Postprint of the Study on Periodic Characteristics of High-Energy Electrons Based on Maximum Entropy Spectral Estimation

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Abstract

This paper investigates the periodic characteristics of high-energy electron flux ($ $2.0MeV) from the FY-2D satellite using the maximum entropy spectral estimation method. Based on the autoregressive (AR) model, this algorithm infers that the high-energy electron flux from FY-2D exhibits periods of 13.87d and 27.8d through analysis of the maximum entropy power spectrum. The optimal order is determined according to the FPE and AIC criteria to calculate the AR model parameters, and the Levinson-Durbin and Burg algorithms are compared with the maximum entropy spectral estimation method, revealing that the maximum entropy spectral estimation method demonstrates advantages in periodic characteristic studies. This result is crucial for investigating the spatial distribution of high-energy electrons in geosynchronous orbit, forecasting high-energy electron enhancement events, and providing early warnings for deep dielectric charging events.

Full Text

Preamble

Study on the Periodic Characteristics of High-Energy Electrons Based on Maximum Entropy Spectral Estimation

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Abstract

This paper investigates the periodic characteristics of high-energy electron flux ($ $2.0 MeV) observed by the FY-2D meteorological satellite using the maximum entropy spectral estimation method. Based on an autoregressive (AR) model, the analysis of the maximum entropy power spectrum reveals that the high-energy electron flux exhibits periods of 13.87 days and 27.8 days. The optimal model order is determined using the Final Prediction Error (FPE) and Akaike Information Criterion (AIC), and AR model parameters are subsequently calculated. A comparative analysis between the Levinson-Durbin algorithm, Burg algorithm, and maximum entropy spectral estimation demonstrates that the maximum entropy method offers distinct advantages for studying periodic characteristics. These findings are crucial for understanding the spatial distribution of high-energy electrons in geosynchronous orbit, forecasting high-energy electron enhancement events, and providing early warnings for deep dielectric charging events.

Keywords: High-energy electrons; Maximum entropy spectral estimation; Periodic characteristics; Power spectrum estimation

Introduction

The space radiation environment composed of high-energy charged particles represents one of the most critical factors affecting the safe operation of spacecraft in orbit. These particles impact spacecraft through three primary mechanisms: First, when high-energy charged particles penetrate materials, they induce charging and discharging reactions on surfaces or within spacecraft materials, causing electromagnetic interference that can lead to system failures. Second, high-energy electrons generate current pulses that alter charge distributions within electronic components, resulting in logic state confusion, malfunctions, or even complete failure. Third, the radiation effects of high-energy charged particles degrade material and component performance through ionization until eventual failure, while also posing significant health risks to astronauts [1].

The Fengyun-2 series represents China' s first generation of geostationary meteorological satellites, which, together with polar-orbiting meteorological satellites, constitute China' s meteorological satellite application system [2]. The FY-2D satellite was successfully launched on December 8, 2006, operating at an altitude of 35,783–35,788 km. The satellite' s space particle detector, part of its space environment monitor, measures high-energy protons, electrons, and helium ions across seven energy channels. Positioned at 86.5°E above the equator, FY-2D has an orbital period of 1,436.04 minutes, an inclination of 1.84°, and maintains spin-stabilized operation at 100 rpm. Together with the previously launched FY-2A satellite, FY-2D enables dual-satellite observations that achieve synergistic benefits greater than the sum of their individual contributions, increasing observation frequency, expanding coverage, and enabling stereoscopic and dynamic monitoring [3]. This configuration provides more precise and reliable data for studying the periodic characteristics of high-energy particles and offers valuable references for investigating the influence of solar and geomagnetic activities on the space environment.

This study employs maximum entropy spectral estimation to analyze the periodic variation characteristics of near-Earth high-energy electrons ($ $2.0 MeV electron flux) and compares the advantages and disadvantages of this method with the Levinson-Durbin and Burg algorithms. The dataset spans from December 19, 2006, to May 21, 2012, with a temporal resolution of 5 minutes. Missing data segments were interpolated to ensure continuity.

1 Research Methods

1.1 Maximum Entropy Spectral Estimation

While Fourier transform is a commonly used time-frequency analysis method [4], this study adopts maximum entropy spectral estimation for two primary reasons. First, Fourier transform lacks time-frequency localization capability; it is a global transformation that cannot reflect how signals change in specific time intervals or provide temporal information about when particular frequencies occur. Second, Fourier transform has limitations when analyzing non-stationary signals. The high-energy electron flux studied here exhibits time-varying frequency characteristics, and Fourier analysis can only provide overall frequency variation effects rather than completely reflecting the essential characteristics of the signal at specific moments.

Maximum entropy spectral estimation offers high resolution and short-time characteristics. Its principle can be summarized as follows: using known autocorrelation function values, with maximum entropy as the premise, extrapolating unknown autocorrelation function values from N known ones, and finally performing frequency domain transformation to obtain continuous power spectrum estimation, thereby providing algorithmic support for studying data periodic characteristics.

