Iterative Algorithm for Penalty Function Convex Optimization and Its Application in UAV Path Planning: Postprint

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Abstract

This paper establishes a nonconvex control problem model with time-invariant nonlinear systems, non-affine equality constraints, and nonconvex inequality constraints for UAV path planning, and develops algorithmic design and solution methods for this model. Based on an iterative optimization solution approach, a convex optimization iterative solution method and a penalty function optimization strategy are proposed. The former employs the concave-convex procedure (CCCP) and Taylor' s formula to perform convexification of the model, while the latter imposes the processed terms as penalty terms onto the objective function to resolve feasibility limitations on the initial point. It is proven that the proposed method strictly converges to the Karush-Kuhn-Tucker (KKT) points of the original problem. Simulation experiments validate the feasibility and superiority of the penalty function convex optimization iterative algorithm, demonstrating that the algorithm can plan a compliant flight path for the UAV.

Full Text

Preamble

Penalty Function Convex Optimization Iterative Algorithm and Its Application in UAV Path Planning

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Abstract: This paper establishes a non-convex control model for unmanned aerial vehicle (UAV) path planning that incorporates time-invariant nonlinear systems, non-affine equality constraints, and non-convex inequality constraints, along with an algorithm designed to solve this model. Based on iterative optimization, we propose a convex optimization iteration method and a penalty function optimization strategy. The former employs the concave-convex procedure (CCCP) and Taylor formula to convexify the model, while the latter adds the processed terms to the objective function as penalty terms to address the feasibility limitations of the initial point. It is proven that the proposed method strictly converges to a Karush-Kuhn-Tucker (KKT) point of the original problem. Simulation experiments verify the feasibility and superiority of the penalty function convex optimization iteration algorithm, demonstrating that it can generate a flight path satisfying all constraints for the UAV.

Key words: unmanned aerial vehicle (UAV); path planning; linearization; convex optimization; iterative; penalty function

0 Introduction

UAVs offer numerous advantages in aerial navigation, including fewer obstacles, high efficiency, and low cost, making them widely applicable in both civilian and military domains such as communications, logistics, and missile operations [1]. Path planning represents a core challenge in UAV operations, aiming to generate real-time global trajectories from start to target points that avoid collisions with obstacles while optimizing performance metrics under kinematic and dynamic constraints [2].

The inherent limitations of UAVs, atmospheric turbulence, local environmental information, and sensor capabilities present significant challenges for UAV path planning and optimization. Experts and scholars worldwide have proposed and improved trajectory planning algorithms from various perspectives to address these issues [3]. Current approaches fall into two main categories: intelligent algorithms and numerical algorithms. Intelligent algorithms such as genetic algorithms [4], ant colony algorithms [5], particle swarm optimization [6], and dynamic programming [7] can achieve desired optimality in their respective domains, but they suffer from inherent drawbacks. For instance, the computational complexity, coding difficulty, and time complexity of most intelligent algorithms grow exponentially with factors such as increased search space, refined grid resolution, and larger numbers of nodes [8].

In contrast, numerical algorithms yield results with higher precision. Most numerical methods model UAV path planning as non-convex programming problems. Due to the complexity of non-convex problems, traditional Pontryagin's minimum principle cannot effectively solve them [9]. With the introduction of DC programming and the DCA algorithm [10], increasing research attention has focused on convexity-concavity properties. Convex problems can be reliably solved to global optimality in polynomial time [11], with common solvers including MISER3 [12] and ECOS [13]. Typically, non-convexity arises from nonlinear dynamics, non-convex state constraints, and non-convex control constraints. For nonlinear systems, successive convexification [14] can be applied; for certain non-convex control constraints, lossless convexification [15] is effective; and for some non-convex state constraints, successive linear approximation [16] can be employed for convexification.

