

## Analysis of the Energy Absorption Capacity of the Caladine-English Folded Plate Energy Absorber

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### Abstract

Building upon the work of Zhang-Yu, this paper conducts a detailed analysis of the energy absorption capacity of the Caladine-English folded plate energy absorber. We developed a general-purpose program using Maple software for the Zhang-Yu energy absorber equations of the Caladine-English folded plate energy absorber, and obtained for the first time the time evolution of the absorber's generalized coordinates (rotation angles). On this basis, a comparative study was performed on the influence of various parameter variations on the energy absorption capacity of the Caladine-English folded plate energy absorber, for which an energy absorption factor was defined to characterize the structure's energy absorption capability. To better understand how parameter variations affect the absorber's motion, we computed the phase portrait of the absorber dynamics. Through extensive numerical simulations, it was found that, from the perspective of energy absorption capacity, the 4-fold plate absorber should be a mass-sensitive'' type absorber, which fully supports Zhang-Yu's conclusion: namely, that the Caladine-English folded plate energy absorber is a mass-sensitive'' type absorber.

### Full Text

#### 3.1 Dynamic Response Analysis of the Calladine-English Model

The dynamic response of the Calladine-English model was analyzed using both rigid-viscoplastic and rigid-plastic formulations. Numerical simulations were performed for three initial folding angles:  $\theta_0 = 1.146^\circ$ ,  $\theta_0 = 2.6^\circ$ , and  $\theta_0 = 4^\circ$ . The governing equations were solved using Maple's rkf45 Runge-Kutta method with appropriate initial conditions.

The system parameters were defined as follows:  $b = 51$  mm,  $h = 1.6$  mm,  $2L = 50$  mm,  $m = 0.016$  kg, and  $M_0 = 9.79$  N·m. The impact energy was fixed at  $U_0 = 122$  J, with the drop weight  $G$  and impact velocity  $V_0$  varied according to seven parameter combinations listed in Table 3. The initial angular velocity was determined from the impact conditions using the relation  $\dot{\theta}_0 = GV_0 \sin \theta_0 / [m/3 + (m + G) \sin^2 \theta_0]$ .

Figure 3 [Figure 3: see original paper] shows the angular displacement  $\theta(t)$  for the rigid-viscoplastic model with  $\theta_0 = 1.146^\circ$  under all seven parameter combinations from Table 3. The corresponding response for the rigid-plastic model is presented in Figure 4 [Figure 4: see original paper]. Similar comparisons for  $\theta_0 = 2.6^\circ$  are shown in Figures 5 [Figure 5: see original paper] (viscoplastic) and 6 [Figure 6: see original paper] (plastic), while Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper] display the results for  $\theta_0 = 4^\circ$ . The Maple code implementation for these simulations is provided in Table 2, which illustrates the piecewise definition of the moment-rotation rate relationship according to the Cowper-Symonds constitutive law.

### 3.2 Parametric Studies of Impact Velocity and Drop Weight

The influence of impact velocity  $V_0$  was investigated while maintaining constant impact energy  $U_0 = 122$  J. Phase diagrams of  $\dot{\theta}$  versus  $d\theta/dt$  were generated for both material models at two initial angles. Figure 27 [Figure 27: see original paper] shows the rigid-viscoplastic response for  $\theta_0 = 1.146^\circ$  with  $V_0$  varying from 1 to 10 m/s, while Figure 28 [Figure 28: see original paper] presents the corresponding rigid-plastic case. For  $\theta_0 = 4^\circ$ , the phase diagrams are shown in Figures 29 [Figure 29: see original paper] (viscoplastic) and 30 [Figure 30: see original paper] (plastic). These results demonstrate that higher impact velocities produce more rapid initial rotation rates, though the total rotation is constrained by the fixed impact energy.

The effect of drop weight  $G$  was similarly examined. Figures 31 [Figure 31: see original paper] and 32 [Figure 32: see original paper] compare the  $\dot{\theta}$ - $d\theta/dt$  phase diagrams for viscoplastic and plastic models with  $\theta_0 = 1.146^\circ$  and  $G$  varying as  $4i$  kg ( $i = 1$  to 10). The corresponding results for  $\theta_0 = 4^\circ$  are presented in Figures 33 [Figure 33: see original paper] and 34 [Figure 34: see original paper]. The analysis reveals that for a given impact energy, heavier drop masses produce lower peak rotation rates but similar final angular displacements.

### 3.3 Influence of Structural Mass and Mass Ratio

The structural mass  $m$  was varied independently to assess its effect on the dynamic response. Phase diagrams for  $m = 0.01 \cdot i$  kg ( $i = 1$  to 10) are shown in Figures 35 [Figure 35: see original paper] and 36 [Figure 36: see original paper] for  $\theta_0 = 1.146^\circ$ , and in Figures 37 [Figure 37: see original paper] and 38 [Figure 38: see original paper] for  $\theta_0 = 4^\circ$ , covering both viscoplastic and plastic

formulations. The results indicate that increasing structural mass reduces the angular acceleration for a given impact condition.

The combined effect of mass ratio  $m/G$  was also analyzed. Figures 39 [Figure 39: see original paper] through 42 [Figure 42: see original paper] present phase diagrams for various  $m/G$  ratios, showing how the relative magnitude of structural mass to drop weight influences the energy transfer efficiency and deformation mode. The mass ratio emerges as a critical parameter governing the inertia effects in the Calladine-English model.

## 4 Discussion and Conclusions

The numerical simulations demonstrate several key characteristics of the Calladine-English energy-absorbing structure:

1. **Strain Rate Sensitivity:** The viscoplastic model consistently predicts higher moment resistance than the plastic model due to rate-dependent material behavior, particularly evident at higher rotation rates.
2. **Inertia Effects:** The dynamic response shows significant inertia effects, especially for small initial angles ( $\theta_0 < 2^\circ$ ), where the rotational acceleration is most sensitive to impact parameters.
3. **Energy Conservation:** The total energy absorption capacity remains consistent across different parameter combinations when  $U_0$  is fixed, though the deformation history varies substantially.
4. **Parameter Interdependence:** The initial folding angle  $\theta_0$ , drop weight  $G$ , impact velocity  $V_0$ , and structural mass  $m$  exhibit complex interactions that determine the final deformation mode and energy dissipation distribution.

The results validate the theoretical framework established by Calladine and English [1,27] and extend previous work by Zhang and Yu [2] through comprehensive parametric analysis. The findings are consistent with experimental observations reported in [30,31,33] regarding inertia and strain-rate effects in impact-loaded structures.

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