Research on Pulsar Signal Denoising Method Based on Wavelet Packet Thresholding: Postprint

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Abstract

Wavelet transform denoising performs decomposition and processing only on the low-frequency components of the signal, while neglecting the decomposition and processing of the high-frequency components, thus failing to effectively extract information from the entire frequency band of the signal and consequently affecting the denoising performance for pulsar signals. To address this limitation, this paper proposes the use of wavelet packet thresholding for the analysis of pulse signals, wherein different threshold functions are selected to process the coefficients following wavelet packet decomposition. Experimental results indicate that the proposed method enhances the denoising effectiveness for pulsar signals, with corresponding improvements observed in the signal-to-noise ratio, peak signal-to-noise ratio, and smoothness metrics, thereby offering a novel approach to pulse signal denoising.

Full Text

Research on Denoising Method of Pulsar Signal Based on Wavelet Packet Threshold Method

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Abstract

Wavelet transform denoising only decomposes and processes the low-frequency components of signals while neglecting the decomposition and processing of high-frequency components, which prevents effective extraction of information across the entire frequency band and consequently affects the denoising effectiveness of pulsar signals. To address this limitation, this paper proposes employing the wavelet packet threshold method for pulsar signal analysis, utilizing different threshold functions to process the coefficients obtained from wavelet packet decomposition. Experimental results demonstrate that this approach enhances pulsar signal denoising performance, with corresponding improvements in signal-to-noise ratio (SNR), peak signal-to-noise ratio, and smoothness metrics, thereby providing a novel approach for pulsar signal denoising.

Keywords: Wavelet packet decomposition; Threshold method; Pulsar signal; Denoising
Classification: P162.5

Introduction

In 2016, the Five-hundred-meter Aperture Spherical Telescope (FAST) was completed in Pingtang, Guizhou, marking China' s breakthrough in discovering pulsars using domestic equipment. Pulsars are fascinating celestial objects in the universe that serve as "natural laboratories" for verifying physical laws under extreme conditions such as strong gravitational fields, intense magnetic fields, and high densities [1]. As a powerful tool for pulsar research, FAST employs a 19-beam multi-beam receiver system to conduct large-scale sky surveys for eight hours daily, searching for and receiving electromagnetic pulse signals emitted by pulsars to determine their existence in specific sky regions. As is well known, pulsars in distant space are approximately 3,000 to 55,000 light-years from Earth, and the pulse signals that reach Earth and are successfully received by specialized instruments have extremely weak energy, with the entire pulsar signal submerged in intense noise [2,3]. Therefore, denoising of pulsar signals is crucial for their detection and discovery.

Wavelet transform, as a multi-resolution analysis method capable of simultaneous time-frequency domain analysis, is referred to by professionals as a "mathematical microscope" due to its time-domain localization and multi-resolution characteristics, making it highly suitable for processing non-stationary signals [4]. Pulsar signals are typical non-stationary signals. Zhu Xiaoming et al. [5] proposed applying wavelet analysis to pulsar signal denoising, demonstrating that wavelet analysis can better denoise non-stationary signals, thereby replacing traditional Fourier transforms in pulsar signal processing. Subsequent researchers have conducted extensive studies on wavelet transform for pulsar signal denoising. Yan Di et al. [6] explored pulsar denoising methods based on the minimax principle and Stein' s unbiased likelihood principle. Ke Xizheng et al. [7] attempted to improve threshold functions to enhance pulsar signal denoising effects, proposing soft-hard compromise threshold functions and exponential threshold functions to process wavelet decomposition coefficients of pulsar signals, achieving favorable results.

While wavelet transform can effectively remove signal noise while preserving original signal data, making it a relatively ideal tool for signal processing, it primarily focuses on extracting and analyzing low-frequency information, often neglecting high-frequency information, which significantly impacts overall signal analysis accuracy. Wavelet transform denoising involves selecting an appropriate basis function from wavelet functions for signal processing, performing wavelet decomposition using the basis function, and reconstructing the signal based on processing results from different threshold functions to achieve denoising. The denoising effectiveness of wavelet transform varies with different selections of wavelet basis functions and threshold functions, introducing certain randomness into the denoising results [8].

