Comprehensive Evaluation Algorithm for Halftone Visual Cryptography Algorithms (Postprint)

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Abstract

Existing evaluations of halftone visual cryptography (HVC) algorithms commonly employ Correct Decoding Rate (CDR) or Bit Error Rate (BER), without considering the disturbance inflicted upon cover images by information hiding. This paper proposes a comprehensive evaluation algorithm for halftone visual cryptography that simultaneously accounts for the disturbance to cover images caused by information hiding and the error rate of extracted secret images. The Average Disturbance Per Pixel (ADPP) metric is selected to quantify the disturbance to cover images resulting from information hiding. To ensure consistency between the data ranges of ADPP and BER, the secret image is utilized in conjunction with a hiding threshold T to compute the Maximum Average Disturbance (MAD). ADPP and BER are then integrated with MAD and a coefficient λ through a comprehensive calculation to derive the Integrated Disturbance of HVC (IDHVC). This algorithm exhibits significant advantages in computational complexity compared with existing comprehensive evaluation algorithms. Experimental verification demonstrates that the proposed algorithm is applicable to the comprehensive evaluation of both grayscale and color halftone visual cryptography algorithms.

Full Text

Preamble

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Integrated Evaluation Algorithm for Halftone Visual Cryptography Methods

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Abstract: Existing evaluations of halftone visual cryptography (HVC) methods typically rely on correct decode rate (CDR) or bit error rate (BER), without considering the distortion inflicted upon the carrier image during data hiding. This paper proposes an integrated evaluation algorithm for HVC methods that simultaneously accounts for both the distortion caused by data hiding and the error rate of the extracted secret image. We select average distortion per pixel (ADPP) to measure the interference to carrier images. To ensure consistent data ranges for ADPP and BER, we compute the maximum average distortion (MAD) using the secret image and a hiding threshold T. By combining ADPP and BER with MAD and a coefficient λ through comprehensive calculation, we obtain the integrated distortion of HVC (IDHVC). Compared with existing integrated evaluation algorithms, the proposed method offers significant advantages in computational complexity. Experimental results demonstrate that the algorithm can be applied to evaluate both grayscale and color halftone visual cryptography methods.

Keywords: halftone; visual cryptography; correct decode rate (CDR); bit error rate (BER); average distortion per pixel (ADPP)

0 Introduction

Halftone images represent a special image format that uses only black and white values to represent continuous-tone images. Conventional data hiding algorithms cannot be directly applied to halftone images, making halftone data hiding a distinct subfield of image steganography. Methods for generating halftone images from continuous-tone originals are called halftoning techniques, which primarily include ordered dithering [2], error diffusion [3,4], dot diffusion [5-8], and direct binary search [9-12].

Dot watermarking [13-17] refers to hiding watermark information by modifying local region characteristics in halftone images, and is mainly applicable to halftone images generated by ordered dithering. Since ordered dithering generally produces lower quality halftone images, error diffusion and dot diffusion methods are now more widely adopted. For these two types of halftone images, researchers primarily employ halftone visual cryptography (HVC) algorithms, which hide a binary secret image across multiple halftone images by modifying the relative relationships of corresponding pixels. The most common variant is

(2,2)-HVC [18–24,26,27], where a secret image is hidden in two halftone images and both are required for extraction.

Based on the type of halftone images they support, HVC algorithms can be categorized into grayscale halftone visual cryptography [18-19,23] and color halftone visual cryptography [20,22]. In [18], the authors proposed Data Hiding by Conjugate Error Diffusion (DHCED), where the first halftone image is generated using standard halftoning, and the halftoning process of the second image is modified to embed information. Reference [19] extended DHCED [18] from error diffusion to dot-diffused halftone images, proposing Data Hiding by Conjugate Dot Diffusion (DHCDD), and introduced a bidirectional conjugate data hiding algorithm (DHDCDD) that modifies data from both halftone images to achieve embedding through bidirectional conjugation.

Reference [20] extended DHDCDD [19] to color halftone images, proposing Double-sided Conjugate Data Hiding in Color Dot-diffused images (DCCDD) for embedding secret images in color dot-diffused halftone images. Reference [21] presented a halftone visual cryptography algorithm based on multi-scale error diffusion. Reference [22] further extended DCCDD to color error-diffused halftone images, implementing Double-sided Conjugate Error Diffusion (DCCED) for color images. Additionally, [22] introduced the concept of "new conjugate" for color halftone images and applied this concept to both error diffusion and dot diffusion, proposing New Conjugate Data Hiding by Error Diffusion (NCCED) and New Conjugate Data Hiding by Dot Diffusion (NCCDD).

