A study of a scalar field probes micro space-time

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Abstract

In this work, we try to find a way to describe the physical law of micro-world under the frame of a space-time theory. By introducing a scalar field D(x), we rewrite the action of conventional field theory and the Lagrangian describing the motion of the particle, where a modified space-time relation is obtained. To prove the correctness of this attempt, we derive the Klein-Gordon equation by the Hamilton-Jacobi method in four dimensional form.

Full Text

Preamble

A Study of a Scalar Field Probes Micro Space-Time

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Abstract

In this work, we propose a framework for describing physical laws of the micro-world within a space-time theory. By introducing a scalar field $D(x)$, we modify both the action of conventional field theory and the Lagrangian governing particle motion, which leads to a modified space-time relation. To validate this approach, we derive the Klein-Gordon equation using the Hamilton-Jacobi method in four-dimensional form.

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Introduction

General relativity (GR) [1, 2] stands as a milestone in 20th-century physics, revealing the nature of gravity and profoundly advancing the development of modern cosmology. In particular, the discovery of the accelerated expansion of the universe has spurred the development of various modified gravity theories [3–10], which have made significant progress in addressing problems such as the cosmological constant, inflation, and structure formation. Scalar-tensor theory, one of the most prominent modified theories, has attracted widespread attention. While conventional general relativity is a geometric or metric theory—often called a "tensor theory"—and we typically aim to minimize the degrees of freedom in a theory, this does not preclude the introduction of additional scalar fields.

In the original scalar-tensor theory, the scalar field was associated with a varying gravitational constant, thereby allowing gravity to be modified. Scalar fields can arise from diverse sources, including the dilaton from string theory, scalar fields in brane-world models, or the size of compactified internal spaces [11]. This versatility makes scalar field theory particularly appealing, as it provides considerable freedom for developing new theoretical frameworks.

The variational principle has played a crucial role in the development of modified gravity theories. By starting from the action, one can readily derive the equations of motion for fields or particles, and theoretical modifications can be achieved by altering the action itself.

Since the inception of general relativity, space-time has emerged as a distinct object of study, with its geometry intimately connected to the distribution of matter. While GR is typically applied to large-scale phenomena, particularly in cosmology, its extension to micro-scales may require modified field equations to properly describe physical laws. In cosmology, the fine-tuning problem [12] in particular highlights the urgent need for a gravity theory that remains valid at microscopic scales.

In this article, we attempt to employ scalar-tensor methods and space-time concepts to derive physical laws for the microcosm. While such an endeavor is generally challenging, the space-time perspective of GR—particularly the notion that space-time possesses a microstructure in certain quantum gravity theories [13–21]—provides a viable pathway. Technically, we adopt the perspective that certain space-time microstructures induce a scalar field $D(x)$. Considering a particle moving in a background containing this scalar field, we propose a modified action for fields and a corresponding Lagrangian for particle motion. From this action, we derive a relationship between $D(x)$ and another scalar field $\zeta(x)$. Through Hamiltonian analysis, we obtain a modified space-time relation within general relativity. To test the viability of this framework for describing micro-scale space-time physics, we derive the Klein-Gordon equation [22–24] from our fundamental equations using the four-dimensional Hamilton-Jacobi method. Throughout this paper, we employ the metric signature $(-,+,+,+)$.

II. THE BASIC EQUATIONS

Considering a spin-independent particle moving in an arbitrary background, we choose the action of fields as

$$I = I_0 + \int \sqrt{-g} \left[\lambda \left(g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + \frac{m^{2c^2}}{\hbar^2} D \zeta^2 \right) \right] d^{4x},$$

and the Lagrangian describing the particle's motion is

$$L = L_0 - mc^{2D}.$$

Here I_0 can represent the action of any conventional theory, such as gauge theory [25–30] or various gravity theories. L_0 is the particle Lagrangian in these known theories, m is the particle's proper mass, c is the speed of light, and $\hbar$ is the reduced Planck constant (for convenience, we adopt natural units where $c = \hbar = 1$ for the remainder of this paper). λ is a constant with dimensions of length. The scalar function $D(x)$, which we specifically introduce in this work, originates from certain space-time microstructures and represents a proper time field. Finally, $\zeta(x)$ is a scalar field that can be directly interpreted through the relation $\zeta^2 = \eta$, where η is the proper space density of the particle arising from restrictions on its region of existence. This can also be understood as the proper probability density, as the two interpretations are proportional. It is therefore reasonable to require the space density to satisfy the conservation law

$$\nabla_\mu (\sqrt{-g}\, \eta u^\mu) = 0.$$

Similar to conventional theories, we require the space density to be positive, continuous, finite, and single-valued at every spatial point.

