Postprint of Ultra-Wideband Signal Design and Analysis Based on Branch-Switching and Nested Chaotic Systems

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Abstract

Using high-dimensional complex chaotic systems in chaotic ultra-wideband signals can enhance signal security, but reduces the implementability of chaotic systems. To address the contradiction between complexity and implementability of chaotic systems used in chaotic ultra-wideband, a low-dimensional and easily implementable chaotic system is designed by employing two easily implementable one-dimensional chaotic maps based on chaotic branch switching and nesting. Lyapunov exponent analysis verifies that this system exhibits chaotic behavior over a wide parameter range. Analysis demonstrates that the signals generated by this system cannot reflect a deterministic mapping relationship, which not only enhances the complexity of chaotic signals but also maintains the system' s implementability. Analysis also indicates that this chaotic signal possesses favorable correlation properties. Therefore, this chaotic system is utilized to design and generate more secure chaotic time-hopping ultra-wideband signals. Through comparative power spectrum analysis between this chaotic ultra-wideband signal and conventional pseudo-random sequence time-hopping ultra-wideband signals, it is found to contain fewer high-intensity discrete spectral components that can cause interference to other transmission systems. Consequently, this chaotic ultra-wideband signal will have broader application prospects.

Full Text

Design and Analysis of Ultra-Wideband Signals Based on Branch-Switching and Nested Chaotic Systems

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Abstract: While employing high-dimensional complex chaotic systems in chaotic ultra-wideband (UWB) signals can enhance signal security, it reduces the ease of implementation. To address the trade-off between complexity and implementability in chaotic systems for UWB applications, this paper proposes a low-dimensional, easily implementable chaotic system designed through branch-switching and mutual nesting of two readily realizable one-dimensional chaotic maps. Lyapunov exponent analysis confirms that this system exhibits chaotic behavior across a wide parameter range. Analysis demonstrates that the signals generated by this system do not reflect deterministic mapping relationships, which not only increases the complexity of chaotic signals but also preserves implementability. The analysis further reveals that these chaotic signals possess excellent correlation properties, making them suitable for generating more secure chaotic time-hopping UWB signals. Comparative power spectrum analysis between this chaotic UWB signal and conventional pseudo-random sequence-based time-hopping UWB signals shows that the former contains fewer high-intensity discrete spectral components that could interfere with other transmission systems, thereby enabling broader application prospects for such chaotic UWB signals.

Keywords: ultra-wideband; chaotic map; nesting; branch switching; power spectrum

0 Introduction

Ultra-wideband impulse radio (IR-UWB) technology typically transmits information by directly modulating the amplitude or position of extremely narrow pulses, resulting in very low transmit power, high bandwidth, and low power spectral density that allows signals to be concealed within noise. This provides low probability of interception and high security, making it attractive for indoor wireless communication products and wireless sensor network interconnection technologies. To distinguish terminals in these networks, IR-UWB systems employ multiple access techniques such as time-hopping (TH-UWB) and direct-sequence (DS-UWB) [?]. The quality of the multiple access sequence codes directly affects the smoothness of the IR-UWB signal' s power spectrum and its multiple access capability [?]. A smoother power spectrum with fewer high-amplitude discrete spectral components reduces interference to other systems, while perfectly random sequence codes with good correlation properties minimize multi-user interference.

Chaotic systems exhibit extreme sensitivity to parameters and initial conditions, where infinitesimal changes can generate numerous distinct signals with white-noise-like characteristics and broad power spectra, making them widely applicable in information security. Since discrete chaotic maps such as the logistic map and Bernoulli map can be easily implemented using digital devices like DSP [?] and FPGA [?], they are frequently employed in digital chaotic secure communications.

The integration of chaotic mapping with UWB transmission represents an excellent approach for UWB signal generation. For instance, chaotic pulse position modulation UWB (CPPM-UWB) directly utilizes chaotic signals to create random-like variations in pulse intervals to differentiate users, while also determining whether additional delay variations should be applied based on symbol values. Chaotic UWB signals not only enable multi-access information transmission but also enhance security through the parameter and initial condition sensitivity of chaotic systems, providing strong anti-decryption capabilities when the receiver lacks an identical chaotic system.

However, existing chaotic UWB signal generation methods face limitations in balancing complexity and implementability. Many approaches employ simple one-dimensional maps with deterministic relationships, such as Tent or Bernoulli maps [?], which are vulnerable to prediction-based cryptanalysis. To address this, researchers have designed constructed maps based on these simple mappings [?], offering improved chaotic complexity and wider parameter ranges that can be optimized to enhance signal complexity.