Reference [23] implemented and compared various grayscale HVC algorithms, concluding from experimental results that bidirectional conjugate algorithms outperform unidirectional conjugate algorithms. Reference [24] introduced the concept of block conjugate to improve HVC performance and proposed a high-quality halftone secret pattern hiding algorithm based on block conjugate. Reference [25] presented halftone self-hiding algorithms based on both dot diffusion and error diffusion, enabling secret image hiding within a single halftone image.

Most existing HVC algorithms evaluate performance using correct decode rate (CDR). However, since halftone data hiding algorithms deliberately modify the halftoning process to embed information, these modifications inevitably introduce interference to the halftone image data. Generally, greater interference leads to better hiding effects and higher CDR values for extracted secret images. Conversely, higher CDR values often result in lower quality of the generated halftone images. Therefore, evaluating HVC algorithm performance using CDR alone is inadequate.

References [26,27] proposed a comprehensive interference evaluation formula for halftone data hiding algorithms, establishing an optimization theory for such methods and introducing unidirectional and bidirectional error diffusion data hiding algorithms. However, these works focused on proposing new algorithms rather than applying the formula to evaluate both their own and other existing algorithms.

This paper proposes an integrated evaluation algorithm for HVC methods that considers both the distortion caused to carrier images and the BER of extracted secret images. We adopt average distortion per pixel (ADPP) [22] to measure interference to carrier images. First, we compute the error rate of the extracted secret image (BER = 1 - CDR). Then we calculate the ADPP caused by data hiding. To ensure consistent data ranges for ADPP and BER, we compute the maximum average distortion (MAD) using the secret image and hiding thresholds. Finally, we combine ADPP and BER with MAD and coefficient λ to obtain the integrated distortion of HVC (IDHVC), where smaller values indicate better algorithm performance. Experimental verification demonstrates that IDHVC comprehensively reflects both the interference to carrier images and the error rate of extracted secret images, making it applicable for evaluating both grayscale and color HVC algorithms.

This paper comprises five sections: (1) review of major existing algorithms; (2) the proposed integrated evaluation algorithm for HVC; (3) experimental results and analysis; and (4) conclusion.

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1 Existing Work

References [18-23] present eight major HVC algorithms: DHCED [18] and DHDCED [23] for grayscale error-diffused halftone images; DHCDD [19] and DHDCDD [19] for grayscale dot-diffused halftone images; DCCED [22] and NCCED [22] for color error-diffused halftone images; and DCCDD [20] and NCCDD [22] for color dot-diffused halftone images.

We first introduce the main HVC algorithms. Let X_1 and X_2 denote the original images, Y_1 and Y_2 the halftone images after data hiding, and W the secret image to be embedded.

1.1 DHDCDD and DHDCED

Both DHDCDD [19] and DHDCED [23] employ bidirectional conjugate methods to hide a secret image in two grayscale halftone images. They share the same implementation process but differ in their halftoning control methods: DHDCDD [19] uses dot diffusion while DHDCED [23] uses error diffusion. The implementation flow is as follows: (a) When $x_1(i,j)$ and $x_2(i,j)$ enter the system, diffusion errors are accumulated to generate $u_1(i,j)$ and $u_2(i,j)$. (b) $u_1(i,j)$ and $u_2(i,j)$ undergo trial quantization to produce trial values $y_{1,trial}(i,j)$ and $y_{2,trial}(i,j)$, implemented through formulas (1)-(3). (c) The system checks whether $y_{1,trial}(i,j)$ and $y_{2,trial}(i,j)$ satisfy the required conditions: when secret image data $w(i,j)$ is 0 (black), $y_2(i,j)$ must be conjugate to $y_1(i,j)$; when $w(i,j)$ is 1 (white), $y_2(i,j)$ must match $y_1(i,j)$. (d) If the trial values meet the requirements, they are directly passed to $u_1'(i,j)$

