A Fast Tracking Algorithm for Nearby Targets Based on Probability Hypothesis Density (Postprint)

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Date: 2020-09-28T00:00:00+00:00

Abstract

To address the issues where the standard Probability Hypothesis Density (PHD) filter struggles to correctly estimate target states and suffers from high computational complexity when multiple targets are in close proximity within clutter scenarios, this paper proposes a fast tracking algorithm for closely spaced targets based on Probability Hypothesis Density. The proposed algorithm first employs an adaptive gating technique to extract observations originating from real targets from the sensor observation set, then utilizes these observations to update the predicted intensity, and finally applies a detection-oriented weight correction method for closely spaced targets to selectively reassign inaccurate component weights in the posterior intensity at each discrete time step. Experimental results demonstrate that the proposed algorithm not only achieves efficient target tracking but also exhibits good robustness.

Full Text

Preamble

Vol. 38 No. 3
Application Research of Computers (ChinaXiv Partner Journal)

Fast Close-Spaced Target Tracking Algorithm Based on Probability Hypothesis Density

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Abstract: To address the problems that the standard probability hypothesis density (PHD) filter struggles to correctly estimate target states and suffers from high computational complexity when multiple targets move close to each other in clutter environments, this paper proposes a fast close-spaced target tracking algorithm based on PHD. The proposed algorithm first employs adaptive gating technology to partition the sensor measurement set into measurements originating from real targets, then uses the real target measurement set to update the predicted intensity, and finally applies a detection-guided close-spaced target weight correction method to selectively reallocate inaccurate component weights in the posterior intensity at each discrete time step. Experimental results demonstrate that the proposed algorithm not only achieves efficient target tracking but also exhibits good robustness.

Keywords: close-spaced target tracking; probability hypothesis density; weight redistribution; state estimation; computational efficiency

0 Introduction

In recent years, Mahler [?] proposed a multi-target tracking algorithm based on finite set statistics using point process theory. Multi-target Bayes filters within the finite set statistics framework exhibit favorable tracking performance, yet suffer from high computational complexity. To address this issue, [?] introduced the probability hypothesis density (PHD) filter using random finite set and point process theory. Under linear Gaussian noise dynamic systems, the Gaussian mixture PHD (GMPHD) provides a closed-form solution to the PHD filter [?].

Unlike most traditional data association filters [?, ?, ?], the PHD filter does not require explicit data association (i.e., measurement-to-target association) [?, ?, ?, ?, ?]. For target detection and tracking in clutter environments, Baisa et al. [?] proposed a multi-type multi-target filtering (MTMTF) algorithm based on GMPHD. By employing a target classification update strategy, the MTMTF algorithm overcomes the mutual interference between different target type measurements in the GMPHD filter update step. However, when targets of the same type move close to each other in the scenario, the MTMTF algorithm yields low accuracy in both target state and number estimation.

Under the standard Bayes framework, Aoki et al. [?] theoretically analyzed issues such as close-spaced target identity recognition and measurement-to-target association in clutter environments, and detailed the label switching phenomenon in close-spaced targets. However, [?] only provided theoretical analysis and proof of identity uncertainty for close-spaced targets without offering a concrete implementation. Based on the maximum likelihood probabilistic multi-hypothesis method, Schoenecker et al. [?] proposed a multi-target filter within a non-Bayesian framework. This filter analyzed and verified the influence of target spacing on the distinguishability of close-spaced targets. The introduction of multi-hypothesis methods makes this filter computationally intensive in strong clutter scenarios, making its iteration efficiency difficult to meet the demands of real tracking applications. Considering the trade-off between target estimation accuracy and computational cost for multiple close-spaced targets, Zhang et al. [?] proposed a weight-reallocated GMPHD (WA-GMPHD) filter. By iteratively performing penalty-based multiple updates on target weights at each filtering time, the WA-GMPHD filter improves upon the shortcomings of the standard GMPHD filter when tracking close-spaced targets.