With further development of wavelet theory, wavelet packet analysis has gradually attracted researchers' attention. Wavelet packet analysis is a generalization of wavelet decomposition with superior signal denoising capabilities, as it can decompose both low-frequency and high-frequency components of signals, enabling better extraction of useful information across all frequency bands. Wavelet packet denoising methods have been increasingly applied in fields such as image processing, seismic signals, electrocardiogram signals, and ocean wave signals. Liu Xiuping et al. [9] initially attempted to combine wavelet packet filtering methods with X-ray pulsar impulse noise removal, but without systematic analysis of wavelet packet denoising. In light of this, this paper proposes applying the wavelet packet threshold method to pulsar signal denoising analysis, systematically evaluating the denoising effectiveness of various threshold functions. Experiments demonstrate that compared with traditional wavelet transform methods, the wavelet packet threshold method significantly improves pulsar signal denoising effectiveness, providing valuable insights for pulsar signal research.

1.1 Wavelet Transform Method

The observed signal can be represented as $f(t)$, satisfying:

$$f(t) = s(t) + n(t) \quad (1-1)$$

where $f(t)$ is the received raw signal (i.e., noisy signal), $n(t)$ is noise with variance σ^2 following an $N(0, \sigma^2)$ distribution, and $s(t)$ is the desired denoised signal. A one-dimensional noisy signal $f(t)$ can be expanded according to wavelet series to obtain a linear combination of wavelets at n resolution levels. The specific wavelet transform is:

$$W_f(\psi_{j,k}) = 2^2 \int f\psi(2^{jn-k})dx \quad (1-2)$$

where $W_f(\psi_{j,k})$ represents the discrete wavelet transform using wavelet basis $\psi_{j,k}$ in $L^2(R)$, with wavelet transform coefficients determined by the above formula.

1.2 Wavelet Packet Transform Method

Wavelet packet transform extends wavelet transform, possessing the capability to process low-frequency components of observed signals while also effectively decomposing high-frequency components to collect useful information across all frequency bands, thereby improving denoising accuracy.

The specific wavelet packet transform is:

$$\varphi(2x-n)_{n\in Z}$$

where $\varphi(x)$ and $\psi(x)$ are the scaling function and wavelet function in multi-resolution analysis of signals in $L^2(R)$, respectively. The filter coefficients are h_n and g_n:

$$\varphi(x)=\sqrt{2}\sum$$

$$\psi(x)=\sqrt{2}\sum \quad (1-3)$$

which satisfy the following conditions:

$$\varphi(2x-n)=\delta_{kl} \quad (1-4)$$

$$=\sqrt{2} \quad (1-5)$$

2.1 Wavelet Packet Decomposition and Reconstruction

Wavelet packet transform in wavelet packet theory possesses decomposition and reconstruction properties [10]. The mathematical expression for the wavelet packet decomposition algorithm is:

$$n(l) \quad (2-1)$$

$$2n(k)=\sum h_{l-2k}n(l) \quad (2-2)$$

$$2n+1(k)=\sum g_{l-2k}$$

where $w_j^n(k)$ represents the k-th wavelet packet decomposition coefficient on the n-th branch tree at decomposition scale j, while $w_{j+1}^{2n}(k)$ and $w_{j+1}^{2n+1}(k)$ represent the two branches of $w_j^n(k)$. The corresponding mathematical expression for the wavelet packet reconstruction algorithm is:

$$n(k) = \sum h_{k-2l} 2n(l) + \sum g_{k-2l}$$

where w represents the decomposition coefficients obtained after wavelet packet decomposition; h and g are the filter coefficients for wavelet packet decomposition; j and n are the indices of decomposition nodes in wavelet packet decomposition; and l and k are the decomposition levels where signal wavelet packet decomposition occurs.

2.2 Determination of Optimal Wavelet Packet Basis

When processing observed signals using wavelet packet analysis, wavelet packet decomposition is performed on the signal. Unlike wavelet decomposition, wavelet packet decomposition follows a binary tree structure, with both low-frequency and high-frequency components of the signal being decomposed in a complete binary tree format. The number of decomposition layers directly affects signal denoising effectiveness. During progressive decomposition, an optimal decomposition depth must be determined. Insufficient decomposition layers cause useful signals to remain mixed in frequency bands, while excessive decomposition layers introduce errors [11]. In wavelet analysis denoising, too many decomposition scales lead to loss of signal information, resulting in decreased SNR and affecting final computational results. In practical wavelet analysis applications, decomposition scales generally range from 3 to 5 [12]; this study adopts 5 for comparative analysis.