and $u_2'(i,j)$ as $u_1'(i,j) = u_1(i,j)$ and $u_2'(i,j) = u_2(i,j)$. (e) If $y_{1,trial}(i,j)$ and $y_{2,trial}(i,j)$ fail to meet the requirements, either $y_{1,trial}(i,j)$ or $y_{2,trial}(i,j)$ must be swapped. Two swapping schemes exist: swapping $y_{1,trial}(i,j)$ with calculated interference Δu_1, or swapping $y_{2,trial}(i,j)$ with interference Δu_2. The scheme causing smaller interference Δu is selected. (f) Δu_1 and Δu_2 calculations depend on swap direction: if $y_{1,trial}(i,j)$ or $y_{2,trial}(i,j)$ changes from $255\$\rightarrow 0, useequation(4); ifchangingfrom0\rightarrow\$255$, use equation (5). (g) If $\min(\Delta u_1, \Delta u_2) \leq thresholdT$, the swap with smaller interference is executed. (h) If $\min(\Delta u_1, \Delta u_2) > T$, no swap occurs. (i) After processing the entire images X_1 and X_2 using this procedure, halftone images Y_1 and Y_2 are generated.

The algorithm' s advantage lies in introducing the bidirectional conjugate concept, but its limitation is applicability only to grayscale halftone images.

1.2 NCCDD and NCCED

Reference [22] defines a new conjugate property (NC) for color halftone images. The NC value can be calculated using equation (6), where $Numof255$ counts the number of color components equal to 255 among three channels. If two or three components equal 255, the NC value is 1; if zero or one component equals 255, the NC value is 0.

Let Y_1 and Y_2 represent two color halftone images, with $NC_{y1}(i,j)$ and $NC_{y2}(i,j)$ denoting the NC values of $y_1(i,j)$ and $y_2(i,j)$. If $NC_{y1}(i,j) \oplus NC_{y2}(i,j) = 1$, then $y_1(i,j)$ and $y_2(i,j)$ are new conjugates; if $NC_{y1}(i,j) \oplus NC_{y2}(i,j) = 0$, they are identical.

When $x_1(i,j)$ and $x_2(i,j)$ enter the system, trial quantization values $y_{1,trial}(i,j)$ and $y_{2,trial}(i,j)$ are calculated using equations (1)-(3). Their NC values $NC_{y1,trial}(i,j)$ and $NC_{y2,trial}(i,j)$ are computed using equation (6). If these NC values are not as required, $y_{1,trial}(i,j)$ and $y_{2,trial}(i,j)$ must be swapped.

1.3 CDR Comparison of HVC Algorithms

1) Grayscale HVC Algorithm CDR Comparison Reference [23] evaluates grayscale HVC algorithm performance by computing the correct decode rate (CDR) of extracted secret images using equation (7), where E is the extracted secret image, W is the original secret image, and both have resolution $W \times H$.

Experimental results show that for grayscale HVC algorithms: (a) Among error diffusion-based algorithms, DHDCED(Jarvis) [23] performs best; (b) Among dot diffusion-based algorithms, DHDCDD [19] performs best; (c) Regardless of whether error diffusion or dot diffusion is used, bidirectional conjugate algorithms achieve higher CDR than unidirectional conjugate algorithms.

2) Color HVC Algorithm CDR Comparison Algorithms DCCED [22] and DCCDD [20] use equation (8) to compute average CDR. Algorithms NCCED [22] and NCCDD [22] use equation (9) to compute the extracted secret image E, then calculate CDR using equation (7).

Reference [22] concludes from experimental results that for color HVC algorithms: (a) NCCED [22] achieves significantly higher CDR than DCCED [22]; (b) NCCDD [22] achieves significantly higher CDR than DCCDD [20]; (c) NCCED [22] delivers the highest CDR among all algorithms.

1.4 Comprehensive Interference Evaluation Formula

References [26,27] propose a comprehensive interference evaluation formula for halftone data hiding algorithms, expressed as equation (10), where D_h represents interference to carrier images, D_w represents error rate of extracted secret images, and λ is the comprehensive calculation coefficient. Smaller comprehensive values indicate better algorithm performance.

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2 Integrated Evaluation Algorithm for HVC

This paper proposes an integrated evaluation algorithm for halftone visual cryptography methods, with the algorithm flow shown in Figure 1 [Figure 1: see original paper]. Following the approach in [26,27], we need to obtain the interference to carrier images (D_h) and the error rate of extracted secret images (D_w), then perform comprehensive calculation.

We select maximum average distortion per pixel (ADPP) [22] as D_h and use the bit error rate (BER) of extracted secret image E as D_w. For comprehensive calculation, we must consider the value ranges of ADPP and BER, requiring computation of maximum average distortion (MAD) using secret image W and threshold T. Finally, to emphasize the importance of BER, we introduce coefficient λ to complete the comprehensive calculation.

