A Consensus Protocol Based on Switching Topology and Event-Triggered Mechanism: Postprint

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Abstract

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Full Text

A Consensus Protocol Based on Switching Topology and Event-Triggering Mechanism

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Abstract: This paper investigates a novel event-triggered consensus protocol for multi-agent systems (MASs) based on switching topology. The protocol designs two event triggers: a convergence event trigger caused by agent state evolution and a topology event trigger caused by topology changes. Stability analysis and theoretical proof of a common Lyapunov function related to the event triggers are provided. Additionally, a network connectivity algorithm based on constraint sets is designed to enhance network connectivity. Simulation results demonstrate that the proposed protocol enables system convergence to consensus, effectively reduces the update frequency of agent controllers, decreases system energy consumption, improves network connectivity, and provides theoretical support for subsequent research on event-triggered control.

Keywords: multi-agent systems; switching topology; event-triggered control; distributed control; Lyapunov function

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Multi-agent systems (MASs) refer to networked systems composed of multiple agents with certain perception, communication, computation, and execution capabilities that collaborate to accomplish group tasks through limited local information exchange [1]. The consensus problem is a hot research topic in MASs [2,3]: through well-designed consensus protocols, all members' state values converge to the same value [4,28].

In practical engineering applications of continuous-time MASs, periodic sampling is primarily employed, leading to excessively frequent controller updates, high system energy consumption, and excessive demands on communication resources and computational capabilities [5]. To address these issues, event-triggered control was proposed in [6]. In event-triggered control, system controllers update only when special events occur, significantly reducing controller updates and effectively lowering system energy consumption without requiring higher-performance processors. This approach has become an active research topic [7~9].

Meng et al. [8] proposed a distributed event-triggered consensus protocol for discrete-time MASs with fixed topology, where each agent checks its event trigger condition only at every sampling period, greatly reducing both the number of trigger condition evaluations and controller updates.

Event-triggered control is mainly divided into three categories: traditional event-triggered control [10,11], sampling-period-based event-triggered control [8], and self-triggered control [10]. Traditional event-triggered control applies to continuous-time MASs, including: a) event-triggered control for first-order integrator systems [9,10,12]; b) event-triggered control for second-order integrator systems [13,14]; c) event-triggered control for linear systems [15~17]; d) event-triggered control for nonlinear systems [18]. The other two categories apply to discrete-time MASs, including: e) sampling-period-based event-triggered control [8,19,20]; f) self-triggered control [10,21]; g) event-triggered control for MASs with time delays [15,22].

Current research on event-triggered control focuses on designing consensus protocols and defining event-triggering functions that enable agents to avoid continuous communication while achieving consensus and ensuring the closed-loop system is free of Zeno phenomena [7,10]. However, existing event-triggered control typically requires fixed topology. Research shows that networked systems with switching topology better reflect practical engineering applications [23,24], making it more important to design MASs that accommodate network switching

[25].

This paper investigates the consensus problem for first-order discrete-time MASs with switching topology under event-triggered control from the perspective of combining event-triggered control with switching topology. We propose a consensus protocol for discrete-time MASs based on sampling-period event-triggered control. The event trigger employs a hybrid approach to reduce controller update frequency—two event triggers coexist, referred to as the convergence event trigger and topology event trigger. At each discrete sampling instant, agents check whether they satisfy the event trigger conditions. When either trigger condition is met, the agent updates its controller; otherwise, it does not. Additionally, considering that the system network has switching topology and may become disconnected during evolution, this paper adopts a constraint-set-based connectivity method to ensure real-time system connectivity.

1.1 Graph Theory and Matrix Theory

This paper models an MAS with n agents as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ with n vertices. Here $\mathcal{V} = \{1, \dots, n\}$ is the vertex set representing ordered agent labels. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, where $(i, j) \in \mathcal{E}$ indicates that vertices i and j are neighbors that can receive information from each other. $A = [a_{ij}]_{n\times n}$ is the non-negative weighted adjacency matrix of graph $\mathcal{G}$, whose elements are associated with edges: $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$. The common neighbor set of vertices i and j is defined as $N_{ij} = N_i \cap N_j = \{k \mid (i, k), (j, k) \in \mathcal{E}\}$. The degree matrix D of graph $\mathcal{G}$ is defined as $D = \mathrm{diag}\{d_1, \dots, d_n\}$, where $d_i = \sum_{j\in N_i} a_{ij}$ is the degree of vertex i.

