The cosmological constant from space-time discreteness

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Abstract

We regard the space-time background as a physical system composed of discrete volume elements at the Planck scale and obtain the internal energy of space-time through the Debye model. A temperature-dependent minimum energy limit for particles is proposed from the thermal motion component of the internal energy. As the temperature decreases due to the expansion of the universe, an increasing number of particles would be "released" because of the change in the energy limit; we regard these new particles as a source of dark energy. The minimum energy limit also leads to a corrected particle number in the universe and a modified conservation equation. According to the modified conservation equation, an effective cosmological constant consistent with the observed value is obtained.

Full Text

Preamble

The Cosmological Constant from Space-Time Discreteness

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Abstract

We regard the background of space-time as a physical system composed of discrete volume elements at the Planck scale and obtain the internal energy of space-time using the Debye model. A temperature-dependent minimum energy limit for particles is proposed based on the thermal motion component of the internal energy. As the temperature decreases due to the expansion of the universe, more and more particles are "released" because of the change in the energy limit, and we regard these new particles as a source of dark energy.

The minimum energy limit also leads to a corrected number of particles in the universe and a modified conservation equation. According to the modified conservation equation, an effective cosmological constant consistent with its observed value is obtained.

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Introduction

Astrophysical data obtained from high redshift surveys of supernovae [?, ?] reveals that the expansion of the universe is accelerating. This acceleration is attributed to an unknown entity called dark energy (DE). In the past few decades, various modifications of general relativity have been proposed to explain its origin. Some of the most studied models include the cosmological constant model, $f(R)$ gravity [?], scalar-tensor theories [?, ?], quintessence models [?, ?], and phantom models [?, ?], among others. Unfortunately, as one of the most successful interpretations of DE, the value of the cosmological constant Λ calculated by the cosmological constant model is 60-120 orders of magnitude smaller than that given by quantum field theory [?]. This indicates that there is an irreconcilable contradiction between general relativity (GR) and quantum field theory (QFT).

To eliminate this contradiction between the two theories, various quantum gravity theories have been proposed in recent years [?]. As an approach to finding a suitable quantum gravitational theory, some physicists have considered the effect of the discreteness of space-time at very small scales. In these theories, the fundamental description of space-time is only an approximation to a manifold. For instance, causal set theory [?] describes space-time as a set of discrete points and develops the quantum dynamics of particles at these points. The points fixed on the background manifold are treated as the positions of particles; therefore, the discreteness of the points will affect the trajectory of the particles at a tiny scale. A recent work by Alejandro Perez and Daniel Sudarsky [?] interpreted the cosmological constant as the accumulation of small violations of energy conservation derived from discreteness at the Planck scale. In these studies, the background of the universe was considered to have discrete microstructures. We follow this view and further hypothesize that the discrete points on the background have properties similar to particles, thus allowing one to study the dynamics or thermodynamics of the background. This assumption is equivalent to figuring out the type of interaction between volume elements (voxels) of space-time, not just whether it is discrete or continuous. If there is a suitable micro-model to describe the background, it will provide possibilities for researching the energy exchange between the background and the particles on it, which is exactly the focus of this article.

In this paper, we regard the background as a thermodynamic system with strong interaction and discrete structure at the Planck scale, so the Debye model [?] can be applied. By calculating the internal energy expression of the background under thermal equilibrium conditions, we obtain a temperature-dependent energy limit. Considering the influence of the thermal motion of the background on the particles, this energy limit is assumed to be the lowest energy of the particles. As the energy limit decreases, more and more particles contribute to gravity. As time goes on, this new energy will dominate the expansion of the universe. We regard it as a source of dark energy and calculate the cosmological constant corresponding to the energy density of these new particles, which is consistent with the observed value of the cosmological constant [?].