The proposed evaluation algorithm proceeds as follows: (a) X_1 and X_2 are original images with resolution $W \times H$. (b) Through HVC algorithms, a secret image W is hidden in X_1 and X_2 to generate halftone images Y_1 and Y_2. Any HVC algorithm from [18-23] may be used. The hiding process is controlled by threshold T, where T_b and T_w correspond to black and white regions in the secret image. (c) During hiding, ADPP is calculated using equation (11) or (12), computed separately for Y_1 and Y_2 to obtain $ADPP_{Y1}$ and $ADPP_{Y2}$. (d) Y_1 and Y_2 undergo extraction operations to obtain extracted secret image E. (e) Comparing E with original secret image W using equations (7)-(9) yields CDR, from which we compute BER ($BER = 1 - CDR$) with range [0,1]. (f) The maximum of the two ADPP values gives $ADPP_{MAX}$, with range [0, MAD], where MAD is the maximum average distortion determined by secret image W and thresholds T_b and T_w. (g) Combining W, T_b, and T_w, MAD is calculated using equation (13). (h) Finally, $ADPP_{MAX}$, BER, MAD, and coefficient λ are combined using equation (14) to obtain the integrated distortion value (IDHVC). Smaller IDHVC values indicate better algorithm performance.

Four aspects of the proposed algorithm require explanation:

2.1 Selecting ADPP as Carrier Image Distortion

Reference [22] defines average distortion per pixel (ADPP) for color HVC algorithms using equation (11). ADPP effectively measures interference caused by halftone data hiding algorithms and features simple computation—only requiring interference statistics during the hiding process. This significantly reduces computational complexity compared to the human visual peak signal-to-noise ratio (HPSNR) used in [26], as analyzed in Section 3.3. Therefore, we select ADPP as the carrier image distortion measure D_h.

2.2 Computing ADPPMAX

During data hiding, ADPP is calculated separately for Y_1 and Y_2, yielding $ADPP_{Y1}$ and $ADPP_{Y2}$. Our algorithm uses the maximum ADPP value rather than the average for three reasons: (a) For unidirectional hiding algorithms, $ADPP_{Y1}$ is 0 while $ADPP_{Y2}$ is large; (b) For bidirectional algorithms, one ADPP may be small while the other is large, with larger ADPP indicating greater interference and quality degradation; (c) Average ADPP cannot reflect the actual impact on generated Y_1 or Y_2. Therefore, we adopt $ADPP_{MAX}$ as the basis for integrated distortion calculation.

2.3 Additional Parameters for Comprehensive Calculation

We select $ADPP_{MAX}$ as D_h and BER as D_w. Before applying equation (10), we analyze consistency between ADPP and BER: (1) **Calculation basis**: BER counts total error points divided by secret image points $W \times H$; ADPP sums interference values divided by carrier image points $W \times H$. Since secret and carrier images share the same dimensions, they are consistent. (2) **Value range**: BER ranges in [0,1], while ADPP ranges in [0, MAD]. MAD depends on hiding thresholds T, where T_b and T_w correspond to black and white regions in W. MAD is computed by first calculating black and white percentages in W, then applying equation (13). Dividing $ADPP_{MAX}$ by MAD normalizes its range to [0,1]. (3) **Emphasizing BER importance**: While both BER and ADPP are important, BER must be prioritized to ensure hiding effectiveness. Therefore, we introduce a multiplicative coefficient λ ($\lambda > 2$) for BER in the comprehensive calculation.

2.4 Integrated Distortion Calculation

Based on the above analysis, the integrated distortion combines $ADPP_{MAX}$, BER, MAD, and coefficient λ using equation (14) to obtain IDHVC:

$$IDHVC = \frac{ADPP_{MAX}}{MAD} + \lambda \times BER$$

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3 Experimental Results and Analysis

Figure 2 [Figure 2: see original paper] shows our test images and secret image, where (a-d) are grayscale test images, (e–h) are color test images, and (i) is the secret image. All images have resolution 512 × 512. The proposed evaluation algorithm applies to both grayscale and color HVC methods.

3.1 Grayscale HVC Algorithm Evaluation

We evaluate four algorithms: DHCDD [19], DHDCDD [19], DHCED [18], and DHDCED [23]. During experiments, thresholds T_b and T_w correspond to black and white regions in the secret image. The Floyd-Steinberg and Jarvis kernels are used for error diffusion, while HVS-based rank matrices are employed for dot diffusion.

Using Figure 2(a) as X_1, Figures 2(b)-(d) as X_2, and Figure 2(i) as secret image W, we create three test sets. Each algorithm is applied across different thresholds, with CDR computed using equation (7) and BER calculated as $BER = 1 - CDR$. Table 1 shows average BER for each algorithm at each threshold across the three test sets.