1.2 Switching Topology

Consider an MAS with n agents whose state values belong to a real set $\mathbb{R}$. Let $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), A(t))$ denote the topology structure corresponding to the agents. If the edge set $\mathcal{E}(t)$ is given at the initial time and changes over time during system state evolution, the MAS network is called a switching topology structure.

1.3 Discrete-Time Consensus Protocol

For agent i, let t_{k_j} denote the last triggering instant of its neighbor j before time t_k. Based on the above expressions, the controller $u_i(t_k)$ depends on both agent i' s triggering instant t_{k_i} and its neighbor j' s triggering instant t_{k_j}. Therefore, agent i' s controller needs to be updated at both its own and its neighbors' triggering instants.

At each sampling period h, agent i communicates with its neighbors and checks whether its state satisfies the convergence event trigger. If the trigger condition is not met, agent i does not update its controller; if the condition is satisfied, agent i updates its controller and communicates with its neighbors to update their controllers. Simultaneously, the measurement error $e_i(t)$ is reset to zero, and the convergence event trigger condition is no longer satisfied.

The triggering time sequence for agent i' s convergence event trigger is defined as $\{t_{k_i}^0, t_{k_i}^1, \dots, t_{k_i}^k, \dots\}$, satisfying the following condition:

Olfati-Saber et al. [26] proposed a distributed consensus protocol for discrete-time MASs with switching topology:

If all agents in the system evolve according to the discrete-time consensus protocol (1), consensus can be considered achieved when the state values of any two agents satisfy equation (2).

2 A Consensus Protocol Based on Switching Topology and Event-Triggered Control

This research combines event-triggered control methods with discrete-time MASs under switching topology, designing two system event triggers for hybrid control. We propose an event-triggered consensus protocol based on switching topology and introduce a constraint-set method to ensure network connectivity. Furthermore, we design a common Lyapunov function and complete stability theoretical analysis and proof for the triggers.

Let $\mathcal{G}(t) = (\mathcal{V}(t), \mathcal{E}(t), A(t))$ denote the switching topology corresponding to the MAS, where $\mathcal{V}(t) = \{1, \dots, n\}$. Let $x(t) = [x_1(t), \dots, x_n(t)]^T$ represent the agents' state vector, where d is the communication range.

2.1.1 Convergence Event Trigger Design

Agent i's real-time state is $x_i(t)$, and its state at each sampling period h is $x_i(kh)$. Let $\{t_{k_i}^0, t_{k_i}^1, \dots, t_{k_i}^k, \dots\}$ be the agent' s triggering time sequence, satisfying the condition $t_{k_i}^0 = 0$, $t_{k_i}^{k+1} = \inf\{t > t_{k_i}^k \mid f_i(t) \geq 0\}$, where $f_i(t)$ is the event-triggering function to be designed.

Agent i' s state at the k-th triggering instant $t_{k_i}^k$ is $x_i(t_{k_i}^k)$. Since the current time t lies in the interval $[t_{k_i}^k, t_{k_i}^{k+1})$, the state at time t can be expressed as $x_i(t) = x_i(t_{k_i}^k + lh)$, where l is an integer multiple of the sampling period h. Therefore, the interval between two consecutive triggering instants of agent i is $t_{k_i}^{k+1} - t_{k_i}^k \geq h$. Since the interval between two triggering instants is always no less than h, the system can fundamentally avoid Zeno phenomena. The purpose is to convert the discrete-time signal $x_i(kh)$ into a continuous-time signal $\hat{x}_i(t)$.

The measurement error is defined as $e_i(t) = \hat{x}_i(t) - x_i(t)$. Agent i' s combined state at current time t between its last triggering instant $t_{k_i}^k$ and current time t is represented as:

Agent i' s convergence event trigger is defined as follows:

The triggering time sequence for agent i' s convergence event trigger $\{t_{k_i}^0, t_{k_i}^1, \dots, t_{k_i}^k, \dots\}$ satisfies the following condition:

2.1.2 Topology Event Trigger Design

Under switching topology, agent i' s neighbor relationships change as the agent' s state evolves. Assuming the connectivity of the MAS switching topology is always maintained, the final switching topology will become a complete graph, and the states of all agents can reach consensus.