This paper proposes a novel approach using the logistic map [?] and a branched Bernoulli-derived map (E-B map) [?], employing branch-switching and mutual nested iteration to create a parameter-variable mapping without deterministic relationships. Lyapunov exponent analysis verifies its chaotic properties across a wide parameter range. This system maintains low dimensionality while achieving super-chaotic behavior through parameter variation. Correlation analysis demonstrates its suitability for generating chaotic time-hopping pulse position modulation UWB (CTH-PPM-UWB) signals with smooth power spectral density and low-amplitude discrete spectra. The continuous variation of initial values creates effectively aperiodic time-hopping sequences, while parameter changes further randomize the sequences and enhance aperiodicity. Power spectrum simulations confirm that this mapping reduces discrete spectral components in UWB signals, thereby decreasing narrowband and multi-access interference in multi-user UWB transmission systems.

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1 Construction of Branch-Switching and Nested Chaotic Mapping

This paper constructs a chaotic mapping using the logistic map and a Bernoulli-derived D-B map.

The logistic map [?] is a structurally simple chaotic map defined by equation (1):

$$x_{n+1} = \mu x_n(1 - x_n), \quad 3.57 < \mu \leq 4$$

When the parameter μ is between 3.57 and 4, the logistic map generates chaotic time series with values in the range [0,1].

A generalized chaotic map derived from the Bernoulli map [**?**] is introduced here for analysis convenience and named the E-B map, as shown in equation (2):

$$y_{n+1} = \begin{cases} \alpha_1 y_n, & 0 \leq y_n \leq \frac{1}{\alpha_1+\alpha_2} \\ \alpha_2(1 - y_n), & \frac{1}{\alpha_1+\alpha_2} < y_n \leq 1 \end{cases}$$

where $\alpha_1, \alpha_2 > 0$ and $\alpha_1 + \alpha_2 > 1$.

From equations (1) and (2), we construct a parameter-variable one-dimensional mapping. First, given an initial value x_n for the logistic map, we iterate to obtain the next value x_{n+1}. This value serves as both the branch selection parameter for the E-B map and its initial value to generate a new mapped value y_{n+1}. This value is then used again as the initial value for the logistic map to produce the next branch selection parameter, and so on, continuously generating the mapping sequence. The construction process is shown in equation (3):

$$\begin{cases} x_{n+1} = \mu x_n(1 - x_n) \\ \text{if } x_{n+1} \leq \frac{1}{\alpha_1+\alpha_2} : y_{n+1} = \alpha_1 x_{n+1} \\ \text{if } x_{n+1} > \frac{1}{\alpha_1+\alpha_2} : y_{n+1} = \alpha_2(1 - x_{n+1}) \\ x_{n+2} = \mu y_{n+1}(1 - y_{n+1}) \end{cases}$$

where parameters μ and α_1 vary. This process generates a class of parameter-variable mappings producing different sequences $x_n, y_n, x_{n+1}, y_{n+1}, \ldots$ For analytical convenience, this constructed mapping is termed the "L-EB map."

The L-EB map exhibits the following characteristics:

a) The logistic map enables chaotic signal-driven switching of the E-B map's branch structure without deterministic patterns.

b) Each generated chaotic sequence serves as the iteration initial value for the other map, achieving irregular initial value switching and mutual nesting of the two mapping relationships.

c) The mapping parameters are variable, allowing construction of not just one but an entire class of mappings. Continuous parameter variation during iteration further increases mapping irregularity.

d) The overall mapping lacks a deterministic relationship, making it resistant to phase-space analysis and chaotic inverse parsing attacks.

These features ensure that the constructed mapping is non-deterministic with irregular iteration and switching patterns, enhancing system security.

2.1 Lyapunov Exponent Analysis

The Lyapunov exponent λ describes the overall rate of trajectory convergence or divergence in phase space. One-dimensional maps typically have a single λ value, while n-dimensional phase spaces have n values. Positive Lyapunov exponents indicate chaotic systems, with their magnitude and quantity reflecting signal complexity. Systems with multiple positive Lyapunov exponents are hyperchaotic, producing more complex trajectories and signals, typically observed in high-dimensional or time-delay chaotic systems. However, high-dimensional systems increase implementation cost. Therefore, achieving high-dimensional complexity using low-dimensional systems would reduce costs while maintaining security.

The proposed L-EB map achieves hyperchaotic-like effects. Different combinations of parameters μ and α_1 yield varying positive Lyapunov exponents, and parameter variation effectively produces hyperchaotic results. Since the L-EB map lacks a deterministic relationship, the "Benettin" method [?] is typically used for Lyapunov exponent analysis, as shown in equation (4):

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \ln\left|\frac{dx_{k+1}}{dx_k}\right|$$

where x_0 and x_k represent the initial value and the k-th iteration value, respectively.