The paper is organized as follows. Section II briefly reviews the method of obtaining the Debye model from canonical ensemble theory, then replaces the ordinary atoms in a solid with Planck-sized voxels and obtains the expression for the internal energy of the space-time background. In Section III, the relationship between the number of "real" particles and the total number of particles in the universe is obtained using the lowest energy limit. Then we derive a new conservation equation and estimate the value of the cosmological constant in Section IV. Finally, Section V presents conclusions and discussions. We will use natural units in which $\hbar = c = k_B = 1$ throughout this article.

II. The Debye Model of the Background of the Universe

To treat the background of space-time as an object of research physically, we need an appropriate model to describe this system. Assuming space-time is discrete, meaning that space-time has internal structure, we can study it using the theory of statistical physics. Intuitively, a system used to describe the background of the universe must not be a thermodynamic system with weak interactions like an ideal gas, because such a system is not stable enough and would "tear" when the energy is too high. Even a particle at rest may move chaotically when embedded in such a background. Therefore, the background system should be dense, with strong interaction between discrete voxels. Fortunately, in statistical physics, there is an appropriate example to describe such a system: the Debye model.

In general solids, there is strong interaction between atoms, which makes them have equilibrium positions and perform micro-vibrations near these positions. Assuming that there are N atoms in the solid and each atom has 3 degrees of freedom, then the solid has $3N$ degrees of freedom. The total potential energy ϕ can be simplified as $\phi = \phi_0 + \frac{1}{2}\sum_{l,s} \frac{\partial^2\phi}{\partial\xi_l\partial\xi_s}\xi_l\xi_s$, where ξ_l is the displacement of the l-th degree of freedom from equilibrium, and ϕ_0 is the potential energy of the system when all atoms are in equilibrium. In addition to potential energy, each degree of freedom has corresponding micro-vibration energy. Therefore, the total energy of the system can be obtained as $E = \sum_l \frac{p_l^2}{2m} + \frac{1}{2}\sum_{l,s} \frac{\partial^2\phi}{\partial\xi_l\partial\xi_s}\xi_l\xi_s + \phi_0$, where $p_l^2/2m$ is the kinetic energy of the l-th degree of freedom. In canonical coordinates, the above formula can be expressed as the sum of $3N$ independent simple harmonic motions. As is well known, in quantum mechanics, the energy of such $3N$ independent simple harmonic motions can be expressed as $E = \phi_0 + \sum_i \hbar\omega_i(n_i + \frac{1}{2})$, where ω_i is the normal frequency and n_i is the quantum number describing the i-th simple harmonic motion. In this way, we obtain a workable form of energy.

In statistical physics, it is convenient to use canonical ensemble theory to study systems with strong interactions. The partition function of the above system can be expressed as $Z = e^{-\beta\phi_0} \prod_i \frac{e^{-\beta\hbar\omega_i/2}}{1-e^{-\beta\hbar\omega_i}}$. According to canonical ensemble theory, the internal energy of the system can be obtained as $U = -\frac{\partial}{\partial\beta}\log Z = U_0 + \sum_i \frac{\hbar\omega_i}{e^{\beta\hbar\omega_i}-1}$, where $U_0 = \phi_0 + \sum_i \frac{\hbar\omega_i}{2}$. Generally, ϕ_0 is negative, and its absolute value is greater than the zero-point energy, so we obtain a negative binding energy U_0 which is independent of temperature. The second term on the right side of the equation represents the energy of the thermal motion of atoms. To obtain the specific result for internal energy, we need to find the frequency spectrum of the simple harmonic motions. Debye regarded the solid as a continuous elastic medium and gave a spectrum of the system. The number of simple harmonic motions in the range of ω to $\omega + d\omega$ is $D(\omega)d\omega = B\omega^{2d}\omega$, where c_1 and c_2 represent the propagation velocities of longitudinal and transverse waves respectively. In addition, since such a system has only $3N$ simple harmonic vibrations, there must be an upper frequency limit, so the following formula holds: $\int_0^{\omega_D} B\omega^{2d}\omega = 3N$. From this, we get $\omega_D = \left(\frac{9N}{B}\right)^{1/3}$. As can be seen from this expression, ω_D is related to the density of atoms and the velocity of elastic waves, a fact that is important for understanding the relationship between scale and ω_D when we apply it to the background of the universe.