Agent i's topology event trigger is defined as follows: When agent i receives state information from a new neighbor (an agent it has not previously communicated with), the topology event trigger is activated, and agent i updates its controller.

The triggering time sequence for agent i's topology event trigger $\{t_{p_i}^0, t_{p_i}^1, \dots, t_{p_i}^k, \dots\}$ satisfies the following condition:

In summary, agent i' s triggering time sequence is $\{t_0^i, t_1^i, \dots, t_k^i, \dots\} = \{t_{k_i}^0, t_{k_i}^1, \dots\} \cup \{t_{p_i}^0, t_{p_i}^1, \dots\}$, which is the union of the triggering time sequences of the two event triggers.

2.2 Event-Triggered Consensus Protocol Design

Yang et al. [27] proposed the following distributed consensus protocol for continuous-time MASs with switching topology:

Yang noted that paying attention to information from neighboring agents with fewer common neighbors facilitates system consensus. If the MAS switching topology is a connected graph at the initial time, then under ideal continuous-time conditions, consensus protocol (10) can ensure that the system topology remains always connected and that all agents' states eventually converge to a balanced value α, where α is the global geometric center of graph $\mathcal{G}$, satisfying $\alpha = \frac{1}{n}\sum_{i=1}^{n} x_i(0)$.

In practical engineering applications, ideal continuous-time consensus protocols are often unrealizable due to hardware constraints. Therefore, after discretizing consensus protocol (10), we obtain the following distributed consensus protocol for discrete-time MASs with switching topology, where h is the sampling period:

To further reduce the controller update frequency of agents in the discrete-time consensus protocol (11) based on switching topology, we combine the two event triggers designed above with discrete-time consensus protocol (11) to obtain the distributed event-triggered consensus protocol for discrete-time MASs with switching topology designed in this paper (12):

Under event-triggered control, agent i' s controller is not updated at every sampling period. At each sampling period h, agent i first checks whether it satisfies either of the two event triggers. If either condition is met, the agent updates its controller; otherwise, it maintains the current controller until one of the triggers is satisfied again.

2.3 Lyapunov Stability Theory Analysis

In event-triggered control, the design of event triggers is intimately related to Lyapunov stability theory. After proposing the event-triggered consensus protocol (12) based on switching topology, we present stability analysis based on a common Lyapunov function and proof of MAS consensus.

2.3.1 Lyapunov Stability Analysis When Switching Topology Remains Unchanged If the parameter σ_i satisfies $\sigma_i \in (0, 0.0625)$, then it can be guaranteed that when the switching topology remains unchanged, for any $k \in \{0, 1, 2, ...\}$ and $t \in [kh, (k+1)h)$, we have $(V(x(t))) \leq 0$.

2.3.2 Lyapunov Stability Analysis When Switching Topology Changes We now discuss time-varying switching topology $\mathcal{G}(t)$. If the parameter $\sigma_{\max}$ satisfies $\sigma_{\max} \in (0, 0.0625)$ and the parameter h satisfies $0 < h \leq 0.125$, then it can be guaranteed that when the switching topology changes, for any $k \in \{0, 1, 2, ...\}$ and $t \in [kh, (k+1)h)$, we have $(V(x(t))) \leq 0$.

The time-varying switching topology can be represented as a discrete set of fixed topologies $\{\mathcal{G}_1, \ldots, \mathcal{G}_m\}$. Let $\bar{x}(t) = \frac{1}{n}\sum_{i=1}^{n} x_i(t)$ denote the average value of all agents. Considering that $\mathbf{1}^T L_{\mathcal{G}(t)} = 0$, based on the event-triggered consensus protocol (12), we have:

Therefore, based on the event-triggered consensus protocol (12), the average state of all agents remains constant during state evolution.

Define the disagreement vector $\delta(t) = x(t) - \bar{x}(t)\mathbf{1}$, which reflects the current convergence status of the system. When the system reaches asymptotic average consensus, the disagreement vector tends to $\mathbf{0}$. Consider $V(x(t)) = \frac{1}{2}\|\delta(t)\|^2 = \frac{1}{2}x(t)^T L_{\mathcal{G}(t)} x(t)$ as the common Lyapunov function for the system.