To address UAV path planning, this paper considers a minimum-fuel control problem model with time-invariant nonlinear systems, non-affine equality constraints, and non-convex inequality constraints. Our work comprises three main components: (a) To solve the non-convex optimal control model, we utilize the concave-convex procedure (CCCP) and Taylor formula for convex optimization; (b) To resolve the initial point feasibility limitations arising from the Taylor expansion linearization process, we incorporate the processed terms as penalty terms into the objective function, thereby proposing a penalty function convex optimization iteration algorithm; and (c) We conduct convergence analysis of the algorithm' s processing and optimization procedures, proving that the proposed algorithm converges to a KKT point of the original problem and demonstrating its universality for solving non-convex optimal control problems.

Finally, simulation experiments on the UAV path planning model demonstrate that the proposed algorithm can converge to an optimal path satisfying all requirements, making it suitable for path planning problems of autonomous vehicles in complex terrain.

1.1 Non-Convex Optimal Control Model

This paper considers the following non-convex optimal control model:

(P0):

$$\min_{u(\cdot)} \sum_{j=1}^{m} \int_0^{t_f} C_j \|u_j(t)\| dt$$

subject to:

$$\dot{x}(t) = f(x(t), u(t))$$

$$g_i(x(t), u(t)) \le 0, \quad i = 1, \dots, q$$

$$h_i(x(t), u(t)) = 0, \quad i = 1, \dots, p$$

$$x(0) = x_0, \quad x(t_f) = x_f$$

$$x(t) \in X, \quad u(t) \in U$$

where Equation (1) represents the total fuel consumption, with C_j being proportional coefficients; Equation (2) represents the state equation of the nonlinear time-invariant system, where $x(t)$ is an n-dimensional state vector and $u(t)$ is an m-dimensional control vector; Equation (3) represents state and control constraints; Equation (4) represents non-linear non-convex inequality constraints, where functions g_i are at least twice continuously differentiable; and Equation (5) represents initial and terminal state constraints.

Without loss of generality, problem (P0) can be written in standard nonlinear optimization form:

(P1):

$$\min_z f(z)$$

$$\text{s.t.} \quad g_i(z) \leq 0, \quad i = 1, \dots, q$$

$$h_i(z) = 0, \quad i = 1, \dots, p$$

$$z \in Z$$

where $z = [x^T, u^T]^T \in X \times U$, Equation (6) is affine, Equation (7) is non-convex, Equation (8) is non-affine, and both g_i and h_i are twice differentiable. Equation (9) represents the feasible domain constraint.

1.2 Convexification

It is known that convex optimization problems must satisfy three requirements: the objective function must be convex; inequality constraint functions must be convex; and equality constraint functions must be affine [16].

For non-convex inequality constraints, based on the concave-convex procedure (CCCP) [18], a function g_i with bounded Hessian matrix can be expressed as the sum of a convex function $g_i^{\text{vex}}(z)$ and a concave function $g_i^{\text{cave}}(z)$:

$$g_i(z) = g_i^{\text{vex}}(z) + g_i^{\text{cave}}(z)$$

This decomposition is not unique. When $g_i^{\text{cave}}(z)$ is twice continuously differentiable near the solution z_k at the k-th iteration, based on the iterative optimization approach, we apply first-order Taylor expansion linearization at z_k to obtain the following inequality constraint:

$$g_i^{\text{vex}}(z) + g_i^{\text{cave}}(z_k) + \nabla g_i^{\text{cave}}(z_k)^T (z - z_k) \leq 0$$

Since $g_i^{\text{vex}}(z)$ is convex and the linear approximation term is affine, Equation (11) constitutes a convex inequality constraint.