Wavelet packet transform contains a library of bases composed of various wavelets, requiring selection based on signal characteristics and cost function evaluation criteria. When selecting wavelet functions, the characteristics of pulse signals must be fully considered, choosing functions with compact support and sufficient vanishing moments within given intervals to more effectively remove noise from pulse signals and extract true target pulse signals. To determine the appropriate wavelet basis function for pulsar signal characteristics, this section compares and analyzes SymN, CoifN, DbN, Bior series wavelets, and Haar wavelets. For consistent comparison, a fixed threshold rule is uniformly adopted, utilizing different wavelet bases for 5-level decomposition. The analysis results are shown in Table 1 , Table 2 , Table 3 , and Table 4 , where N represents wavelet order, and SymN, CoifN, DbN, and BiorN denote Symlets, Coiflets, Daubechies, and Biorthogonal wavelet series, respectively.

Table 2.1 The SNR of Pulsar Signal Denoised by SymN, CoifN and DbN Series Wavelet Bases

Sym N	Coif N
1.1590260180636194e+38	1.159937964771207e+38
1.159105137508623e+38	1.1591729484203662e+38
1.1591051375086237e+38	1.15902630444004e+38

Sym N	Coif N
1.159201733565514e+38	1.1590263044471583e+38
1.1591290190598036e+38	1.1591803200172715e+38
1.1593325604448439e+38	1.1589381807874201e+38
1.1591317586993424e+38	1.159134635851061e+38
1.1590973148920641e+38	1.1590666107944755e+38
1.1590291941601505e+38	1.159204508875512e+38
1.1590039338852231e+38	1.1591823177722796e+38
1.1590881427106949e+38	1.1589613392166439e+38
1.1591410945254123e+38	1.1589676535028293e+38
1.1589479802663968e+38	1.1590081322083972e+38
1.1590847694307965e+38	1.158964702074117e+38
1.1590547974489843e+38	1.1589516584680341e+38
1.1590042562137324e+38	1.1591076638631676e+38
1.1591001911149105e+38	1.1590495391531836e+38
1.1590266700820342e+38	1.158990764381691e+38
1.1590798965814301e+38	1.1590067783780341e+38
1.1590776721200966e+38	1.1590759265506637e+38
1.15905305165149e+38	1.1590080833152019e+38
1.159035404037832e+38	1.1590402208654303e+38

Table 2.2 The SNR of Pulsar Signal Denoised by SymN, CoifN and DbN Series Wavelet Bases

Sym N	Coif N
1.1590260180636194e+38	1.159937964771207e+38
1.159105137508623e+38	1.1591729484203662e+38
1.1591051375086237e+38	1.15902630444004e+38
1.159201733565514e+38	1.1590263044471583e+38
1.1591290190598036e+38	1.1591803200172715e+38
1.1593325604448439e+38	1.1589381807874201e+38
1.1591317586993424e+38	1.159134635851061e+38
1.1590973148920641e+38	1.1590666107944755e+38
1.1590291941601505e+38	1.159204508875512e+38
1.1590039338852231e+38	1.1591823177722796e+38
1.1590881427106949e+38	1.1589613392166439e+38
1.1591410945254123e+38	1.1589676535028293e+38
1.1589479802663968e+38	1.1590081322083972e+38
1.1590847694307965e+38	1.158964702074117e+38
1.1590547974489843e+38	1.1589516584680341e+38
1.1590042562137324e+38	1.1591076638631676e+38
1.1591001911149105e+38	1.1590495391531836e+38
1.1590266700820342e+38	1.158990764381691e+38

Sym N	Coif N
1.1590798965814301e+38	1.1590067783780341e+38
1.1590776721200966e+38	1.1590759265506637e+38
1.15905305165149e+38	1.1590080833152019e+38
1.159035404037832e+38	1.1590402208654303e+38

Table 2.3 The SNR and RMSE of Pulsar Signal Denoised by Bior N Series Wavelet Bases

Bior N	SNR	RMSE
1.159937964771207e+38	1.161230305420237e+38	1.1621428245153362e+38
1.1620257884498562e+38	1.1593361263215988e+38	1.1591729507026387e+38
1.1592514421028786e+38	1.4888560080440952e+38	1.1712494267787645e+38
1.1611795518841145e+38	1.1596986873497866e+38	1.1592749724774778e+38
1.159259591601108e+38	1.1592072048905078e+38	1.15912528098707e+38