This function satisfies: $V(x(t)) \geq 0$ and $V(x(t)) = 0 \Leftrightarrow x(t) = \bar{x}(t)\mathbf{1}$. As the agents' states evolve, the value of the common Lyapunov function gradually decreases and eventually converges to its minimum value, correspondingly, the MAS also reaches average consensus. An effective event-triggered consensus protocol should make the Lyapunov function value decrease continuously over time.

In the sampling period time interval $[kh, (k+1)h)$, we have:

Here, for further inequality relaxation, let $\|e(kh)\|_{\max} = \max\{\|e_i(kh)\|\}$ and $\|z(kh)\|_{\max} = \max\{\|z_i(kh)\|\}$. Using the basic inequality, we obtain:

According to the convergence event trigger (6), we know that $\|e_i(t_{k_i}^k + lh)\|^2 + \|z_i(t_{k_i}^k + lh)\|^2 \leq \sigma_i \|z_i(t_{k_i}^k + lh)\|^2$.

According to the properties of the Laplacian matrix, each disk' s center $l_{ii}(t)$ satisfies $l_{ii}(t) > 0$, and each disk' s radius is the distance from the center to the origin. Therefore, each disk lies in the right half of the complex plane.

Based on the above formulas, we know that the maximum value of the main diagonal elements of $L_{\mathcal{G}(t)}$ must not exceed 2. The number of agents in the left neighbor region of agent i is $p = |X^i_{\text{left}}(t)|$, and the number of agents in the right neighbor region is $q = |X^i_{\text{right}}(t)|$. It is easy to see that agents within each neighbor region are neighbors of each other. Therefore, the weight $l_{ij}(t)$ of agents in the left neighbor region must not exceed p, and the weight term of agents in the right neighbor region must not exceed q. If there are two agents in the left and right neighbor regions that are neighbors, the weights of these two agents are also less than p (or less than q).

$l_{ii}(t)$ is the sum of the weights of all neighbors of agent i. In summary, $l_{ii}(t) \leq \max\{p, q\} \leq \max\{p + q\} \leq \max\{n - 1\}$. Therefore, if the parameter $\sigma_{\max}$ satisfies $\sigma_{\max} \in (0, 0.0625)$ and the parameter h satisfies $0 < h \leq 0.125$, then it can be guaranteed that under switching topology, for any $k \in \{0, 1, 2, ...\}$ and $t \in [kh, (k + 1)h)$, we have $(V(x(t))) \leq 0$.

The time-varying Laplacian matrix $L_{\mathcal{G}(t)}$ can be expressed as a discrete set of fixed Laplacian matrices $\{L_{\mathcal{G}_1}, ..., L_{\mathcal{G}_m}\}$. According to Gersgorin' s theorem [29], $L_{\mathcal{G}(t)}$ lies in the union of the following n disks:

Combining the definition of measurement error, agent i' s state evolution after its k-th triggering instant at time $t^k_{k_i} + lh$ can be rewritten in vector form as:

Therefore, based on the event-triggered consensus protocol (12), during the agents' state evolution, the average state of all agents remains constant.

Define the disagreement vector $\delta(t) = x(t) - \bar{x}(t)\mathbf{1}$, which reflects the system' s current convergence status. When the system reaches asymptotic average consensus, the disagreement vector tends to $\mathbf{0}$. Consider $V(x(t)) = \frac{1}{2}\|\delta(t)\|^2 = \frac{1}{2}x(t)^T L_{\mathcal{G}(t)} x(t)$ as the system' s common Lyapunov function.

This function satisfies: $V(x(t)) \geq 0$ and $V(x(t)) = 0 \Leftrightarrow x(t) = \bar{x}(t)\mathbf{1}$. As the agents' states evolve, the value of the common Lyapunov function gradually decreases and eventually converges to its minimum value, correspondingly, the MAS also reaches average consensus. An effective event-triggered consensus protocol should make the Lyapunov function value decrease continuously over time.

In the sampling period time interval $[kh, (k + 1)h)$, we have:

Here, for further inequality relaxation, let $\|e(kh)\|_{\max} = \max\{\|e_i(kh)\|\}$ and $\|z(kh)\|_{\max} = \max\{\|z_i(kh)\|\}$. Using the basic inequality, we obtain:

According to the convergence event trigger (6), we know that $\|e_i(t^k_{k_i} + lh)\|^2 + \|z_i(t^k_{k_i} + lh)\|^2 \leq \sigma_i \|z_i(t^k_{k_i} + lh)\|^2$.