For non-affine functions $h_i(z)$, when they are twice continuously differentiable near z_k, we apply Taylor' s formula to obtain the linear approximation:

$$h_i(z_k) + \nabla h_i(z_k)^T (z - z_k) = 0$$

Since this is an affine function, the resulting constraint is convex. The iterative optimization model is then formulated as:

(P2):

$$\min_z f(z)$$

$$\text{s.t.} \quad F_i^k(z) \le 0, \quad i = 1, \dots, q$$

$$H_i^k(z) = 0, \quad i = 1, \dots, p$$

$$z \in Z$$

$$\|z - z_k\| \le \varepsilon$$

where $F_i^k(z) = g_i^{\text{vex}}(z) + g_i^{\text{cave}}(z_k) + \nabla g_i^{\text{cave}}(z_k)^T(z - z_k)$ and $H_i^k(z) = h_i(z_k) + \nabla h_i(z_k)^T(z - z_k)$. The objective function $f(z)$ is convex, the equality constraints $H_i^k(z)$ are affine, Equation (16) represents the feasible domain constraint, and Equation (17) represents the trust region constraint with ε being a constant. Therefore, problem (P2) is a convex optimization problem.

1.3 Convergence Analysis

Lemma 1. If convex problem (P2) has a minimum point, then the iterative process is monotonically non-increasing and convergent.

Proof. Since problem (P2) has a minimum point, for any k, we have $f(z_k) \ge \inf f(z)$, meaning the sequence $\{f(z_k)\}$ is bounded below. Moreover, z_{k+1} is the optimal solution of the $(k+1)$-th iteration, which serves as the initial solution for the k-th iteration, while z_k is the optimal solution of the k-th iteration. Therefore, $f(z_{k+1}) \le f(z_k)$. With the sequence being bounded below and monotonically non-increasing, it follows that $\{f(z_k)\}$ is monotonically non-increasing and convergent.

Lemma 2. The iterative sequence $\{z_k\}$ of convex problem (P2) converges to its KKT point.

Proof. Since a convex problem necessarily has a unique minimum point z^*, Lemma 1 shows that the sequence $\{f(z_k)\}$ is monotonically non-increasing and convergent, and it converges to this KKT point.

Theorem 1. The global minimum point of the original problem (P1) is also a global minimum point of convex problem (P2), and the global minimum point of convex problem (P2) is a KKT point of the original problem (P1).

Proof. (a) If z^* is the global minimum point of the original problem (P1), then for any z, we have $f(z^*) \le f(z)$. Since $g_i^{\text{cave}}(z)$ has a negative semidefinite Hessian matrix, we have:

$$g_i^{\text{cave}}(z) \leq g_i^{\text{cave}}(z^*) + \nabla g_i^{\text{cave}}(z^*)^T (z - z^*)$$

Therefore, z^* is the global minimum point of convex problem (P2).

(b) Assume the global minimum point of convex problem (P2) is z^*. Since a global minimum point must be a KKT point, i.e., z^* satisfies the KKT conditions, and combining this with the fact that $g_i(z) = g_i^{\text{vex}}(z) + g_i^{\text{cave}}(z)$, we obtain the KKT conditions of the original problem (P1). Thus, z^* is a KKT point of the original problem (P1).

Theorem 2. The convex optimization process (P2) globally converges to a KKT point of the original non-convex optimal control problem (P1).

Proof. Starting from the initial point z_0, convex problem (P2) generates an iterative sequence $\{z_k\}$.

(a) If $\{z_k\}$ converges, assume $\lim_{k\to\infty} z_k = z^*$. For the k-th iteration, assuming z_{k+1} is its global optimum, we have:

$$F_i^k(z_{k+1}) \leq 0, \quad H_i^k(z_{k+1}) = 0$$

Taking the limit on both sides as $k \to \infty$, we obtain $g_i(z^*) \leq 0$ and $h_i(z^*) = 0$. Therefore, z^* is a feasible point of convex problem (P2). Since z^* is the global minimum point of convex problem (P2), and by Theorem 1, z^* is a KKT point of the original problem (P1).