The specific steps are [5]: Calculate initial values rx(0) = f(n) = g(n) = x(n); determine the prediction mean square error pm recursion formula; solve for AR model reflection coefficients km; compute forward and backward prediction errors, then sequentially estimate reflection coefficients km:

f(n) = f(n) + kmgm−1(n−1)
gm(n) = fm−1(n−1) + gm−1(n−1)

Through Levinson recursion, determine AR model parameters am(i) = am−1(i) + kmam−1(m−1) for i = 1, 2, ···, m−1 when order m = 2, with am(m) = km. Repeat this process until m reaches the required AR model order, obtain all AR model parameters, and predict the maximum entropy of the time series:

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where k = 1, 2, ···, p represents p-order linear prediction filter coefficients, and σ^2 is the prediction error power of the filter.

1.2 Levinson-Durbin and Burg Algorithms

Both Levinson-Durbin and Burg algorithms [6] are linear prediction methods that calculate power spectrum estimates from known time signal sequences through recursive algorithms. The Burg algorithm utilizes forward filtering error and backward filtering error to minimize filter error power, then calculates AR model parameters according to the Levinson-Durbin algorithm. AR model power spectrum estimation requires the Yule-Walker equations:

am(i) = am−1(i) + kmam−1(m−i), i = 1, 2, ···, m−1

The (m+1)th parameter G of an M-order AR model satisfies:

G^2 = ppm

where pm is the prediction power error, yielding the recursion formula:

$(1-k^2)$

Using equation (8) for recursion, we obtain the p+ parameters of the AR model characterizing the random signal, and finally calculate the power spectral density of the random signal:

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1.3 Determination of AR Model Order

Determining the appropriate AR model order is crucial in maximum entropy spectral estimation and Levinson-Durbin and Burg algorithms. If the order is too small, the resulting power spectrum becomes overly smoothed, failing to effectively resolve periodic components of the time series. If the order is too large, it affects the stability of maximum entropy estimates. The optimal AR model order must be determined during the recursion process. Using the Levinson recursion algorithm, each parameter set from low to high order and the model' s minimum prediction error power pmin are decreasing. Theoretically, when the p value reaches the expected value or no longer changes, optimal applicability is achieved, and the corresponding order is the best parameter.

Two commonly used order applicability test criteria are [7]:

(1) Final Prediction Error Criterion (FPE)

The FPE criterion considers the error between predicted and actual values, minimizing error corresponds to a single order value r. FPE(r) is defined as:

(1))(1)rNrFPErpNr （

where N is the sample size of high-energy electron flux data.

(2) Akaike Information Criterion (AIC)

The AIC criterion is based on entropy concepts and can determine both model complexity and goodness-of-fit, defined as:

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When the order r changes, the values of FPE(r) and AIC(r) also change. When r takes a certain value where both FPE(r) and AIC(r) reach their minima, this r is the optimal order value.

In practical calculations, when data is short, the obtained optimal order tends to be low, and both criteria yield essentially the same optimal order value, satisfying:

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2 High-Energy Electron Flux Periodicity

2.1 Data Selection

This study analyzes high-energy electron flux in the $ $2.0 MeV energy channel, representing extremely energetic charged particles. Since the original data contains gaps during December 6–30, 2009, February 2–11, 2011, and August 22–24, 2011, interpolation was performed using the Fillmissing function in MATLAB to avoid impact from missing data on periodicity analysis. [Figure 1: see original paper] shows the variation of $ $2.0 MeV electron flux from December 19, 2006, to May 21, 2012, with a 5-minute sampling frequency (288 data points per day on average). After interpolation, the dataset contains over 500,000 data points.

2.2 Data Processing

As shown in [Figure 1: see original paper], the long time span and enormous data volume make direct analysis difficult for intuitively identifying periodic characteristics due to the dense, highly fluctuating data. Therefore, daily averaging was applied. [Figure 2: see original paper] presents the daily averaged data from December 19, 2006, to May 21, 2012. Since we focus on electron flux periodicities on the order of days and daily averaging smooths short-term temporal characteristics, this approach is justified. The figure reveals that high-energy electron flux exhibits periodic behavior with multiple periods—primary and secondary. Based on the distribution characteristics, fluctuations are more pronounced during December 19, 2006–December 19, 2008, and December 19, 2009–May 21, 2012, while variations are minimal during December 19, 2008–December 19, 2009. To better investigate the periodic characteristics from 2006–2012, maximum entropy spectral estimation [8] was employed.

Since the particle detector on FY-2D has missing data in some high-energy electron flux measurements, we interpolated the raw data and applied normalization to reduce the impact of interpolated data on the original measurements and overall results [9]. [Figure 3: see original paper] shows that the data distribution characteristics are nearly identical to those in [Figure 2: see original paper], demonstrating that using interpolated data for periodicity analysis is reasonable.