According to the properties of the Laplacian matrix, each disk' s center $l_{ii}(t)$ satisfies $l_{ii}(t) > 0$, and each disk' s radius is the distance from the center to the origin. Therefore, each disk lies in the right half of the complex plane.

Based on the above formulas, we know that the maximum value of the main diagonal elements of $L_{\mathcal{G}(t)}$ must not exceed 2. The number of agents in the left neighbor region of agent i is $p = |X^i_{\text{left}}(t)|$, and the number of agents in the right neighbor region is $q = |X^i_{\text{right}}(t)|$. It is easy to see that agents within each neighbor region are neighbors of each other. Therefore, the weight $l_{ij}(t)$ of agents in the left neighbor region must not exceed p, and the weight term of agents in the right neighbor region must not exceed q. If there are two agents in the left and right neighbor regions that are neighbors, the weights of these two agents are also less than p (or less than q).

$l_{ii}(t)$ is the sum of the weights of all neighbors of agent i. In summary, $l_{ii}(t) \le \max\{p, q\} \le \max\{p + q\} \le \max\{n - 1\}$. Therefore, if the parameter $\sigma_{\max}$ satisfies $\sigma_{\max} \in (0, 0.0625)$ and the parameter h satisfies $0 < h \le 0.125$, then it can be guaranteed that under switching topology, for any $k \in \{0, 1, 2, ...\}$ and $t \in [kh, (k + 1)h)$, we have $(V(x(t))) \le 0$.

$V(x(t))$ will converge to its minimum value $V(x) = 0$. According to the positive definiteness of the Lyapunov function $V(x(t))$, as long as we guarantee $(V(x(t))) \le 0$. Therefore, if the switching topology remains always connected, all agents can achieve average consensus under the event-triggered consensus protocol (12).

2.4 Network Connectivity Algorithm Based on Constraint Sets

In research on MASs with switching topology, maintaining system connectivity is a necessary condition for achieving consensus. However, protocol (12) cannot maintain system network connectivity. J. Cortes et al. [30] proposed an algorithm to maintain connectivity in switching topology networks. This algorithm constrains the agents' original control inputs, where the actual control input is a certain proportion of the original control input. This paper references this method as a supplement to the event-triggered consensus protocol (12) to maintain connectivity of the system' s switching topology network.

Let $x^i_{\text{left}}(t)$ and $x^i_{\text{right}}(t)$ denote the farthest left and right neighbors of agent i in the current sampling period, respectively. The constraint set is $[x^i_{\text{low}}(t), x^i_{\text{high}}(t)] = \left[\frac{x^i_{\text{left}}(t)+x^i_{\text{right}}(t)}{2} - d, \frac{x^i_{\text{left}}(t)+x^i_{\text{right}}(t)}{2} + d\right]$. Let x^i_{goal} denote the target value the agent should obtain in sampling. The algorithm is described as follows:

Algorithm: Workflow of Constraint Set Algorithm
Input: $x_i(t)$, $x^i_{\text{left}}(t)$, $x^i_{\text{right}}(t)$, $u_i(t)$, h
Output: $x_i(t + h)$

Step 1: Compute $x^i_{\text{goal}} = x_i(t) + u_i(t)h$
Step 2: If $x^i_{\text{low}}(t) \le x^i_{\text{goal}} \le x^i_{\text{high}}(t)$, then $x_i(t + h) = x^i_{\text{goal}}$
Else if $x^i_{\text{goal}} < x^i_{\text{low}}(t)$, then $x_i(t + h) = x^i_{\text{low}}(t)$
Else if $x^i_{\text{goal}} > x^i_{\text{high}}(t)$, then $x_i(t + h) = x^i_{\text{high}}(t)$

Step 3: $u_i(t+h) = u_i(t)$

After the algorithm executes, agent i' s next sampling state $x_i(t+h)$ is strictly constrained within the constraint set. If the target point x^i_{goal} is within the constraint set' s range, agent i' s actual obtained state in the next sampling is the target point x^i_{goal}; if the target point x^i_{goal} is outside the constraint set' s range, agent i' s actual obtained state in the next sampling is the boundary value of the constraint set.

