Interfacial Conditions at the Front of Immiscible Gas-Liquid Two-Phase Displacement in Porous Media

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Abstract

Gas-liquid two-phase displacement is ubiquitous in oil and gas reservoir development. Traditional seepage theory holds that pressure and Darcy velocity are continuous at discontinuous interfaces, which we have refuted in previous studies. Building upon prior research on incompressible fluids, this paper investigates the connection conditions for pressure and seepage velocity at moving gas-water sharp interfaces in porous media, using gas-water two-phase displacement as an example. The research results demonstrate that at gas-water two-phase displacement interfaces: (1) total pressure is continuous, but fluid pressure may be discontinuous; (2) the distribution of total flow velocity may be discontinuous but satisfies a specific functional relationship; (3) the seepage velocity of each fluid phase is discontinuous.

Full Text

Preamble

Interface Conditions for Immiscible Gas-Liquid Displacement Fronts in Porous Media

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Abstract

Gas-liquid two-phase displacement phenomena are ubiquitous in oil and gas reservoir development. Traditional seepage theory posits that pressure and Darcy velocity must be continuous across discontinuous interfaces, a claim we

have previously falsified. Building upon our earlier work on incompressible fluids, this paper investigates the connection conditions for pressure and seepage velocity at moving gas-water displacement fronts in porous media, using gas-water two-phase displacement as a representative example. The results demonstrate that at the displacement front: (1) the global pressure remains continuous, while fluid pressures may be discontinuous; (2) the total Darcy velocity distribution can be discontinuous but must satisfy a specific functional relationship; and (3) the Darcy velocity of each fluid phase is discontinuous.

Gas-liquid two-phase displacement occurs extensively in reservoir engineering contexts, including edge and bottom water gas reservoirs, aquifer-based underground gas storage, CO_2 geological sequestration, gas injection for enhanced oil recovery, gas-cap reservoir development, gas drainage accumulation, and dynamic testing of gas-water relative permeability. The interface conditions for gas-liquid displacement are fundamental for establishing mathematical flow models, tracking interface movement, and calculating production performance under displacement. According to classical seepage theory, fluid phase pressures and velocities remain continuous at the displacement front interface, expressed as:

Based on the mass conservation equation, the change in fluid mass within an infinitesimal element per unit time equals the inflow minus outflow. For a moving interface with zero volume and no source/sink terms, this reduces to inflow minus outflow equals zero. Hassanizadeh and Gray (1989) provided a proof using mass conservation principles. For cases with different capillary pressures across the interface, we previously questioned these traditional interface conditions and derived new interface conditions and cross-interface flow equations based on mass conservation and two-phase flow theory [1, 2]. Gas-liquid displacement fronts exhibit three key characteristics: (1) the displacement interface moves continuously with time; (2) the interface is typically a sharp discontinuity where fluid distributions may be discontinuous; and (3) gas compressibility is significant and must be considered. Therefore, this work extends our previous investigation of jump interface conditions for incompressible fluids to incorporate fluid compressibility.

2. Critique of Traditional Jump Interface Conditions

We present six primary arguments challenging conventional interface conditions:

(1) Interface conditions at displacement fronts must establish pressure continuity for different fluid particles across the interface and connection conditions for fluid velocities (as illustrated in [Figure 1: see original paper]). However, the mass balance for an interfacial element of zero volume refers to the same material point on the interface, indicating that traditional formulations describe different physical processes.

(2) In one dimension, consider a thin layer element containing the interface with thickness $b \to 0$. Let the inflow velocity be v^- and outflow veloc-

ity be v^+. Their relationship constitutes the connection condition derived from mass conservation. Traditional seepage mechanics expresses mass conservation differently, which essentially modifies the fundamental equation. Hassanizadeh and Gray attempted to prove the equivalence of these formulations, but their proof contains errors.

(3) The analytical solution of the Buckley-Leverett model explicitly shows discontinuity in seepage velocity at the front, contradicting traditional formulations. De Neef and Molenaar (1997) demonstrated that fluid phase pressures and capillary pressure cannot simultaneously satisfy continuity, proposing modified conditions where the wetting phase pressure remains continuous while the non-wetting phase pressure may be discontinuous.

(4) For moving interfaces, traditional interface conditions contradict the Rankine-Hugoniot conditions, which are rigorously derived from mass conservation equations. For equation (2), the Rankine-Hugoniot condition is:

(5) Based on traditional interface conditions and two-phase Darcy' s law, transmissibility at discontinuities should be the harmonic average of both sides. This approach fails to capture actual flow behavior and has been superseded by single-point upstream weighting (SPU) and higher-order upstream methods. Our literature review reveals that although condition (1) appears extensively in publications, it has not been genuinely applied in practice.

