio between the rest clock and the moving clock is:

$$\frac{\tau'}{\tau} = \sqrt{1 - \beta^2}$$

where $\beta^2 = v^2/c_0^2$, v is the speed of the light clock, c_0 is the light speed, τ' and τ are the periods of the moving and rest clocks respectively. L_x, L_y and L'_x, L'_y are components of the rest and moving equipment respectively. We assume:

$$\frac{L'_x}{L_x} = g(v), \quad \frac{L'_y}{L_y} = h(v)$$

The period ratio between rest and moving clocks becomes:

$$\frac{\tau'}{\tau} = \frac{g(v)}{\sqrt{1-\beta^2}}$$

We assume the period ratio between a rest and moving clock depends only on the magnitude of velocity, giving:

$$\frac{\tau'}{\tau} = \sqrt{1-\beta^2} = \frac{g(v)}{\sqrt{1-\beta^2}}$$

From this we obtain $g(v) = 1 - \beta^2$.

B. Period Motion II: One-Dimensional Mass Sphere Motion

For a one-dimensional moving sphere as a clock (the entire equipment is named M), the rest and moving clocks are shown in left frame (A) and right frame (B) respectively. The period ratio is:

$$\frac{\tau'}{\tau} = \frac{v_0 L'}{v' L} = \frac{L'}{L}\sqrt{1-\beta^2}$$

Equation (25) is just the special case of equation (21) and serves as a test for our previous hypotheses.

C. Period III: Circling Motion

We now consider circling motion as a clock (see FIG. 4 [Figure 4: see original paper]). The circling motion is in the $y-z$ plane. Assuming the rest mass of the moving sphere is m_0, the radius of the circle is r and r' corresponding to case A and case B respectively, we have:

$$m = m_0(1 - \omega^2 r^2/c_0^2)^{-1/2}, \quad m' = m_0(1 - \omega {a'}^2 {r'}^2/c_0^2)^{-1/2}$$

Since angular momentum is invariant and there is no force in the $y-z$ plane:

$$m\omega r^2 = m'\omega' {r'}^2$$

For the period, we can write:

$$\frac{2\pi/\omega'}{2\pi/\omega} = \frac{r'}{r}\sqrt{1-\beta^2}$$

Combining the three thought experiments, we conclude:

1. Length does not change in directions where the velocity component is zero, and contracts by $\sqrt{1-\beta^2}$ in the moving direction.
2. The period of a moving clock dilates by $1/\sqrt{1-\beta^2}$, independent of velocity direction.

V. Lorentz Transformation

In this section, we derive the Lorentz transformation based on the previous length contraction and time dilation results. First, we assume the transformation between different inertial frames shown in Fig. 5 [Figure 5: see original paper] has the simple form:

$$\begin{gathered} x_0 = a_{11}x + a_{14}t \\ y_0 = a_{22}y \\ z_0 = a_{33}z \\ t_0 = a_{41}x + a_{44}t \end{gathered}$$

The velocity transformation becomes:

$$v(x_0) = \frac{a_{11}v_x + a_{14}}{a_{41}v_x + a_{44}}$$

where parameters with subscripts represent quantities in the static reference frame. For length perpendicular to the moving direction, we have $a_{22} = a_{33} = 1$.

In the S_0 frame, S_0 will be at $x_0 = vt_0$, yielding:

$$a_{14} = a_{44}v$$

From our previous conclusion, in the moving frame the length contracts by $\sqrt{1-\beta^2}$. Assuming the rule contracts the same for all objects, the length of a moving object observed in the static frame contracts by $\sqrt{1-\beta^2}$ compared to the same object in the static frame. We have:

$$x_2 - x_1 = \frac{a_{44}(x_{20} - x_{10})}{a_{11}a_{44} - a_{14}a_{41}} = (x_{20} - x_{10})\sqrt{1-\beta^2}$$

where x_2 and x_1 are the x coordinates of the two endpoints of the object in the S frame, and x_{20} and x_{10} are the x coordinates of the two endpoints in the S_0 frame at the same time.

Similarly, the time of a moving clock observed in the static frame dilates by $1/\sqrt{1-\beta^2}$ compared to the time in the moving frame. When the two frames coincide at $t = t_0 = 0$, the time between clocks at $x = 0$ or $x_0 = vt_0$ in the S frame and clocks in the S_0 frame gives:

$$t_0 = a_{44}t = \frac{t}{\sqrt{1-\beta^2}}$$

For the general case, a clock runs from t_1 to t_2, and in the S_0 frame it will be:

$$t_{20} - t_{10} = a_{44}(t_2 - t_1) = \frac{t_2 - t_1}{\sqrt{1-\beta^2}}$$

To obtain the final equations, we consider light frequency and wavelength transformation. In the static frame $c_0 = \omega_0/k_0$. The phase is invariant in different frames:

$$\mathbf{r} \cdot \mathbf{k} - \omega t = \mathbf{r}_0 \cdot \mathbf{k}_0 - \omega_0 t_0$$

Substituting the transformation equations and using the arbitrariness of x, y, z, t, we obtain:

$$\begin{aligned} k_x &= a_{11}k_{x0} - a_{41}\omega_0 \\ k_y &= a_{22}k_{y0} \\ k_z &= a_{33}k_{z0} \\ \omega &= a_{44}\omega_0 - a_{14}k_{x0} \end{aligned}$$

In the static frame, for light along the y_0 axis ($k_z = k_{z0} = k_{x0} = 0$), we get:

$$\begin{aligned} a_{11} &= a_{44} = \frac{1}{\sqrt{1-\beta^2}} \\ a_{22} &= a_{33} = 1 \\ a_{14} &= a_{41} = \frac{v}{\sqrt{1-\beta^2}} \end{aligned}$$

Finally, we obtain the Lorentz transformation:

$$x_0 = \frac{x + vt}{\sqrt{1-\beta^2}}, \quad y_0 = y, \quad z_0 = z, \quad t_0 = \frac{t + vx/c_0^2}{\sqrt{1-\beta^2}}$$

The wave vector transformation is:

$$k_x = \frac{k_{x0} - v\omega_0/c_0^2}{\sqrt{1-\beta^2}}, \quad k_y = k_{y0}, \quad k_z = k_{z0}, \quad \omega = \frac{\omega_0 - vk_{x0}}{\sqrt{1-\beta^2}}$$

VI. Conclusion and Discussion

In this work, we reproduce important results of special relativity—mass-velocity and mass-energy relations—under the simplest ideal gas model of vacuum hypothesis, where the inertial mass of a particle comes from its drift mass. Our model has a free parameter γ called the adiabatic index, and when $\gamma = 3$, the speed of sound in the rest fluid equals the maximum speed of the fluid ($a_0 = c_0$). We also reproduce time dilation and length contraction based on the mass-velocity relation derived from the ideal gas model, and finally reproduce the Lorentz transformation.

We must emphasize that all conclusions are obtained under the assumption of the existence of ether, i.e., that vacuum is an ideal gas with a static state. Additionally, all velocities are relative to the static vacuum, and the foundation is classical physics of the 19th century. Of course, this ideal gas model of vacuum is very simple and cannot describe the complex structure of elementary particles. Therefore, this model can help us understand the nature of vacuum, special relativity, and the relationship between them. It also represents an exploration of the origin of inertial mass different from the Higgs mechanism obtained from quantum physics.

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Note: Figure translations are in progress. See original paper for figures.