3 Periodicity Analysis of High-Energy Electron Flux

To estimate the periodic characteristics of high-energy electron flux [10], this study analyzes daily averaged data from December 19, 2006, to May 21, 2012, using both maximum entropy spectral estimation and Levinson-Durbin/Burg algorithms for comparative accuracy analysis.

3.1 AR Model Order Selection and Determination

As described in Section 2.2.1, we generally employ FPE and AIC criteria to determine the appropriate AR model order. Selecting the correct AR order is critical in maximum entropy spectral estimation, as its magnitude affects identification of spectral peaks in the entropy spectrum and consequently influences periodicity results.

Based on the FPE and AIC calculation formulas, optimal order selection was implemented in MATLAB [11]. [Figure 4: see original paper] illustrates the relationship between order and parameter estimation for FPE (left) and AIC (right) criteria. Using least squares method and FPR/AIC criteria for AR model parameter estimation, the corresponding functional relationships were plotted.

Comparison of the two panels in [Figure 4: see original paper] reveals that when the AR model order is 10, both FPE and AIC functions reach their minimum parameter estimation values of 1,277,145.757 and 33,018.72179, respectively. According to the established criteria, these results satisfy equation (11) in Section 2.3, confirming that the optimal AR model order is 10. This determined order applies to both maximum entropy spectral estimation and Levinson-Durbin/Burg algorithms.

3.2.1 Maximum Entropy Spectral Estimation

Based on the optimal order determined in Section 4.1, maximum entropy spectral analysis was performed [12]. Using daily averaged data, the maximum entropy power spectrum of high-energy electron flux was generated, as shown in [Figure 5: see original paper] for order 10. High-energy electron flux periods consist of primary and secondary components [13]. According to the power spectrum probability distribution, multiple peaks are evident. This study focuses on the highest and second-highest peaks. The primary period corresponds to a frequency of 0.0625 d^{-1} with maximum power spectral density of 14.16, yielding a period of 13.87 days. The secondary peak occurs at 0.1641 d^{-1} with power spectral density of 13.71, corresponding to a period of 27.8 days. These results are consistent with literature [14] that identified 27-day and 13-day periods in high-energy electron flux using wavelet analysis, validating the accuracy of maximum entropy spectral estimation for this application.

Although the method is reasonable and considers numerous factors, interpolation introduces an estimated error of approximately 0.03–0.08 days in the calculated results. Additionally, the multiple spectral peaks clearly indicate non-unique and unstable periodicities [15].

3.2.2 Burg and Levinson Algorithms

With the optimal order determined as 10, daily averaged data were used for periodicity analysis. Power spectra for Burg and Levinson algorithms were plotted according to order 10. Comparison with [Figure 5: see original paper] shows that the left and right panels in [Figure 6: see original paper] are essentially consistent, confirming that order 10 is optimal. However, the primary period frequency is 0.011 d^{-1} and the secondary period frequency is 0.128 d^{-1}, yielding primary and secondary period values that differ from maximum entropy spectral estimation results.

[Figure 6: see original paper] demonstrates that high-energy electron flux indeed exhibits periodic behavior with multiple periods. The primary and secondary periods are 13.87 days and 27.8 days, respectively. Comparison of maximum entropy spectral estimation with Burg and Levinson algorithms indicates that maximum entropy spectral estimation produces results closer to true values and better reflects actual conditions.

Literature [16, 17] shows that the 13-day and 27-day periods of electron flux are closely related to solar activity. During solar minimum years, trans-equatorial coronal holes develop high-speed solar wind that causes electron flux variations in the outer radiation belt. As the Sun rotates, these extended coronal holes complete one rotation every 27 days, imparting a 27-day periodicity to electron flux variations. The 13-day period emerges alongside solar wind speed variations, with the 27-day cycle containing three phases: rapid rise, peak, and decline. This symmetric variation process forms the 13-day period, meaning the 13-day periodic component generally exists within the 27-day electron flux cycle.

4 Conclusion

Based on high-energy electron flux data detected by FY-2D satellite, this study investigated periodic characteristics using maximum entropy spectral estimation, yielding the following conclusions:

1. Maximum entropy spectral estimation reveals primary periods of 13.87 days and 27.8 days for high-energy electron flux, with non-unique and unstable periodic behavior.

2. Comparison between maximum entropy spectral estimation and Levinson-Durbin/Burg algorithms for high-energy electron periodicity analysis demonstrates that maximum entropy spectral estimation is more suitable

for this research [18], producing results that better approximate real conditions.

3. Solar activity influences high-energy electron periods, which vary accordingly. Analyzing periodic variations in electron flux can indirectly predict solar activity trends, providing valuable references for future solar activity research.

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Note: Figure translations are in progress. See original paper for figures.