ADPP is calculated using equation (12) during hiding, with average ADPP values for each algorithm at each threshold shown in Table 2 . Taking the maximum of the two ADPP values yields $ADPP_{MAX}$, presented in Table 3 . Combining secret image W with different thresholds T, MAD is calculated using equation (13) and shown in Table 4 . Finally, $ADPP_{MAX}$, BER, MAD, and coefficient λ are combined using equation (14) to compute IDHVC, with results in Table 5 ($\lambda = 3$).

Table 5 reveals that for grayscale HVC algorithms: (a) Among error diffusion-based methods, DHDCED(Jarvis) [23] performs best; (b) Among dot diffusion-based methods, DHDCDD [19] performs best; (c) Bidirectional conjugate algorithms achieve higher CDR than unidirectional conjugate algorithms. These conclusions align with [23], though IDHVC comparisons show less pronounced differences than CDR-only evaluations.

3.2 Color HVC Algorithm Evaluation

We evaluate four algorithms: DCCDD [20], NCCDD [22], DCCED [22], and NCCED [22]. The Jarvis kernel is used for error diffusion, and HVS-based rank matrices for dot diffusion.

Using Figure 2(e) as X_1, Figures 2(f)-(h) as X_2, and Figure 2(i) as W, we create three test sets. Each algorithm is applied across thresholds, with CDR computed using equations (7)-(9) and BER calculated accordingly. Table 6 shows average BER results. ADPP is calculated using equation (11) during hiding, with average ADPP values in Table 7 .

$ADPP_{MAX}$ values are shown in Table 8 . Since W and thresholds T remain unchanged, MAD values are the same as in Table 4. Combining $ADPP_{MAX}$, BER, MAD, and $\lambda = 3$ using equation (14) yields IDHVC values in Table 9 .

Table 9 shows that for color HVC algorithms: (a) For color error-diffused halftone images, NCCED [22] significantly outperforms DCCED [22]; (b) For color dot-diffused halftone images, NCCDD [22] significantly outperforms DCCDD [20]; (c) Among all color algorithms, NCCED [22] achieves the best performance. These conclusions match [22], as NCCED [22] and NCCDD [22] exhibit both lower BER and significantly smaller ADPP than DCCED [22] and DCCDD [20], resulting in superior IDHVC values.

3.3 Comparison with Existing Algorithms

Table 10 compares our algorithm with evaluation methods in [22] and [26]: (a) [22] uses CDR and ADPP for evaluation but without comprehensive integration; (b) Both our method and [26] employ comprehensive evaluation, requiring computational complexity comparison; (c) Both use BER as D_w, making D_w computation identical; (d) Our method uses ADPP for D_h while [26] uses HPSNR. The key comparison lies in computational complexity between ADPP and HPSNR.

For grayscale halftone images, ADPP is calculated using equation (12), while HPSNR uses equation (15). Table 10 compares D_h computational complexity, where W and H are image dimensions. Our algorithm' s D_h complexity is shown in column 3, while [26] requires filtering each pixel with a specialized 7$×$7 filter to generate an estimated image before PSNR calculation, making the process significantly more complex (column 5). Table 10 demonstrates that our algorithm offers clear computational advantages over existing comprehensive evaluation methods.

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4 Conclusion

This paper proposes an integrated evaluation algorithm for halftone visual cryptography methods. We compute the extracted secret image' s error rate (BER), calculate the average distortion per pixel (ADPP) caused by data hiding, determine the maximum average distortion (MAD) using the secret image and thresholds, and combine ADPP and BER with MAD and coefficient λ to obtain the integrated distortion of HVC (IDHVC). Smaller IDHVC values indicate better algorithm performance. Compared with existing comprehensive evaluation algorithms, our method demonstrates significant advantages in computational complexity. Experimental verification shows that IDHVC comprehensively reflects both HVC algorithm interference to carrier images and error rates of extracted secret images, making it applicable for evaluating grayscale and color halftone visual cryptography methods.

IDHVC comparisons sometimes reveal less pronounced differences between algorithms than CDR-only evaluations, providing more objective performance assessment. In practical algorithm implementation, one cannot solely pursue higher CDR or lower BER; comprehensive consideration of carrier image interference is essential. Reducing carrier image interference represents a valuable direction for algorithm improvement.

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Note: Figure translations are in progress. See original paper for figures.