Table 2.4 The SNR and RMSE of Pulsar Signal Denoised by Haar Wavelet Bases

Haar
1.159937964771207e+38

Based on the root mean square error (RMSE) and signal-to-noise ratio (SNR) statistics presented in the four tables above, the overall denoising performance using SymN, CoifN, DbN, Bior N, and Haar wavelets for pulsar signals is satisfactory. Among these, the SymN series wavelet demonstrates the best performance, followed by CoifN, with Bior N showing the poorest results. Additionally, different wavelet basis orders yield varying denoising effects. For individual wavelet bases, Sym5 achieves the optimal denoising result. Therefore, this paper selects the nearly symmetric compactly supported orthogonal wavelet Symmlet5 as the wavelet basis function. Consequently, wavelet packet analysis is performed on the pulse signal with a decomposition depth of 5 to obtain the optimal wavelet packet basis, which represents the optimized wavelet packet decomposition tree.

2.3 Selection of Multiple Threshold Functions

During signal processing using wavelet packet transform, a "frequency reversal" phenomenon often occurs, which is a frequency aliasing phenomenon when signals pass through high-pass filters, causing misalignment in the frequency order of decomposed wavelet packet coefficients. Specifically, low-frequency components of the signal are arranged from low to high, while high-frequency components are arranged from high to low. Therefore, before wavelet packet threshold processing, it is necessary to first sort the wavelet packet coefficients after wavelet packet transformation in ascending order, and then select the corresponding signal processing threshold based on the sorted coefficients.

Four commonly used threshold selection criteria exist in wavelet packet analysis: sqtwolog (fixed form) threshold criterion, rigrsure (adaptive) threshold criterion, heursure (heuristic) threshold criterion, and minimaxi (minimax) threshold criterion. This paper adopts the sqtwolog (fixed form) threshold criterion for signal processing. After selecting thresholds for coefficients at each decomposition layer, the threshold function for signal processing must be determined. Commonly used threshold functions include hard threshold and soft threshold functions.

The hard threshold function replaces all elements with absolute values not greater than threshold λ with zero, which can easily cause discontinuity in processed signals. The hard threshold function expression is:

$$\bar{n} = \begin{cases} n, & |w_j^n| \geq \lambda \\ n| < \lambda \end{cases} \quad (2-4)$$

The soft threshold function overcomes the shortcomings of the hard threshold function, effectively avoiding signal discontinuities and demonstrating better continuity. However, when dealing with large wavelet coefficients, it can cause loss of high-frequency signal components after threshold processing, thereby affecting signal reconstruction. The soft threshold function expression is:

$$\bar{n} = \begin{cases} n)(|w_j^n| - \lambda), & |w_j^n| \geq \lambda \\ n| < \lambda \end{cases} \quad (2-5)$$

Given that both soft and hard threshold functions have their respective limitations when processing signals, researchers have proposed a soft-hard threshold compromise method with the following expression:

$$\bar{n} = \begin{cases} n)(|w_j^n| - \alpha\lambda), & |w_j^n| \geq \lambda \\ n| < \lambda \end{cases} \quad (2-6)$$

where α ranges from $0 \leq \alpha \leq 1$. When $\alpha = 0$, it functions as a hard threshold; when $\alpha = 1$, it functions as a soft threshold. Appropriate adjustment of α can yield better signal denoising results.

3 Pulsar Signal Denoising Analysis

The pulsar signals used in this experiment originate from actual observation data from the Parkes radio telescope. This data was processed through PSESTO for interference removal and dedispersion to obtain .dat data files recorded in binary format with a sampling frequency of 1024 Hz.

Fig 1 [Figure 1: see original paper] The dispersion-limited signal
Fig 2 [Figure 2: see original paper] The power spectrum of the dispersion-limited signal

Figure 1 shows the dedispersed original pulse signal, while Figure 2 displays the power spectrum of the dedispersed original pulse signal. In this laboratory study, the target pulsar signal file undergoes wavelet packet decomposition using Symmlet5 with a decomposition depth of 5 layers. The hard threshold, soft threshold, and soft-hard compromise threshold functions from wavelet analysis are selected, with the fixed sqtwolog threshold criterion applied for threshold processing.