3 Simulation Experiments

3.1 Experiment 1 (Comparison Between Discrete-Time Consensus Protocol and Event-Triggered Consensus Protocol Under Switching Topology)

In Experiment 1, we compare the performance of the discrete-time consensus protocol under switching topology from [27] (referred to as Protocol 1) and the event-triggered consensus protocol under switching topology proposed in this paper (referred to as Protocol 2). Protocol 1 uses traditional periodic control, while Protocol 2 uses sampling-period-based event-triggered control. Both protocols incorporate the constraint-set method. Four sets of different data were designed for simulation.

The agents' communication range $d = 0.05$, and the system' s initial topology is connected. The main differences in the data lie in the sparsity and distribution of the initial topology. The cases with $\sigma_i = 0.05$ and $h = s$ are used as four groups of comparative data, with left and right figures corresponding to Protocol 1 and Protocol 2, respectively.

a) **Protocols 1 and 2:** 10 agents' state values are uniformly distributed in the interval $[0, 5]$. During system evolution, the network remains connected. After approximately 10s, the agents' state values under both protocols reach consensus at the convergence point 2.35 (Figures 1 [Figure 1: see original paper] and 2).

b) **Protocols 1 and 2:** 10 agents' state values are uniformly distributed in the interval $[0, 9]$. During system evolution, the network remains connected. After approximately 15s, the agents' state values under both protocols reach consensus at the convergence point 4.95 (Figures 3 [Figure 3: see original paper] and 4 [Figure 4: see original paper]).

c) **Protocols 1 and 2:** 10 agents'state values are randomly distributed in the interval $[0, 5]$. During system evolution, the network remains connected. After approximately 10s, the agents' state values under both protocols reach consensus at the convergence point 2.23 (Figures 5 [Figure 5: see original paper] and 6 [Figure 6: see original paper]).

d) **Protocols 1 and 2:** 10 agents'state values are randomly distributed in the interval $[0, 7]$. During system evolution, the network remains connected.

After approximately 15s, the agents' state values under both protocols reach consensus at the convergence point 3.31 (Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper]).

Simulation Results Analysis: From Figures 1, 3, 5, and 7, we observe that the state evolution of agents under Protocols 1 and 2 is essentially the same. From Figures 2, 4, 6, and 8, which show the agents' control input evolution, we see that the controller updates are no longer smooth curves but piecewise linear segments, indicating that event-triggered control prevents agents from updating their controllers at every sampling period. Therefore, based on Experiment 1, we conclude that given the same initial topology, the event-triggered consensus protocol under switching topology can effectively reduce the controller update frequency compared to the discrete-time consensus protocol under switching topology.

3.2 Experiment 2 (Comparison Between Event-Triggered Consensus Protocol Under Switching Topology and Event-Triggered Consensus Protocol Under Fixed Topology)

In Experiment 2, we compare the performance of the event-triggered consensus protocol under switching topology proposed in this paper (Protocol 2) and the event-triggered consensus protocol under fixed topology from [8] (Protocol 3). Both Protocols 2 and 3 use sampling-period-based event-triggered control. Four sets of different data were designed for simulation.

In all four data sets, the agents' communication range $d = 0.05$, sampling period $h = s$, and the system's initial topology is connected. The main differences lie in the sparsity and distribution of the initial topology. The cases with $\sigma_i = 0.03$ for Protocol 2 and $\sigma_i = 0.05$ for Protocol 3 are used as four groups of comparative data, with left and right figures corresponding to Protocol 2 and Protocol 3, respectively.

e) **Protocols 2 and 3:** 10 agents' state values are uniformly distributed in the interval $[0, 5]$. During system evolution, the network remains connected. After approximately 10s, the agents' state values under both protocols reach consensus at the convergence point 2.35, with left and right figures corresponding to Protocol 2 and Protocol 3, respectively (Figures 9 [Figure 9: see original paper] and 10 [Figure 10: see original paper]).

f) **Protocols 2 and 3:** 10 agents' state values are uniformly distributed in the interval $[0, 9]$. During system evolution, the network remains connected. Protocol 2 reaches consensus after approximately 15s, while Protocol 3 reaches consensus after approximately 40s, both at the convergence point 4.95 (Figures 11 [Figure 11: see original paper] and 12 [Figure 12: see original paper]).

g) **Protocols 2 and 3:** 10 agents'state values are randomly distributed in the interval $[0, 5]$. During system evolution, the network remains connected.