The Lyapunov exponent spectrum of the chaotic map is shown in Figure 1 [Figure 1: see original paper]. Figures 1(a) and 1(b) depict the variation of the L-EB map' s Lyapunov exponent with parameters μ and α_1, demonstrating positive values across certain ranges and confirming its chaotic nature. Figure 1(a) shows the spectrum with fixed $\mu = 3.7$ and varying α_1, while Figure 1(b) shows the spectrum with fixed $\alpha_1 = 0.46$ (the value yielding maximum positive exponent from Figure 1(a)) and varying μ.

The results indicate that the L-EB map achieves positive Lyapunov exponents across a relatively wide parameter range. Notably, while the original logistic map exhibits chaos only for $3.57 < \mu \leq 4$, the constructed map maintains chaotic behavior for $0.2 < \mu \leq 1.48$ when $\alpha_1 = 0.46$, demonstrating that this construction extends the parameter range for chaotic behavior. Particularly, if parameters can be varied dynamically, the system effectively possesses multiple positive Lyapunov exponents, achieving hyperchaotic properties suitable for generating time-hopping codes in UWB communications.

2.2 Correlation Analysis

Based on the Lyapunov exponent spectrum in Figure 1, with initial values of 0.3 and 0.31, parameters producing chaotic signals are selected to generate L-EB map sequences x_1 and x_2. The autocorrelation of sequence x_1 is calculated (sequence x_2 shows similar behavior) and shown in Figure 2 Figure 2: see original paper. For comparison, the autocorrelation of a white noise sequence is shown in Figure 2(b). The comparison reveals that the L-EB map exhibits sharp autocorrelation properties similar to white noise signals.

Figure 3 [Figure 3: see original paper] compares the cross-correlation of white noise sequences with that of L-EB map sequences under different parameters and initial values. The L-EB map produces low cross-correlation fluctuations across a wide parameter range. If parameter pairs yielding positive Lyapunov exponents are pre-stored in FPGA or other digital devices and switched at intervals during branch-switching iteration, the randomness and complexity of the generated chaotic signals can be further enhanced. The number of parameter pairs, selection intervals, and selection patterns can all be used to adjust signal complexity, though this may slightly increase implementation difficulty compared to existing methods.

2.3 Relationship Between Signal Complexity and Implementability of L-EB Map

One-dimensional simple chaotic maps like logistic and E-B maps are easily implemented using FPGA digital devices [?] with high efficiency [?]. Reference [?] describes a physically unclonable FPGA implementation of the logistic map that enhances anti-attack capabilities. The L-EB map is constructed through branch-switching and nesting of these easily implementable maps, requiring only simple switching and control logic in addition to existing implementation methods. Reference [?] demonstrates multi-chaotic system switching using microcontroller control and FPGA, producing time-varying, diverse, and complex chaotic signals. However, this involves continuous high-dimensional chaotic systems requiring discretization and precise clock synchronization across tightly coupled dimensions, increasing implementation difficulty. In contrast, the L-EB map can be directly implemented using digital device switching and control, with its uncertain mapping relationships and irregular branch-switching ensuring time-varying, diverse, and complex signal generation.

3.1 Chaotic Time-Hopping UWB Signal Model

Direct modulation of nanosecond-width narrow pulses by information data creates binary 2PPM-UWB signals, as shown in equation (5):

$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT_s - b_k \varepsilon)$$

where T_s is the average pulse repetition period, b_k is the information data (0 or 1), and ε is the offset when $b_k = 1$.

The second derivative of the Gaussian pulse [**?**] is commonly used for UWB signal shaping as it contains no DC component. Equation (5) employs this pulse shape, shown in equation (6):

$$p(t) = A \left[1 - 4\pi \left(\frac{t}{\tau} \right)^2 \right] e^{-2\pi \left(\frac{t}{\tau} \right)^2}$$

where A is pulse amplitude and τ is the time decay constant. Smaller τ compresses pulse width and increases bandwidth.

To achieve UWB spectral characteristics, pseudo-random sequence codes further spread the spectrum. For PPM modulation, time-hopping causes approximately random pulse position variations within short time slots, enhancing transmission security. The binary 2PPM-TH-UWB signal is expressed in equation (7):

$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT_s - c_k T_c - b_k \varepsilon)$$

where c_k is the PN code and T_c is the chip duration, creating approximately random pulse offsets.

Time-hopping code selection critically affects UWB system performance: (a) PN codes must differentiate users in multi-access scenarios, requiring strong randomness and low cross-correlation to minimize multi-access interference, while maintaining good autocorrelation for synchronization; (b) UWB systems can interfere with co-band systems. Periodic or weakly periodic pulses produce strong discrete spectral lines in the power spectrum, causing significant interference. Therefore, random time shifts are necessary to smooth and reduce discrete spectra. However, conventional pseudo-random sequence-based time-hopping codes exhibit periodicity. While increasing code length reduces discrete spectra, complete randomness cannot be achieved, and longer codes increase implementation cost.