(b) If $\{z_k\}$ diverges, since convex problem (P2) is bounded below, by the Weierstrass accumulation point theorem, there exists at least one accumulation point in the bounded set $\{z_k\}$, denoted as z^*. Consequently, there exists a subsequence $\{z_{k_n}\}$ in $\{z_k\}$ that converges to z^*, and z^* is the optimal point of convex problem (P2) within the set. This is proved by contradiction: assume z^* is not a KKT point of the original problem (P1), then for any sufficiently small $\delta > 0$, there exists a feasible point $\bar{z}$ of the original problem (P1) such that $f(\bar{z}) < f(z^*)$. Since $F_i^k(z)$ and $H_i^k(z)$ are continuous, we have $F_i^{k_n}(\bar{z}) \leq 0$ and $H_i^{k_n}(\bar{z}) = 0$ for sufficiently large k_n. Therefore, $\bar{z}$ is a feasible point of convex problem (P2) at iteration k_n, which implies $f(\bar{z}) \geq f(z_{k_n+1})$. Taking the limit as $k_n \to \infty$, we obtain $f(\bar{z}) \geq f(z^*)$, contradicting $f(\bar{z}) < f(z^*)$. Thus, the assumption is false, and z^* is a KKT point of the original problem (P1).

In summary, the convex optimization process (P2) globally converges to a KKT point of the original non-convex optimal control problem (P1).

1.4 Penalty Function Optimization

The convex optimization process employs Taylor formula approximation and linearization, which involves discarding second-order terms in the constraint functions, particularly in the equality constraints. Consequently, some infeasible initial solutions may lead to convergence failure. To address this issue, this paper applies penalty function optimization to convex problem (P2). By imposing penalties on infeasible iteration points, the algorithm forces these points toward the feasible domain, thereby reducing the feasibility requirements for initial point selection.

For convex problem (P2), let $F_i^k(z) = \max\{0, F_i^k(z)\}$ and $H_i^k(z) = |H_i^k(z)|$. Incorporating the processed terms as penalty terms into the objective function yields the penalty problem:

(P3):

$$\begin{gathered}\min_z f(z) + \sigma_k \left(\sum_{i=1}^{q} \max\{0, F_i^k(z)\} + \sum_{i=1}^{p} |H_i^k(z)| \right) \\ \text{s.t.} \quad z \in Z \\ \|z - z_k\| \le \varepsilon\end{gathered}$$

where σ_k is the penalty factor with $\sigma_k > 0$ and $\sigma_{k+1} > \sigma_k$.

Regarding the relationship between the penalty problem and the original constrained optimization problem, we have the following theorem:

Theorem 3. Let z^* be the global minimum point of convex problem (P2), and let the penalty factor sequence $\{\sigma_k\}$ diverge to $+\infty$. If z_k^* is the global minimum point of penalty problem (P3), then any accumulation point of the sequence $\{z_k^*\}$ converges to a KKT point of the original problem (P1).

Proof. Let $P(z,\sigma) = f(z) + \sigma \left(\sum_{i=1}^{q} \max\{0, F_i^k(z)\} + \sum_{i=1}^{p} |H_i^k(z)| \right)$. Denote z_1^* and z_2^* as the optimal solutions of the penalty function $P(z, \sigma_1)$ and $P(z, \sigma_2)$, respectively, with $\sigma_2 > \sigma_1 > 0$. Then:

$$\begin{cases} f(z_1^*) + \sigma_1 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_1^*)\} + \sum_{i=1}^{p} |H_i^k(z_1^*)| \right) \le f(z_2^*) + \sigma_1 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_2^*)\} + \sum_{i=1}^{p} |H_i^k(z_2^*)| \right) \\ f(z_2^*) + \sigma_2 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_2^*)\} + \sum_{i=1}^{p} |H_i^k(z_2^*)| \right) \le f(z_1^*) + \sigma_2 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_1^*)\} + \sum_{i=1}^{p} |H_i^k(z_1^*)| \right) \end{cases}$$

Adding these two inequalities yields:

$$(\sigma_2 - \sigma_1) \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_2^*)\} + \sum_{i=1}^{p} |H_i^k(z_2^*)| \right) \le (\sigma_2 - \sigma_1) \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_1^*)\} + \sum_{i=1}^{p} |H_i^k(z_1^*)| \right)$$