Although the WA-GMPHD filter achieves relatively high state estimation accuracy, its computational efficiency remains relatively low. Based on a new close-spaced target component fusion method, Nie et al. [?] proposed an improved GMPHD (IPA-GMPHD) filter. By utilizing target weights, means, and covariances, the IPA-GMPHD filter avoids erroneous component fusion for close-spaced targets to some extent, thereby improving tracking accuracy for close-spaced targets. However, the IPA-GMPHD filter exhibits relatively low state estimation accuracy in close-spaced target scenarios with dense clutter, and its computational cost is high.

To address the aforementioned problems of low state estimation accuracy and high computational cost in close-spaced target tracking algorithms, this paper proposes a fast close-spaced target tracking algorithm within the GMPHD framework. The algorithm first uses an adaptive gate method to filter real target measurements from the measurement set, then employs a detection-guided close-spaced target component weight correction method to selectively reallocate inaccurate component weights in the posterior intensity, and finally validates the approach through simulation.

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1 GMPHD Filter Based on Random Finite Sets

The PHD filter models both target states and sensor measurements as finite sets. At time k, the target state set and measurement set are denoted as $\mathbf{X}_k = \{\mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,M_k}\}$ and $\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \dots, \mathbf{z}_{k,N_k}\}$, respectively, where M_k and N_k represent the number of targets and measurements.

The PHD filter is an approximation method for random finite set multi-target Bayes filtering. It achieves target state and number estimation by iteratively propagating the target intensity at each time step. However, the PHD filter iteration requires set integration operations, making it impossible to obtain a closed-form solution directly. Under linear Gaussian white noise dynamic systems, the GMPHD filter provides an analytical solution to the PHD filter. Through PHD filter iteration, the GMPHD filter approximates the target intensity using a weighted sum of Gaussian components. The GMPHD filter consists primarily of a prediction step and an update step.

Prediction Step: Assuming the Gaussian mixture representation of the posterior intensity at time $k-1$ is

$$\mathbf{v}_{k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(\mathbf{x}; \mathbf{m}_{k-1}^{(i)}, \mathbf{P}_{k-1}^{(i)})$$

where $w_{k-1}^{(i)}$ is the weight of the i-th Gaussian component, $\mathcal{N}(\cdot; \mathbf{m}, \mathbf{P})$ denotes a Gaussian density function with mean $\mathbf{m}$ and covariance $\mathbf{P}$, and J_{k-1} is the number of Gaussian components. Then the predicted intensity at time k is

$$\mathbf{v}_{k|k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i)}, \mathbf{P}_{k|k-1}^{(i)})$$

where $w_{k|k-1}^{(i)} = p_{S,k} w_{k-1}^{(i)}$ is the survival probability, $\mathbf{m}_{k|k-1}^{(i)} = \mathbf{F}_{k-1}\mathbf{m}_{k-1}^{(i)}$, and $\mathbf{P}_{k|k-1}^{(i)} = \mathbf{F}_{k-1}\mathbf{P}_{k-1}^{(i)}\mathbf{F}_{k-1}^{T} + \mathbf{Q}_{k-1}$ with $\mathbf{F}_{k-1}$ being the state transition matrix and $\mathbf{Q}_{k-1}$ the process noise covariance matrix.

Update Step: Based on the predicted intensity $\mathbf{v}_{k|k-1}(\mathbf{x})$ and measurement set $\mathbf{Z}_k$, the posterior intensity at time k is

$$\mathbf{v}_k(\mathbf{x}) = (1 - p_{D,k})\mathbf{v}_{k|k-1}(\mathbf{x}) + \sum_{\mathbf{z} \in \mathbf{Z}_k} \mathbf{v}_{k|k-1}(\mathbf{x}) \sum_{i=1}^{J_{k|k-1}} w_{k|k}^{(i)}(\mathbf{z}) \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k}^{(i)}(\mathbf{z}), \mathbf{P}_{k|k}^{(i)})$$

where the right-hand side first term represents the missed detection component and the second term represents the measurement update component. Here $p_{D,k}$ is the detection probability, and the weight $w_{k|k}^{(i)}$, mean $\mathbf{m}_{k|k}^{(i)}$, and covariance matrix $\mathbf{P}_{k|k}^{(i)}$ are given by

$$w_{k|k}^{(i)} = \frac{p_{D,k} w_{k|k-1}^{(i)} q_k^{(i)}(\mathbf{z})}{\kappa_k(\mathbf{z}) + \sum_{j=1}^{J_{k|k-1}} p_{D,k} w_{k|k-1}^{(j)} q_k^{(j)}(\mathbf{z})}$$