Using the Debye spectrum, the internal energy can be expressed as $U = U_0 + B\int_0^{\omega_D} \frac{\hbar\omega^3}{e^{\hbar\omega/kT}-1}d\omega$. In statistical physics, this integral has different results at different temperatures. When the temperature of the system is much higher than the characteristic temperature $\theta_D = \hbar\omega_D/k$, i.e., $T \gg \theta_D$, the result is $U = U_0 + 3NkT$. At low temperatures $T \ll \theta_D$, the internal energy of the system is approximately equal to $U = U_0 + \frac{3\pi^4}{5}NkT\left(\frac{T}{\theta_D}\right)^3$. The result at low temperature indicates that the thermal motion part of the internal energy is limited by the characteristic temperature θ_D, a property that guarantees the stability of strongly interacting systems. Similarly, stability can also be guaranteed when we apply the above formula to the background of the universe.

Now we have reviewed the process of obtaining the internal energy expression for a strongly interacting system in statistical physics. As mentioned above, we regard space-time as a physical object similar to a strongly interacting system; therefore, the expression for internal energy will be used to represent the internal energy of the background. We will see that the thermal motion part of the internal energy plays a crucial role in the production of dark energy.

III. Minimum Energy Limit

Let us study the internal energy expression from the viewpoint of cosmology. As mentioned above, space-time is treated as a discrete system with strong interaction between each small voxel, similar to general solids; thus the internal energy expression can also represent the internal energy of space-time. The first term on the right side, U_0, is the binding energy of the background of the universe. We assume that, in general, this part of the energy does not interact with the particles moving on the background, so it does not contribute to gravity. The last term represents the energy of the thermal motion of the voxels, where T is the temperature when the background and particles are in thermal equilibrium. This means that there is energy exchange between the background and the particles. Assuming space-time is composed of N voxels, then the thermal motion energy of one voxel is $\epsilon_{voxel} = \frac{\pi^4}{5} T \left(\frac{T}{\theta_D}\right)^3$, where θ_D is determined by the number density of voxels and the parameter B.

In this article, the scale of voxels is at the order of the Planck scale ℓ_p, thus the corresponding number density $n = T_p^3$, where T_p is the Planck temperature. Then the expression for θ_D can be re-expressed as $\theta_D = \left(\frac{9N}{B}\right)^{1/3} \frac{\hbar}{k}$. We suppose that the speeds of transverse and longitudinal waves c_1 and c_2 are equal to the speed of light, i.e., $c_1 = c_2 = c = 1$. In fact, as long as c_1 and c_2 are of order c, the choice of their values has little effect on the result. As we can see from the above formula, $\theta_D \approx T_p$, which means that the relationship $T \ll \theta_D$ always holds during most of the universe's history, so the low-temperature expression is the correct one for the background of the universe. Substituting the result into the voxel energy expression, a more specific energy expression can be obtained: $\epsilon_{voxel} = \frac{\pi^4}{5} \frac{T^4}{T_p^3}$. This means that, on cosmological scales, there is a vibration of order T^4/T_p^3 at every point in the background. Unlike the general case where only matter exists, the vibration of voxels in the background will have an important effect on the movement of particles because the particles are embedded in space-time. Therefore, it is reasonable to believe that there is a minimum value for the energy of particles given by the vibration of the background. Particles may not appear below this energy limit.