Protocol 2 reaches consensus after approximately 10s, while Protocol 3 reaches consensus after approximately 40s, both at the convergence point 2.23 (Figures 13 [Figure 13: see original paper] and 14 [Figure 14: see original paper]).

h) **Protocols 2 and 3:** 10 agents'state values are randomly distributed in the interval $[0, 7]$. During system evolution, the network remains connected. Protocol 2 reaches consensus after approximately 15s, while Protocol 3 reaches consensus after approximately 40s, both at the convergence point 3.31 (Figures 15 [Figure 15: see original paper] and 16 [Figure 16: see original paper]).

Simulation Results Analysis: From Figures 9, 11, 13, and 15, we observe that the time for agents'states to converge to consensus differs between Protocols 2 and 3. From Figures 10, 12, 14, and 16, we see that the time for agents' control inputs to reach zero also differs. In terms of system convergence time, Protocol 2 is significantly faster than Protocol 3. This is because Protocol 2 uses switching topology, where as the agents' states evolve, the number of neighbors for each agent increases, the network connectivity gradually improves, making the agents' control inputs larger than those under fixed topology, and the system' s convergence speed is significantly accelerated. Therefore, based on Experiment 2, we conclude that given the same initial topology, the event-triggered consensus protocol under switching topology can effectively reduce system convergence time and accelerate convergence speed compared to the event-triggered consensus protocol under fixed topology.

3.3 Experiment 3 (Evolution of Common Lyapunov Function Under Event-Triggered Consensus Protocol with Switching Topology)

In Experiment 3, we demonstrate the evolution of the system' s common Lyapunov function $V(x(t)) = \frac{1}{2}x(t)^T L_{\mathcal{G}(t)} x(t)$ under the event-triggered consensus protocol with switching topology (Protocol 2). Four sets of different data were designed for simulation. The agents' communication range $d = 0.05$, and the event-triggered parameter $\sigma_i = 0.05$. The main differences in the data lie in the distribution of the initial topology. The cases with $h = s$ are used as two groups of comparative data.

i) **Protocol 2:** 10 agents' state values are uniformly distributed in intervals $[0, 5]$ and $[0, 9]$. The function values of the system' s common Lyapunov function show a monotonic decreasing trend, reaching a stable value after some time (Figure 17 [Figure 17: see original paper]).

j) **Protocol 2:** 10 agents' state values are randomly distributed in intervals $[0, 5]$ and $[0, 7]$. The function values of the system' s common Lyapunov function show a monotonic decreasing trend, reaching a stable value after some time (Figure 18 [Figure 18: see original paper]).

Simulation Results Analysis: Based on Experiment 3, we observe that under the event-triggered consensus protocol with switching topology, the system's common Lyapunov function $V(x(t)) = \frac{1}{2}x(t)^T L_{\mathcal{G}(t)} x(t)$ decreases over time. This is because the convergence event trigger designed in this paper is based on ensuring that the time derivative of the Lyapunov function is negative definite. The experimental values of the event-triggered parameters and sampling period also satisfy the two parameter conditions obtained in the Lyapunov stability analysis section: $\sigma_{\max} \in (0, 0.0625)$ and $0 < h \leq 0.125$. Therefore, it can be guaranteed that under switching topology, for any time t, we always have $(V(x(t))) \leq 0$, and the system can achieve asymptotic stability.

4 Conclusion

This paper proposes an event-triggered consensus protocol under switching topology for multi-agent systems with limited communication range. The protocol employs a hybrid event-triggered mechanism, designing two event triggers: a convergence event trigger caused by agent state evolution and a topology event trigger caused by topology changes. We also introduce a constraint-set method to maintain network connectivity under switching topology. Based on this event-triggered consensus protocol under switching topology, agents can asymptotically converge to the average of their initial states, achieving Lyapunov stability, effectively reducing the update frequency of system controllers, decreasing system energy consumption, saving communication and computational resources, enhancing network connectivity, and further accelerating system convergence speed.

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