Chaotic mapping sequences offer excellent correlation properties and easy generation, making them ideal replacements for pseudo-random sequences. Using L-EB chaotic map sequences in 2PPM-TH-UWB creates 2PPM-CTH-UWB signals. Parameter variations in the L-EB map produce complex output changes and system unidentifiability, further enhancing security. The chaotic UWB signal transmission model based on equation (7) is shown in Figure 4 [Figure 4: see original paper].

In Figure 4, information data consists of binary symbols (0 or 1). The repetition encoder with rate $1/N_s$ repeats each symbol N_s times. The offset selector implements PPM modulation, adding offset ε for symbol 1. The time-hopping selector applies time-hopping offsets $c_k T_c$ with chip time T_c. The L-EB map chaotic time-hopping code generator produces integer time-hopping codes from L-EB sequences, generating nearly infinite random-like codes as parameters and initial values continuously change. The pulse shaper with impulse response from equation (6) convolves with the encoded data to produce the final UWB signal.

3.2 Power Spectrum Simulation of Chaotic Time-Hopping UWB Signals

To compare power spectra between conventional 2PPM-TH-UWB signals using pseudo-random time-hopping codes and 2PPM-CTH-UWB signals using L-EB map-generated codes, parameters are set based on equation (7):

Transmit power: -30 dBm; sampling rate: 50$×10^{9}$ samples/s. Since information code quantity and values do not affect the time-hopping UWB waveform, only 5 binary symbols are used for clear visualization. Repetition coding: $N_s = 5$, converting one symbol into 5 repeated symbols requiring 5 pulses. Assuming symbol duration T_b before repetition coding, each pulse occupies $T_s = T_b/N_s$ without reducing symbol rate. PPM offset: $\varepsilon = 0.25 \times 10^{-9}$ s.

Pulse shaping parameter: $\tau = 0.25 \times 10^{-9}$ s. When pulse non-zero duration is 2.2τ, truncation error is 50 dB below original energy. This paper uses $2\tau =$ 0.5$×10^{-9}$ s. From equation (7), each pulse occurs at $t = kT_s + c_k T_c + b_k \varepsilon$. To prevent pulse overlap, the sum of time-hopping offset and PPM offset must be less than chip time T_c. Therefore, frame time T_s is divided into slots by chip time T_c. Two parameter sets are used: (1) $T_s = 10 \times 10^{-9}$ s, $T_{c1} = 2 \times 10^{-9}$ s; (2) T_s unchanged, $T_{c2} = 1 \times 10^{-9}$ s. Set 1 divides one frame into 5 slots, set 2 into 10 slots.

Using set 1 parameters, L-EB map sequences are mapped to integer time-hopping codes in range [0,4]; using set 2, codes are in range [0,9]. The resulting chaotic UWB signals are shown in Figure 5 Figure 5: see original paper for set 1 and Figure 5(c) for set 2. Figure 5(a) shows the UWB signal using pseudo-random sequence-based time-hopping codes with period 5. The L-EB map-generated codes exhibit no periodicity—the code length can match any repeated symbol length without periodicity, reducing high-intensity discrete spectra as shown in Figure 6 [Figure 6: see original paper].

Figure 6 demonstrates that pseudo-random sequence-based time-hopping codes produce obvious discrete spectral lines due to their periodicity, with significantly stronger intensity than those using L-EB map-based codes. Longer L-EB map-generated time-hopping codes with more integer states produce more random pulse positions per frame, resulting in fewer high-intensity discrete spectral lines, smoother power spectra, and reduced interference to other systems.

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4 Conclusion

While simple one-dimensional maps are easily implementable, they generate insufficiently complex chaotic signals. This paper addresses this limitation by using the logistic map to perform branch-switching and mutual nesting with a Bernoulli-derived E-B map, constructing a class of parameter-variable, non-deterministic one-dimensional L-EB mappings. Lyapunov exponent analysis verifies the system' s chaotic properties, resolving the conflict between implementability and complexity. Embedding information security within the transmitted signal itself, without requiring separate encryption algorithms, represents a superior approach. For short-range wireless networks using time-hopping UWB transmission, chaotic system-generated time-hopping codes provide strong anti-decryption capabilities due to sensitivity to initial conditions and parameters. Simulation analysis confirms that the proposed chaotic UWB signals exhibit good correlation properties that reduce multi-access interference while more effectively smoothing discrete spectra, thereby decreasing interference to other systems. The proposed chaotic UWB signal scheme demonstrates strong practical significance for secure communication in indoor wireless products and sensor networks.

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Note: Figure translations are in progress. See original paper for figures.