$$\mathbf{m}_{k|k}^{(i)} = \mathbf{m}_{k|k-1}^{(i)} + \mathbf{K}_k^{(i)}(\mathbf{z} - \mathbf{H}_k \mathbf{m}_{k|k-1}^{(i)})$$

$$\mathbf{P}_{k|k}^{(i)} = [\mathbf{I} - \mathbf{K}_k^{(i)}\mathbf{H}_k]\mathbf{P}_{k|k-1}^{(i)}$$

with $\kappa_k(\cdot)$ being the clutter intensity, $\mathbf{H}_k$ the observation matrix, $\mathbf{R}_k$ the observation noise covariance matrix, and

$$\mathbf{q}_k^{(i)}(\mathbf{z}) = \mathcal{N}(\mathbf{z}; \mathbf{H}_k \mathbf{m}_{k|k-1}^{(i)}, \mathbf{H}_k \mathbf{P}_{k|k-1}^{(i)} \mathbf{H}_k^T + \mathbf{R}_k)$$

$$\mathbf{K}_k^{(i)} = \mathbf{P}_{k|k-1}^{(i)}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}^{(i)}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

In close-spaced target scenarios, the posterior intensity component weights calculated by equation (5) are often incorrect, meaning the posterior intensity $\mathbf{v}_k(\mathbf{x})$ has low accuracy. Consequently, target states cannot be correctly estimated from the posterior intensity $\mathbf{v}_k(\mathbf{x})$.

The WA-GMPHD filter improves close-spaced target state estimation accuracy by reallocating weights of Gaussian components in the posterior intensity. However, in close-spaced target scenarios with dense clutter, the WA-GMPHD filter requires weight correction for Gaussian components at almost every time step, resulting in high computational cost. Furthermore, when reallocating Gaussian component weights, the WA-GMPHD filter selects components for correction based on current component weights, which does not accurately reflect the relationship between Gaussian components and their true corresponding measurements. Therefore, the target state estimation accuracy of this filter is relatively low.

To address these two shortcomings of the WA-GMPHD filter in tracking close-spaced targets, this paper proposes a detection-guided close-spaced target component weight correction method. Whether component weights need reallocation at each filtering time is determined by a detection strategy. If reallocation is needed, the close-spaced target component weight correction strategy is applied; otherwise, the Gaussian component weights in the posterior intensity remain unchanged.

At time k, the weight $w_{k|k}^{(i)}$ of the Gaussian component with state $\mathbf{x}$ in the posterior intensity $\mathbf{v}_k(\mathbf{x})$ can be calculated by equation (6), and its unnormalized weight is $\tilde{w}_{k|k}^{(i)}$. The weight matrix $\mathbf{W}_k$ and unnormalized weight matrix $\tilde{\mathbf{W}}_k$ corresponding to all Gaussian components in the posterior intensity $\mathbf{v}_k(\mathbf{x})$ are expressed as

$$\mathbf{W}_k = \begin{bmatrix} w_{k|k}^{(1,1)} & \cdots & w_{k|k}^{(1,N_k)} \\ \vdots & \ddots & \vdots \\ w_{k|k}^{(J_{k|k-1},1)} & \cdots & w_{k|k}^{(J_{k|k-1},N_k)} \end{bmatrix}, \quad \tilde{\mathbf{W}}_k = \begin{bmatrix} \tilde{w}_{k|k}^{(1,1)} & \cdots & \tilde{w}_{k|k}^{(1,N_k)} \\ \vdots & \ddots & \vdots \\ \tilde{w}_{k|k}^{(J_{k|k-1},1)} & \cdots & \tilde{w}_{k|k}^{(J_{k|k-1},N_k)} \end{bmatrix}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{R}_k$ is the observation noise covariance matrix.

2.1 Observation Set Partitioning Based on Adaptive Gate Method

In target tracking scenarios with clutter interference, the measurement set obtained by the sensor at each time includes both target measurements and clutter.

