Postprint: Research on Interpolation Accuracy of BeiDou Satellite Precise Ephemeris

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Abstract

The non-sliding Lagrange interpolation method and sliding Lagrange interpolation method are employed to interpolate the precise ephemeris of the BeiDou Satellite Navigation System (Beidou Satellite Navigation Syetem, BDS), and the relationship between BDS satellite interpolation accuracy and the two methods as well as interpolation order is investigated through extensive experiments. Experimental results demonstrate: (1) When employing non-sliding and sliding Lagrange interpolation respectively, the interpolation errors of MEO and IGSO satellites approximately exhibit "U"-shaped and "L"-shaped distributions with increasing order, while the interpolation error of GEO satellites shows gradually increasing and approximately stable patterns; (2) At lower interpolation orders, both methods exhibit certain regularities in interpolation errors, but the regularity of GEO satellite interpolation errors is weaker than that of MEO and IGSO satellites; (3) The optimal interpolation orders for each satellite using the two methods differ by less than 1, and at the optimal interpolation order, the sliding Lagrange interpolation accuracy improves by 11.96%–44.01% compared to the non-sliding method, with GEO satellite interpolation accuracy outperforming MEO and IGSO satellites.

Full Text

Research on Interpolation Accuracy of BeiDou Satellite Precise Ephemeris

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Abstract

This study investigates the interpolation of BeiDou Satellite Navigation System (BDS) precise ephemeris using both non-sliding and sliding Lagrange interpolation methods. Through extensive experiments, we examine the relationship between interpolation accuracy and interpolation order for BDS satellites. The results demonstrate that: (1) For both non-sliding and sliding Lagrange interpolation, the interpolation errors of MEO and IGSO satellites exhibit approximately U-shaped and L-shaped distributions with increasing order, respectively, while GEO satellite errors show gradual increase and approximate stability; (2) At lower interpolation orders, both methods produce errors with certain regular patterns, though GEO satellites display weaker regularity compared to MEO and IGSO satellites; (3) The optimal interpolation orders for each satellite differ by less than 1 between the two methods, and under optimal order conditions, sliding Lagrange interpolation achieves 11.96%–44.01% higher accuracy than non-sliding interpolation, with GEO satellites demonstrating superior interpolation accuracy compared to MEO and IGSO satellites.

Keywords: Lagrange interpolation; sliding method; BDS precise ephemeris; error distribution law; optimal interpolation order

Introduction

On June 23, 2020, the completion of the global BeiDou Satellite Navigation System (BDS) constellation marked China's official entry into providing global navigation services. Precise positioning—a critical function of satellite navigation systems—relies on accurate satellite positions typically obtained from precise ephemerides. However, the International GNSS Service (IGS) provides precise ephemerides at 15-minute or 5-minute intervals, which cannot satisfy practical requirements for continuous positioning. Consequently, interpolation of precise ephemerides is necessary to obtain satellite positions at arbitrary epochs [1-2].

Numerous scholars have investigated ephemeris interpolation methods, including Lagrange interpolation, Newton interpolation, spline function interpolation, and Chebyshev interpolation [3-7]. However, most research has focused on GPS satellite ephemerides, with relatively limited attention to BDS. Unlike the GPS constellation, BDS comprises three orbital types: Geostationary Earth Orbit (GEO), Inclined Geosynchronous Satellite Orbit (IGSO), and Medium Earth Orbit (MEO) satellites. Each satellite type exhibits different responses to interpolation methods and orders. The current satellite distribution is summarized in Table 1 . Existing literature primarily examines interpolation method accuracy without analyzing the specific interpolation performance across the three satellite types, and typically uses only one or three satellites (one per type) for algorithm validation—providing insufficient data support for robust conclusions

[8-10].

Building upon previous research, this study employs both sliding and non-sliding Lagrange interpolation methods to investigate the interpolation accuracy of BDS' s three satellite types. We first analyze the relationship between interpolation accuracy and interpolation order for each satellite type, examining how accuracy varies with order. Second, we investigate the statistical regularity of interpolation errors at lower orders. Finally, we determine the optimal interpolation order for each experimental satellite under both methods and compare the interpolation accuracy and optimal results across satellite types. Due to space constraints and data quality considerations, we selected 32 satellites with PRN numbers between C01 and C37 for experimentation (bold numbers indicate selected experimental satellites).