(6) Some scholars argue that fluid saturation cannot be discontinuous, thereby requiring pressure and velocity continuity. Our perspective is twofold: First, at the microscopic scale, fluid distribution interfaces can be discontinuous, and macroscopic behavior represents a statistical average, thus saturation discontinuities are permissible macroscopically. Second, from a computational standpoint, the narrow transition zone contributes negligible flow resistance compared to the pressure drop across the interface. Representing a steep but continuous distribution as a discontinuous interface yields similar results while significantly reducing computational requirements.

[Figure 3: see original paper] illustrates this concept, showing the simplification of a steep but continuous fluid distribution (left) to a discontinuous interface (right).

3. Immiscible Gas-Water Flow Equations

For immiscible gas-water systems, the mass conservation equations are:

where Bg is the formation volume factor (dimensionless), Z is the gas deviation factor (dimensionless), Sw is water saturation, v is Darcy velocity (m/day), and is the angle with the horizontal axis. Fluids obey Darcy' s law in porous media:

4.1 Connection Condition for Total Flow Velocity at the Displacement Front

In one-dimensional coordinates, equation (7) becomes:

Consider the region near the displacement front, denoted by Γ. During displacement, over an infinitesimal time period: (1) the front moves with unchanged geometry, and (2) porosity can be considered constant if its distribution is continuous.

Integrating equation (9a) over the interval $[x , x]$ yields:

Assuming constant parameters, we obtain:

Equation (15) represents the Rankine-Hugoniot condition for one-dimensional gas-water displacement fronts. Similarly, from (9b) we derive:

Combining (15) and (16) yields at the displacement front:

These relationships (15)-(17) also hold in radial coordinate systems.

[Figure 4: see original paper] illustrates edge/bottom water radial displacement.

Substituting (21) into (17) gives:

This demonstrates that total seepage velocities are unequal across the displacement front. For incompressible fluids, total velocity equality holds at the front, showing that compressible and incompressible fluids exhibit different interface continuity conditions, with the latter being a special case of the former.

4.2 Connection Condition for Total Pressure at the Displacement Front Interface

Defining total pressure as:

and differentiating with respect to x yields:

Since the fractional flow curve varies smoothly, we can approximate:

From (21), we obtain:

Substituting (26) into (20) gives:

Rearranging (27) yields:

Integrating (28) over the interval $[x , x]$:

From (30) or (31), when $x \rightarrow x\Gamma^-$ and $x \rightarrow x\Gamma^+$, we have $\Phi = \Phi$, indicating total pressure continuity at the displacement front interface.

4.3 Pressure Connection Conditions at the Displacement Front Interface

Combining (30) and (32) yields the new interface conditions:

Based on (33), which reveals the connection relationships for total seepage velocity and total pressure at the displacement front, and using Darcy' s law (21) along with the total pressure definition, we can calculate the connection conditions for gas and water velocities at the interface. The pressure connection conditions are:

4.4 Calculation of Gas-Water Flow Rates on Both Sides of the Displacement Front

Solving (22) and (31) simultaneously yields the total seepage velocities v and v on both sides of the front.

Substituting (35) into (21) calculates v w, v g, v w, and v g.

Substituting (36) into (22) yields p w, p g, p w, and p g.

Discussion

(1) This study demonstrates that fluid pressures and seepage velocities can be discontinuous at displacement fronts. Seepage velocity discontinuity arises because it represents flow rate divided by cross-sectional area; even if actual molecular velocities are continuous across the interface, seepage velocities may differ. While pressure fields should be continuous for continuously distributed fluids, at the microscopic scale, two-phase bodies are easily truncated at pore throats within the sharp interface region.

(2) The possibility of discontinuous fluid distribution at sharp interfaces remains debated. Some scholars argue against discontinuity based on traditional interface conditions. Our perspective is that, regardless of this debate, the transition zone is extremely narrow and contributes minimal flow resistance. Representing a steep continuous distribution as a discontinuous interface using our proposed connection conditions yields comparable results while substantially reducing computational requirements.

(3) In most cases, fractional flow coefficients vary weakly with pressure and can be neglected (ω 0).

(4) Comparison between moving displacement fronts and stationary property discontinuities reveals different connection condition characteristics. Moving sharp interfaces exhibit different conditions than static ones, requiring careful application. For instance, the interface conditions developed herein are unsuitable for calculating flow across scales or different medium types.

(5) This study uses gas-water systems without considering solubility. Solubility would alter the mass conservation equations and consequently the interface conditions. Additionally, only one-dimensional cases are considered; multidimensional interface conditions are more complex.

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Note: Figure translations are in progress. See original paper for figures.