Clutter not only reduces the accuracy of the GMPHD filter' s posterior intensity but also increases the filter' s iteration burden. This paper adopts an adaptive gate-based target measurement partitioning method to separate measurements originating from real targets from the measurement set $\mathbf{Z}_k$, thereby reducing the number of clutter components in the iteration process.

At time k, the predicted intensity $\mathbf{v}_{k|k-1}(\mathbf{x})$ is represented by equation (2). The adaptive gate-based target measurement partitioning method divides the sensor measurement set $\mathbf{Z}_k$ into a target measurement set $\mathbf{Z}_k^{\text{real}}$ and a clutter set $\mathbf{Z}_k^{\text{clutter}}$, where the target measurement set is

$$\mathbf{Z}_k^{\text{real}} = \{\mathbf{z}_{k,j} \in \mathbf{Z}_k \mid d_{k,j}^{(i)} \leq \eta_{th} + \alpha, \forall i\}$$

with

$$d_{k,j}^{(i)} = (\mathbf{z}_{k,j} - \mathbf{H}_k \mathbf{m}_{k|k-1}^{(i)})^T (\mathbf{H}_k \mathbf{P}_{k|k-1}^{(i)} \mathbf{H}_k^T + \mathbf{R}_k)^{-1} (\mathbf{z}_{k,j} - \mathbf{H}_k \mathbf{m}_{k|k-1}^{(i)})$$

where η_{th} is an empirical gate threshold and α is an adjustment factor. As seen in equation (11), the upper bound on the right side automatically scales with changes in $\mathbf{H}_k \mathbf{P}_{k|k-1}^{(i)} \mathbf{H}_k^T + \mathbf{R}_k$. That is, different components have different thresholds when associating their corresponding measurements. The adaptive gate-based target measurement partitioning method can significantly improve measurement loss caused by empirical gate threshold methods.

2.2 Detection-Guided Close-Spaced Target Component Weight Correction

At time k, based on the predicted intensity $\mathbf{v}_{k|k-1}(\mathbf{x})$ and target measurement set $\mathbf{Z}_k^{\text{real}}$, the posterior intensity $\mathbf{v}_k(\mathbf{x})$ is calculated using equation (5). Since $\mathbf{Z}_k^{\text{real}}$ contains almost only measurements originating from targets, the posterior intensity $\mathbf{v}_k(\mathbf{x})$ has relatively high accuracy. In close-spaced target scenarios, measurements from multiple close-spaced targets are also adjacent to each other, so measurements from different targets may approach one of the multiple close-spaced targets. Based on the assumption of uncertain measurement origins in real tracking scenarios and the update mechanism of the GMPHD filter, the weights of some Gaussian components in the posterior intensity $\mathbf{v}_k(\mathbf{x})$ may be inaccurate.

Whether the weights of Gaussian components in the posterior intensity need correction can be detected using equation (16). Let $\mathcal{I}_k = \{i \mid w_{k|k}^{(i)} > \omega_{th}, i = 1, \dots, J_k\}$ denote the index set of components whose weights exceed the state extraction threshold ω_{th} [**?**]. When $\mathcal{I}_k$ is non-empty, the weights of Gaussian components in the posterior intensity $\mathbf{v}_k(\mathbf{x})$ need reallocation. The detailed steps for component weight reallocation are as follows:

a) Retrieve the Gaussian component with the largest weight in the posterior intensity $\mathbf{v}_k(\mathbf{x})$, with its weight index in the weight matrix $\mathbf{W}_k$ being β.

b) If the weight sum $\sum_{i=1}^{r_k} w_{k|k}^{(i)}$ does not exceed 1, the weights of Gaussian components with state $\mathbf{x}$ do not need correction, and the process proceeds to step (f); otherwise, the weights need reallocation.

c) Calculate the distance between the Gaussian component with state $\mathbf{x}$ and its corresponding target at time $k-1$, retrieve the component with the minimum distance value, and denote the Euclidean distance between the target at time $k-1$ and time k as $d_{k-1,k}^{(i)}$.

d) Update the weight of the Gaussian component with state $\mathbf{x}$ in the unnormalized weight matrix $\tilde{\mathbf{W}}_k$.

e) If the weight sum does not exceed 1, proceed to step (f); otherwise, return to step (d).

f) Update the set $\mathcal{I}_k$, i.e., delete β from $\mathcal{I}_k$.

g) If $\mathcal{I}_k$ is empty, the close-spaced target component weight correction process ends; otherwise, return to step (a).