Since $\sigma_2 > \sigma_1$, we have:

$$\sum_{i=1}^{q} \max\{0, F_i^k(z_2^*)\} + \sum_{i=1}^{p} |H_i^k(z_2^*)| \leq \sum_{i=1}^{q} \max\{0, F_i^k(z_1^*)\} + \sum_{i=1}^{p} |H_i^k(z_1^*)|$$

This shows that $P(z_k^*, \sigma_k)$ is monotonically non-increasing with respect to σ_k. From the second inequality, we obtain:

$$f(z_2^*) - f(z_1^*) \leq \sigma_2 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_1^*)\} + \sum_{i=1}^{p} |H_i^k(z_1^*)| \right) - \sigma_1 \left(\sum_{i=1}^{q} \max\{0, F_i^k(z_2^*)\} + \sum_{i=1}^{p} |H_i^k(z_2^*)| \right)$$

Since z_k^* is the global minimum point of convex problem (P2) and z_k^* is also the global minimum point of penalty problem (P3), both z_k^* and z^* are feasible points, meaning $F_i^k(z_k^*) \leq 0$, $H_i^k(z_k^*) = 0$, $F_i^k(z^*) \leq 0$, and $H_i^k(z^*) = 0$. As $P(z_k^*, \sigma_k)$ is monotonically non-increasing and bounded below with respect to σ_k, the limits exist. Let $\lim_{k\to\infty} z_k^* = \bar{z}$ and $\lim_{k\to\infty} \sigma_k = +\infty$.

For the accumulation point $\bar{z}$, we have $F_i^k(\bar{z}) \leq 0$ and $H_i^k(\bar{z}) = 0$, thus $\bar{z}$ is a feasible point. Moreover, since z^* is the global minimum point of convex problem (P2), we have $f(z^*) \leq f(\bar{z})$. Taking the limit yields:

$$\lim_{k\to\infty} P(z_k^*, \sigma_k) = f(\bar{z})$$

Thus, the accumulation point $\bar{z}$ is the global minimum point of convex problem (P2). Combining this with Theorem 2, we conclude that $\bar{z}$ converges to a KKT point of the original problem (P1).

Based on the above process, we present the pseudocode for the penalty function convex optimization iteration algorithm:

Algorithm 1: Penalty Function Convex Optimization Iteration Algorithm

a) **Input parameters:** Terminal time t_f, constraint parameters, tolerance error ε, penalty factor σ, maximum iteration count $k_{\max}$;

b) **Discretization:** Divide the total time horizon into N discrete nodes with time step Δt;

c) **Initialization:** Select any initial point x_0;

d) **Linearization:** Apply Equations (14) and (15) for linearization;

e) **Iterative optimization:** Solve problem (P3) to obtain the optimal solution z_k;

f) **Loop:** If $\|z_{k+1} - z_k\| > \varepsilon$ and $k < k_{\max}$, proceed to step g); otherwise, proceed to step h);

g) **Update:** Set $k \leftarrow k + 1$ and return to step d);

h) **Output:** Optimal value z^* and objective functional.

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2 Simulation Experiments and Results Analysis

The total duration t_f is discretized into N nodes with node spacing Δt to establish the UAV path planning model:

(P4):

$$\min_{x,u} \sum_{i=1}^{N} (w_u \|u_i\|^2 + w_x \|x_i\|^2) + \sigma \sum_{i=1}^{N} \max\{0, F_i^k(x_i)\}$$

subject to:

$$y(i+1) = Ay(i) + Bu(i) + g_i, \quad i = 1, \dots, N-1$$
$$y(i) \in X_R, \quad u(i) \in U_R, \quad i = 1, \dots, N$$
$$y(0) = y_0, \quad y(N) = y_f$$

where w_u and w_x are weight coefficients corresponding to fuel consumption and path length, respectively, σ is the penalty factor, $F_i^k(x_i)$ represents the convexified inequality constraint function from the non-convex obstacle avoidance constraint, c_j and r_j denote the coordinates and radius of the j-th obstacle, and M is the total number of obstacles.