1 Algorithm Principles

1.1 Lagrange Interpolation Method The Lagrange interpolation method can estimate values at arbitrary positions within nodal points based on known data. Given a series of known nodes and their corresponding values, the Lagrange interpolation polynomial can be expressed as [3,11]:

$$L_n(x) = \sum_{k=0}^{n} y_k \cdot l_k(x) = \sum_{k=0}^{n} y_k \cdot \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

Since satellite position information contains three coordinate components (X, Y, Z), the interpolation polynomial for each component at time t can be expressed as:

$$\begin{cases} X(t) = \sum_{k=0}^{n} X_k \cdot \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{t-t_i}{t_k-t_i} \\ Y(t) = \sum_{k=0}^{n} Y_k \cdot \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{t-t_i}{t_k-t_i} \\ Z(t) = \sum_{k=0}^{n} Z_k \cdot \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{t-t_i}{t_k-t_i} \end{cases}$$

where $(X(t), Y(t), Z(t))$ represents the interpolated satellite position at time t, t_k denotes the sample epoch, and (X_k, Y_k, Z_k) represents the satellite position at time t_k.

1.2 Sliding Lagrange Interpolation Strategy The sliding Lagrange interpolation method treats the interpolation interval as a movable "window," shifting the interval with the target point to maintain the interpolation point at the center of the interval [11-12]. Traditional interpolation methods often suffer from Runge' s phenomenon, where oscillations near interval endpoints degrade accuracy. By interpolating only at the center of the interval, the sliding method avoids Runge' s phenomenon and improves accuracy, though with greater computational cost than the non-sliding approach. In practice, both methods can be combined to balance accuracy and efficiency.

2 Experimental Analysis

We utilized IGS precise ephemerides from May 30, 2020, with a 5-minute epoch interval as our original data. Based on Table 1, we extracted precise ephemerides for 32 satellites, each containing 288 three-dimensional positions. To evaluate interpolation accuracy and order, we extracted coordinates at 10-minute intervals as known data, interpolated positions at remaining epochs, and compared them with the precise ephemerides using Root Mean Square Error (RMS) as the accuracy metric. Given the large number of experimental satellites and errors in each coordinate direction, and due to space limitations, all subsequent RMS values represent the comprehensive position interpolation error rather than individual directional components.

2.1 Relationship Between Interpolation Accuracy, Method, and Order

To investigate how interpolation order affects accuracy for both methods, we calculated interpolation precision for orders 4 through 20, with results shown in Figure 1 [Figure 1: see original paper] and Figure 2 [Figure 2: see original paper]. To illustrate the accuracy magnitude across satellite types, Table 2 and Table 3 present detailed statistics for two satellites from each type.

The analysis reveals several key findings:

(1) For all 32 satellites across all orders, sliding Lagrange interpolation consistently outperformed non-sliding interpolation at equivalent orders. At order 4, both methods produced relatively large RMS errors, with average RMS values of 1629.47 mm and 908.13 mm for non-sliding and sliding methods, respectively—representing a 79.43% accuracy improvement with the sliding approach. For higher orders, sliding interpolation maintained superior accuracy, particularly for MEO and IGSO satellites, where improvements typically exceeded 50%.

(2) GEO satellites were significantly less sensitive to interpolation method and order compared to MEO and IGSO satellites. With non-sliding Lagrange interpolation, GEO satellites maintained RMS values below 4 mm at lower orders, with errors gradually increasing with order. At order 20, average RMS values for GEO, MEO, and IGSO satellites were 1972.66 mm, 2050.71 mm, and 1935.66 mm, respectively, with MEO and IGSO errors showing clear U-shaped distributions. With sliding Lagrange interpolation, GEO satellite RMS remained nearly constant across orders, while MEO and IGSO satellites exhibited larger RMS at orders 4–5 with minimal variation thereafter. Notably, optimizing interpolation order for GEO satellites improved accuracy by only approximately 4% (about 0.1 mm) compared to order 4 results.