Remark: Gaussian component merging method. Based on the component fusion method of the GMPHD filter [**?**], the i-th Gaussian component in the posterior intensity $\mathbf{v}_k(\mathbf{x})$ at time k (with state $\mathbf{x}$) is fused into a single Gaussian component with state $\mathbf{x}$ according to the weights of its sub-components. The state is given by equation (5), where the missed detection component corresponding to the Gaussian component with state $\mathbf{x}$ in the target missed detection term is represented.

Experimental Results

The experimental scenario is a two-dimensional surveillance area of $[0, 700] \times [0, 700]$ m containing three real targets that approach each other at time 35. Figure 1 [Figure 1: see original paper] shows a 60-second simulation of the experimental scenario, where the number of clutter points follows a uniform distribution with mean 5. The detection probability is $p_{D,k} = 0.99$, survival probability $p_{S,k} = 0.99$, process noise follows a zero-mean Gaussian distribution with standard deviation 0.5, and observation noise follows a zero-mean Gaussian distribution with standard deviation 25. The sampling interval is 1 second, and the state extraction threshold is $\omega_{th} = 0.5$.

Based on three metrics–OSPA distance [**?**], computational time, and weight reallocation probability–the proposed algorithm is compared with the WA-GMPHD [**?**] and IPA-GMPHD [**?**] filters. The two parameters of OSPA distance are $c = 100$ and $p = 2$. A smaller OSPA distance indicates higher target state estimation accuracy, and vice versa [?]. All experimental results are averaged over 1000 Monte-Carlo simulations.

Figure 2 [Figure 2: see original paper] presents the OSPA distance, computational time, and weight reallocation probability results from 1000 Monte-Carlo experiments. The IPA-GMPHD filter, which only employs component merging strategy, cannot fully resolve the Gaussian component merging problem for close-spaced targets, resulting in large OSPA distance and high computational time. Since it does not require Gaussian component weight reallocation, its weight reallocation probability is zero. When target separation is large, the OSPA distances of the WA-GMPHD filter and the proposed algorithm are nearly identical. However, when targets move close to each other, the OSPA distance of the WA-GMPHD filter is significantly larger than that of the proposed algorithm. Compared with the WA-GMPHD filter, the proposed algorithm has lower computational time and weight reallocation probability. The smaller weight reallocation probability leads to reduced computational time. The excellent filtering performance of the proposed algorithm is attributed to its ability to selectively correct Gaussian component weights at each time step through the application of detection-guided close-spaced target component weight correction strategy and measurement partitioning method, thereby obtaining high-precision posterior intensity.

Figure 3 [Figure 3: see original paper] compares the three filters under different clutter mean scenarios. The OSPA distance and computational time of the IPA-GMPHD filter are significantly affected by clutter density. Although the OSPA distance of the WA-GMPHD filter improves somewhat with increasing clutter, the gradual increase in clutter density causes its weight reallocation probability to rise continuously, leading to increasing computational time. Since the weight reallocation probability of the proposed algorithm is not affected by clutter density, its computational time remains similar across different clutter levels. Compared with the WA-GMPHD filter, the proposed algorithm generally has smaller OSPA distance, indicating that its target state estimation accuracy is less affected by changes in clutter density.

Figure 4 [Figure 4: see original paper] shows the impact of detection probability on each filter' s performance, with clutter mean fixed at 10. The proposed algorithm achieves smaller OSPA distance and less computational time at all detection probability levels compared to both WA-GMPHD and IPA-GMPHD filters. Moreover, as detection probability increases, the weight reallocation probability of the proposed algorithm continuously improves. Lower detection probability makes measurement origins more difficult to distinguish in close-spaced target scenarios, resulting in a high weight reallocation probability for the WA-GMPHD filter. This means the WA-GMPHD filter needs to reallocate Gaussian component weights in the posterior intensity more frequently, leading to higher computational time.