To reflect the energy exchange caused by the minimum energy limit, it is useful to calculate the number of particles. We focus on the period after the reheating of the electroweak transition is completed, when the temperature of the universe is much higher than the rest mass of most particles and radiation is dominant. According to statistical physics, the state density of relativistic particles is given by $D(\epsilon)d\epsilon = \frac{gV}{2\pi^2\hbar^3 c^3} \epsilon^{2d}\epsilon$. Assuming that the particles follow the Boltzmann distribution, in the presence of a minimum energy limit, the new number of particles is expressed as $\tilde{N} = N - \int_0^{\gamma\epsilon_c} D(\epsilon) e^{-\alpha-\beta\epsilon} d\epsilon$, where γ is a coupling constant that indicates the correlation between particles and space-time, with a value around 1, and ϵ_c is the characteristic energy obtained from the voxel energy expression.

To evaluate this formula, the expression for $e^{-\alpha}$ must be known. It can be obtained from the normalization condition $\int_0^\infty D(\epsilon)e^{-\alpha-\beta\epsilon}d\epsilon = N$. Substituting the state density into this equation yields $e^{-\alpha} = \frac{\pi^2}{gV(kT)^3}N$. Then the number of particles with energy above $\gamma\epsilon_c$ is $\tilde{N} = N - N\left[1 - \frac{\tilde{\gamma}T^3}{T_p^3}\right]$, where $\tilde{\gamma} = \frac{\pi^4}{10}\gamma$. The Planck temperature is the upper limit of temperature allowed, and even at the beginning of the time we study, the temperature T, which is approximately 100 GeV, is much lower than the Planck temperature $T_p \approx 10^{19}$ GeV. Ignoring small terms, a simplified modified number of particles is obtained: $\tilde{N} \approx Ne^{-\tilde{\gamma}T^3/T_p^3}$. This formula means that the "real" particles in the universe are not all the particles (if there is a certain number of particles at the beginning of the Big Bang that remains roughly unchanged at later times), but are partly cut off by a very small energy limit that changes with T^3. As the temperature decreases, more and more particles are released, a process accompanied by the accumulation of energy. When $T \to 0$ and $\tilde{N} \to N$, this energy accumulation process is complete. In our study, this part of the energy is considered the source of dark energy, and according to our calculations, we will see that the energy density of this component is consistent with the energy density represented by the observed cosmological constant.

IV. Dark Energy

At the end of the previous section, we obtained a corrected number of particles. In this case, the corresponding particle number density is $\tilde{n} \propto T^{3e^{-\tilde{\gamma}T^3/T_p^3}}$. In the standard cosmological model, the volume of the universe is proportional to $a(t)^3$, which means that the particle number density $n \propto a(t)^{-3}$. Therefore, a corrected Hubble parameter is obtained: $\tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = \frac{\dot{a}}{a} + \frac{\tilde{\gamma}T^2\dot{T}}{T_p^3} = H + \frac{\tilde{\gamma}T^2\dot{T}}{T_p^3}$, where H is the Hubble parameter in the standard cosmological model.

Now we examine the influence of this modified Hubble parameter on the conservation of the energy-momentum tensor. Under the flat Friedmann-Lemaître-Robertson-Walker metric $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$, the conservation equation $\nabla^\mu T_{\mu\nu} = 0$ takes the form $\dot{\rho} + 3H(1+\omega)\rho = 0$, where $T_{\mu\nu}$ is the energy-momentum tensor of the perfect fluid, ρ refers to the energy density (including radiation and matter), and ω is the equation-of-state (EoS) parameter of the perfect fluid. Replacing the Hubble parameter H with the modified expression, one obtains a corrected conservation equation: $\dot{\rho} + 3H(1+\omega)\rho = -\tilde{\gamma}(1+\omega)\frac{\dot{T}}{T_p^3}\rho$.

Comparing with the standard conservation equation, a term representing the energy flow density appears on the right side of the new conservation equation. Defining $\dot{\rho}_\Lambda = -\tilde{\gamma}(1+\omega)\frac{\dot{T}}{T_p^3}\rho$, on the large scale of the universe, the decrease in temperature leads to a negative $\dot{T}$. If there is always $-1 < \omega < \frac{1}{3}$ in every period of the universe, $\dot{\rho}_\Lambda$ is always positive or close to zero. This means that this part of energy will gradually accumulate in some way and contribute to gravity, which we treat as the source of dark energy.