(3) Non-sliding Lagrange interpolation demonstrated greater sensitivity to order than the sliding method. Overall, non-sliding errors followed a U-shaped distribution, with larger errors at low and high orders and minimal errors near orders 7–9. Sliding interpolation produced larger errors only at orders 4–5, with errors stabilizing as order increased.

To examine error patterns, we analyzed interpolation errors at each epoch. Due to space constraints, we selected one satellite from each type (C01, C06, C11). Figures 3 [Figure 3: see original paper] through 5 [Figure 5: see original paper] illustrate error distributions across interpolation epochs, with the Z-axis representing comprehensive position interpolation error at each epoch.

With non-sliding Lagrange interpolation, errors became anomalous at certain epochs when order exceeded 10, with anomaly frequency decreasing but magnitude increasing with order. These anomalies predominantly occurred near interval endpoints, confirming Runge' s phenomenon as the primary cause. Sliding Lagrange interpolation, by maintaining the interpolation point at interval center, effectively avoided Runge' s phenomenon, showing no anomalies and relatively stable error variation across orders.

Further analysis revealed that interpolation errors exhibited periodicity with respect to interpolation epochs, particularly pronounced at lower orders. Power spectrum analysis of three-directional errors confirmed that low-order interpolation error periodicity matches that of the precise ephemeris. This occurs because low-order interpolation uses fewer original data points, making results more susceptible to original data patterns. As order increases, the influence of multiple data points dilutes these patterns, explaining why periodicity diminishes at higher orders. GEO satellite interpolation errors showed weaker periodicity, further confirming their superior interpolation accuracy.

2.2 Comparison of Optimal Interpolation Order Accuracy While non-sliding Lagrange interpolation is more computationally efficient than sliding interpolation at equivalent orders, its accuracy is lower. To compare optimal accuracies, we extracted the optimal interpolation order for each experimental satellite (Table 4) and calculated RMS statistics at these optimal orders (Figure 6 [Figure 6: see original paper]).

Table 4 shows that optimal interpolation orders for both methods ranged from 4 to 8, with differences less than 1 for each satellite. Specifically, 12.5% of satellites had lower optimal orders with non-sliding interpolation, 59.4% had lower optimal orders with sliding interpolation, and 28.1% had equal optimal orders. Among satellite types, GEO satellites primarily had optimal orders of 4–5, lower than IGSO satellites (mainly order 6), while MEO satellites required the highest orders (primarily 6–7, occasionally 8).

Figure 6 demonstrates that at optimal orders, sliding Lagrange interpolation still outperformed non-sliding interpolation, achieving accuracy improvements of 11.96% to 44.01%. Furthermore, GEO satellites attained superior accuracy at lower optimal orders, indicating more stable orbital data, while MEO and IGSO satellites showed similar interpolation accuracy.

Conclusion

This study investigated BDS precise ephemeris interpolation for three satellite types using sliding and non-sliding Lagrange interpolation methods. We visualized the relationship between interpolation accuracy and order in three dimensions, and statistically analyzed accuracy across satellite types at equivalent orders. At low orders, interpolation accuracy was significantly influenced by original ephemeris patterns, with GEO satellites less affected than MEO and IGSO satellites. As order increased, these patterns gradually weakened. Optimal interpolation orders differed by less than 1 between methods for all satellites, with sliding Lagrange interpolation achieving 11.96%–44.01% higher accuracy than non-sliding interpolation at optimal orders. Overall, GEO satellites demonstrated superior interpolation accuracy at both equivalent and optimal orders, with lower optimal orders. Between methods, sliding Lagrange interpolation consistently outperformed non-sliding interpolation. In practical applications with large datasets, the increased computational load of sliding Lagrange interpolation can be mitigated by combining both methods to balance accuracy and efficiency.

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Note: Figure translations are in progress. See original paper for figures.