4 Conclusion

This paper proposes a fast PHD tracking algorithm for close-spaced targets to address the difficulty of standard PHD filters in effectively tracking such targets. Through adaptive gate measurement partitioning and detection-guided close-spaced target component weight correction, the proposed algorithm selectively reallocates Gaussian component weights in the posterior intensity at each filtering time, alleviating the problem of erroneous weight updates for Gaussian components in close-spaced target tracking environments and improving the trade-off between high-precision posterior intensity and low computational cost. Experiments demonstrate that the proposed algorithm exhibits good tracking performance (high target state estimation accuracy and low computational time) and represents a robust close-spaced multi-target PHD tracking algorithm that balances state estimation accuracy and computational burden.

References

[1] Mahler R. Statistical multisource-multitarget information fusion [M]. Norwood, MA: Artech House, 2007: 483-531.

[2] Mahler R. Multi-target Bayes filtering via first-order multi-target moments [J]. IEEE Trans on Aerospace and Electronic Systems, 2003, 39 (4): 1152-1178.

[3] Vo B N, Ma W K. The Gaussian mixture probability hypothesis density filter [J]. IEEE Trans on Signal Processing, 2006, 54 (11): 4096-4104.

[4] Liu Yunxiao, Wang Dongfeng, Cao Lin, et al. A k nearest neighbor-joint probability data association algorithm based on vehicle data [J]. Telecommunication Engineering, 2020, 60 (4): 448-454.

[5] Sheng Hao, Chen Jiahui, Zhang Yang, et al. Iterative multiple hypothesis tracking with tracklet-level association [J]. IEEE Trans on Circuits and Systems for Video Technology, 2019, 29 (12): 3660-3672.

[6] Xie Yifan, Huang Yuan, Song T L. Iterative joint integrated probabilistic data association filter for multiple-detection multiple-target tracking [J]. Digital Signal Processing, 2018 (72): 232-243.

[7] Hong Lei, Chen Shuxin, Wu Hao, et al. Student' s t particle implementations of probability hypothesis density filters [J/OL]. Application Research of Computers. (2019-09-25) [2020-02-26]. https://doi.org/10.19734/j.issn.1001-3695.2018.11.0905.

[8] Hu Qi, Ji Hongbing, Zhang Yongquan. A standard PHD filter for joint tracking and classification of maneuvering extended targets using random matrix [J]. Signal Processing, 2018 (144): 352-363.

[9] Garcia F, Prioletti A, Cerri P, et al. PHD filter for vehicle tracking based on a monocular camera [J]. Expert Systems with Applications, 2018 (91): 135-152.

[10] Jorquera F, Hernandez S, Vergara D. Probability hypothesis density filter using determinantal point processes for multi object tracking [J]. Computer Vision and Image Understanding, 2019 (183): 33-41.

[11] Cheng Xuan, Song Liping, Ji Hongbing, et al. Group target tracking algorithm based on labeled box particle probability hypothesis density [J]. Systems Engineering and Electronics, 2019, 41 (8): 1677-1685.

[12] Baisa N L, Wallace A. Multiple target, multiple type filtering in the RFS framework [J]. Digital Signal Processing, 2019 (89): 49-59.

[13] Aoki E H, Mandal P K, Svensson L, et al. Labeling uncertainty in multi-target tracking [J]. IEEE Trans on Aerospace and Electronic Systems, 2016, 52 (3): 1006-1020.

[14] Schoenecker S, Willett P, Bar-Shalom Y. Resolution limits for tracking closely spaced targets [J]. IEEE Trans on Aerospace and Electronic Systems, 2018, 54 (6): 2900-2910.

[15] Zhang Huanqing, Ge Hongwei, Yang Jinlong, et al. A GM-PHD algorithm for multiple target tracking based on false alarm detection with irregular window [J]. Signal Processing, 2016 (120): 537-552.

[16] Nie Yongfang, Zhang Tao. Improved pruning algorithm for Gaussian mixture probability hypothesis density filter [J]. Journal of Systems Engineering and Electronics, 2018, 29 (2): 229-235.

[17] Schuhmacher D, Vo B T, Vo B N. A consistent metric for performance evaluation of multi-object filters [J]. IEEE Trans on Signal Processing, 2008, 56 (8): 3447-3457.

Note: Figure translations are in progress. See original paper for figures.