The constraints include:

$$\{x_i, v_i\} \in \{\mathbb{R}^6 \mid \|v_i\| \leq V_{\max}\}$$
$$\{u_i\} \in \left\{\mathbb{R}^2 \mid \cos\theta \leq \frac{\hat{n}^T u_i}{\|u_i\|}, \|u_i\| \leq U_{\max}\right\}$$

where $V_{\max}$ and $U_{\max}$ represent the velocity and acceleration limits, respectively, $F_{\max}$ is the maximum thrust, m is the vehicle mass, θ represents the thrust angle constraint, $\hat{n}$ is the unit vector in the positive z-axis direction, and y_0 and y_f denote the start and end points.

We treat obstacles as cylindrical, considering only circumvention from the sides rather than flying over the top. The fuel consumption is equivalently replaced by the 2-norm of acceleration: $\sum_{i=1}^{N} \|u_i\|^2$.

Simulation experiments were conducted on a PC with Windows 7 operating system, Matlab R2017b environment, Intel(R) Core(TM) i3-5005U CPU @ 2.00GHz processor, and 4GB RAM, utilizing the CVX toolbox [19] during the solution process.

To verify the feasibility of the proposed algorithm, experimental parameters were set as shown in .

The planning space is defined as $D = \{(x, y, z) \mid 0 \leq x, y \leq 1000, 0 \leq z \leq 1000\}$ m, with the starting point at $(0, 0, 0)$ and the terminal point at $(1000, 1000, 2.4)$. The straight line connecting these points serves as the initial path. The initial velocity is set to $(0, 0, 0)$ m/s, the terminal velocity to $(0, 0, 0)$ m/s, and the initial acceleration to $[0, 0, -9.81]^T$ m^2/s. The maximum velocity is $U_{\max} = 50$ m/s.

The experimental results are shown in [Figure 1: see original paper]. The results demonstrate that the proposed algorithm can achieve both 2D and 3D path planning for UAVs in complex environments without being constrained by initial path feasibility. Observing [Figure 2: see original paper] reveals that both path length and fuel consumption approach optimal values by the second iteration. Therefore, in practical applications, the tolerance error ε can be appropriately increased or the maximum iteration count $k_{\max}$ reduced to save computational time.

To demonstrate the superiority of the algorithm, we compare it with three common intelligent path planning algorithms: genetic algorithm, ant colony algorithm, and dynamic programming algorithm. To satisfy the feasibility requirements of the comparison algorithms, we adjust the starting point, reduce the radius of the first circular obstacle accordingly, and scale all values proportionally. Experiments are conducted with the same number of nodes (37) and a grid step size of 0.25 km.

The comparison results are shown in [Figure 3: see original paper]. The planned path lengths for the genetic algorithm, ant colony algorithm, dynamic programming algorithm, and the proposed algorithm are 15.9541 km, 13.6725 km, 13.5432 km, and 13.3255 km, respectively. The results indicate that the proposed algorithm can plan shorter paths. Furthermore, unlike ant colony and dynamic programming algorithms that require gridding of the planning space, the proposed algorithm produces smoother trajectories that better reflect actual flight conditions. Additionally, unlike ant colony and genetic algorithms that exhibit randomness and fluctuations in planning results, the proposed algorithm yields more stable paths, making it more suitable for practical applications.

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3 Conclusion

To address UAV path planning problems, this paper proposes a penalty function convex optimization iteration algorithm for solving non-convex optimal control problems from an optimal control perspective. The proposed algorithm is proven to strictly converge to a KKT point of the original problem without being limited by initial point feasibility. Simulation experiments verify the feasibility and superiority of the algorithm in UAV path planning applications.

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Note: Figure translations are in progress. See original paper for figures.