To gain further insight into these formulas, we can write Eqs. (1) and (2) more concretely. For instance, considering a spin-independent particle moving in an electromagnetic field within Einstein gravity, the complete action and Lagrangian take the form

$$I = \int \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + L_m + \lambda \left(g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + \frac{m^{2c^2}}{\hbar^2} D \zeta^2 \right) \right] d^{4x},$$

$$L(x^\mu, \dot{x}^\mu) = m \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} + q A_\mu \dot{x}^\mu - mc^{2D},$$

where R is the curvature scalar, the electromagnetic tensor $F_{\mu\nu}$ satisfies $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$, and A_μ is the four-vector potential. L_m is the matter Lagrangian density that provides sources for the gravitational and electromagnetic fields. In Eq. (6), $\dot{x}^\mu = dx^\mu/d\tau = u^\mu$ is the four-velocity, τ is the particle's proper time, and q is the particle's charge.

For such a particle, the Hamiltonian is

$$H = \pi_\mu \dot{x}^\mu - L = \frac{m}{2}(g_{\mu\nu}u^\mu u^\nu + D),$$

where $\pi_\mu = \partial L/\partial \dot{x}^\mu = mu_\mu + qA_\mu$ is the canonical four-momentum vector. Hamiltonian conservation then leads to

$$g_{\mu\nu}u^\mu u^\nu + D = \text{const.}$$

When $D = 0$, Eq. (9) should reduce to the conventional gravity result $g_{\mu\nu}u^\mu u^\nu = -1$, allowing us to determine the constant and rewrite Eq. (9) as

$$g_{\mu\nu}u^\mu u^\nu + D = -1.$$

To clarify this result, we can express it as

$$g_{\mu\nu}dx^\mu dx^\nu = -d\bar{\tau}^2,$$

where

$$d\bar{\tau} = \sqrt{1 + D}\, d\tau.$$

Here $\bar{\tau}$ is the proper time of the local coordinate system comoving with the particle. We see that while the space-time relation for coordinates in this work remains consistent with standard GR, the particle's proper time differs from the coordinate proper time due to the presence of the proper time field. Consequently, the interpretation of four-velocity and the proper space density of the particle's existence must be adjusted accordingly.

To further understand the proper time field $D(x)$, we examine the equation for the scalar field ζ. Varying the action (5) with respect to ζ yields

$$\nabla_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\zeta) + m^2\sqrt{-g}D\zeta = 0,$$

which demonstrates the relationship between the proper time field and the space density field. Together with Eq. (4) and Eq. (11), this constitutes the fundamental set of equations in our work.

III. KLEIN-GORDON EQUATION

To test the validity of our basic equations for describing physical laws at microscopic scales, we employ the Hamilton-Jacobi method [31]:

$$H(x^\mu, \partial_\mu S) + \frac{\partial S}{\partial \tau} = 0,$$

where S is the Hamilton-Jacobi function and π_μ has been replaced by $\partial_\mu S$. With the ansatz

$$S = W(x^\mu, \pi_\mu) - \alpha h \tau,$$

we obtain

$$H(x^\mu, \partial_\mu W) = \alpha h = -\frac{\partial S}{\partial \tau}.$$

The quantity W is Hamilton's characteristic function, with $\pi_\mu = \partial_\mu W$. Equation (11) can now be rewritten as

$$g^{\mu\nu}(\partial_\mu W - qA_\mu)(\partial_\nu W - qA_\nu) + m^{2D} = -m^2,$$

and Eq. (4) similarly takes the form

$$\nabla_\mu(\sqrt{-g}\,\zeta^2 g^{\mu\nu}(\partial_\nu W - qA_\nu)) = 0.$$

Equations (18), (19), and (14) constitute the new fundamental formulas for describing a particle moving in an electromagnetic field under gravity.

To achieve our goal, we introduce a new function ξ defined by

$$W = \frac{\hbar}{i}\ln\xi,$$

or equivalently

$$\xi = e^{iW/\hbar}.$$

Substituting this into Eqs. (18) and (19) and combining with Eq. (14), we obtain

$$\nabla_\mu(\sqrt{-g}g^{\mu\nu}(\partial_\nu - iqA_\nu)\phi) - ig^{\mu\nu}qA_\mu(\partial_\nu - iqA_\nu)\phi - m^2\phi = 0,$$

which is the Klein-Gordon equation derived from our theory. Here $\phi = \xi\zeta$ is the particle's wavefunction.

Regarding the positive definiteness of the probability density, we define $\gamma = dt/d\tau$ in accordance with Eq. (4) and note that $\phi^*\phi = \eta$. The normalization condition can then be written as

$$\int \sqrt{-g}\,\gamma\,\phi^*\phi\,d^{3x} = 1.$$

We thus see that the wavefunction in the Klein-Gordon equation has obtained a clear physical interpretation. These results demonstrate the viability of our basic equations for describing physical laws in microscopic space-time. Additionally, Eq. (21) may provide a convenient method for calculating $D(x)$, which plays a central role in this theory.

IV. CONCLUSIONS AND OUTLOOK

In this paper, we have introduced a scalar field $D(x)$ to probe physical laws of the micro-world within a space-time theoretical framework, obtaining a modified space-time relation. We have shown that the proper time field $D(x)$ is related to the probability density field. Furthermore, by employing the Hamilton-Jacobi equation, we derived the Klein-Gordon equation from our fundamental equations, demonstrating that this approach to micro space-time is indeed effective.

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Note: Figure translations are in progress. See original paper for figures.