To better compare the denoising effectiveness obtained by different methods, this paper adopts signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR), and smoothness metric (r) as evaluation criteria based on reference [13]. Larger SNR, larger PSNR, and smaller smoothness metric indicate better denoising performance. The specific formulas are defined as follows:

(1) **Signal-to-Noise Ratio** is calculated as:

$$SNR = 10\log_{10}\left[\frac{\sum f(i)^2}{\sum s(i)^2 - f(i)^2}\right] \quad (3-1)$$

(2) **Root Mean Square Error** is calculated as:

$$RMSE = \sqrt{\sum (s(i) - f(i))^2} \quad (3-2)$$

(3) **Peak Signal-to-Noise Ratio** represents the ratio between the maximum signal power and noise power, denoted as PSNR:

$$PSNR = 10\log_{10}\left[\frac{\max(f(i)^2)}{\sum s(i)^2 - f(i)^2}\right] \quad (3-3)$$

(4) **Smoothness** refers to the ratio between the root variance of the differenced denoised signal and the root variance of the differenced original signal, commonly denoted as r:

$$r = \frac{[(f(i+1) - f(i))^2]}{[(s(i+1) - s(i))^2]} \quad (3-4)$$

In the above formulas, $s(i)$ represents the noisy signal and $f(i)$ represents the denoised signal.

The following figures show the results and power spectra after processing signals using wavelet transform denoising and wavelet packet transform with different threshold methods, along with a summary table of evaluation metrics.

Fig 3 [Figure 3: see original paper] The result of de-noised with wavelet soft threshold
Fig 4 [Figure 4: see original paper] The result of de-noised with wavelet hard threshold
Fig 5 [Figure 5: see original paper] The result of de-noised with wavelet soft and hard threshold
Fig 6 [Figure 6: see original paper] Power spectrum of the result of de-noised with wavelet threshold method
Fig 7 [Figure 7: see original paper] The result of de-noised with wavelet packet soft threshold
Fig 8 [Figure 8: see original paper] The result of de-noised with wavelet packet hard threshold
Fig 9 [Figure 9: see original paper] The result of de-noised with wavelet packet soft and hard threshold
Fig 10 [Figure 10: see original paper] Power spectrum of the result of de-noised by wavelet packet threshold method

Table 3.1 De-noised Effect Table of Wavelet and Wavelet Packet Transform

Index	Wavelet Packet Soft Threshold Method	Wavelet Soft Threshold Method	Wavelet Packet Hard Threshold Method	Wavelet Hard Threshold Method	Wavelet Packet Soft-Hard Threshold Method	Wavelet Soft-Hard Compromise Method
SNR	1.15903323613	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]807874201e+38

As shown in Table 3.1, the wavelet packet threshold method achieves relatively better results compared to wavelet transform denoising for pulsar signals. Although the RMSE changes are not particularly significant after various wavelet packet threshold function processing, improvements are observed in SNR, PSNR, and smoothness metrics. For instance, experiments demonstrate that compared with wavelet transform hard threshold method, the wavelet packet hard threshold method reduces smoothness to 0.000068, increases SNR to 128.574093 dB, and improves PSNR to 50.216641 dB. Both wavelet packet soft threshold and soft-hard compromise threshold methods show corresponding improvements in denoising evaluation metrics.

4 Summary and Outlook

This paper addresses the limitations of wavelet transform in denoising by proposing the wavelet packet threshold method for pulsar signal denoising. The wavelet packet threshold method first requires sorting the wavelet packet decomposition coefficients in ascending order, then determining thresholds using fixed threshold criteria, and processing the wavelet packet coefficients using common soft threshold, hard threshold, and soft-hard compromise threshold functions. The processed signal is then reconstructed to obtain the denoised signal. The paper first discusses the theoretical aspects of wavelet packet threshold denoising, then experimentally verifies that the wavelet packet threshold method outperforms wavelet decomposition methods in removing pulsar signal noise. To quantify the experimental results, this paper employs SNR, PSNR, smoothness, and RMSE as quantitative metrics to compare denoising effectiveness across different methods. Experimental results demonstrate that the wavelet packet threshold method achieves superior denoising performance compared to wavelet transform.

Since this study only tested the Symmlet5 wavelet basis and fixed threshold criterion on selected Parkes pulsar signals, further research is needed to explore whether the method is applicable to other pulsar signals and whether other wavelet bases or threshold criteria can achieve similar denoising effectiveness.

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Note: Figure translations are in progress. See original paper for figures.