Now let us calculate the energy density accumulated by $\dot{\rho}_\Lambda$. We can write the following integral: $\rho_\Lambda = -\int_{t_{reh}}^{t_0} \tilde{\gamma}(1+\omega)\frac{\dot{T}}{T_p^3}\rho dt = -\int_{T_{reh}}^{T_0} \tilde{\gamma}(1+\omega)\frac{\rho}{T_p^3}dT$, where t_{reh} is the end time of the reheating period after inflation caused by the electroweak transition, and t_0 is the present time of the universe. T_{reh} and T_0 refer to the thermal equilibrium temperatures of the universe at the corresponding times. At time t_{reh}, the universe has just finished reheating, where the rest mass of most particles is below the temperature T_{reh} and the universe is dominated by radiation, with energy density $\rho = g_* T^4$, where $g_* \approx 100$ is the effective degeneracy factor during the radiation-dominated period [?].

At time t_0, the current universe is dominated by dark energy. Strictly speaking, the integral is not applicable to the non-radiation-dominated universe because the particle number correction was obtained from the condition of relativistic particles. To obtain the correct number of particles in the non-radiation period, one would need to replace the state density in the particle number expression. For example, for non-relativistic particles, the state density is $D(\epsilon)d\epsilon = \frac{gV}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}d\epsilon$. In this way, we would obtain a result slightly different from the radiation-dominated case, and this temperature-affected result would be smaller than during the radiation-dominated period. Today, since the equation-of-state parameter $\omega \to -1$ and the energy density is much smaller than in the early universe, $\rho_\Lambda(T_0)$ in the integral can be ignored (although this is inaccurate for today's universe). Thus the result for ρ_Λ is actually determined by the temperature at the initial moment. Substituting the radiation density and the values of each constant into the integral yields $\rho_\Lambda = \frac{g_*\tilde{\gamma}(1+\omega)}{4}\frac{T_{reh}^4}{T_p^3}$. Therefore, the cosmological constant Λ is $\Lambda = 8\pi G\rho_\Lambda = 2\pi G g_* \tilde{\gamma}(1+\omega)\frac{T_{reh}^4}{T_p^3}$.

This result is consistent with Perez's conclusion in [?], but from a completely different perspective. A slight difference from [?] is that the initial temperature we choose is the temperature after reheating, not the temperature T_{EW} of the electroweak transition. As mentioned above, T_{reh} is of order 100 GeV, but a more accurate value of T_{reh} is important for calculating the energy density of dark energy. Since T_{reh} is the temperature of the universe after reheating is completed, its value should be slightly lower than the temperature T_{EW} because of the expansion of the universe during reheating [?, ?]. According to our calculation, if the value of T_{reh} is between 20 GeV and 30 GeV, we can obtain a result for Λ close to the observed value of the cosmological constant Λ_{obs}.

V. Conclusions and Discussions

In this work, we studied the background of the universe as a physical system and further proposed that the background has microstructure at the Planck scale. Under the assumption that the background is composed of strongly interacting voxels, we used the Debye model and canonical ensemble theory to describe the thermal motion of these voxels. With consideration of the influence of the background on particle motion, we obtained a minimum energy limit for particles from the expression for internal energy. Just as the beach appears with the ebb tide of the sea, particles with lower energy gradually participate in contributing to gravity as the energy limit drops. These newly emerged particles provide a new source of energy for the universe, which we refer to as dark energy. According to the relationship between the particle number density and the scale factor, we obtained a new conservation equation and calculated the energy density of dark energy. The corresponding cosmological constant was also calculated and found to be consistent with the observed value.

The discussion in this article shows that the energy density of dark energy is largely determined by the temperature at the end of the reheating period, which means that dark energy may mainly come from accumulation in the early universe. As the energy density of other components decreases, dark energy gradually dominates the expansion of the universe. Furthermore, such a result arises from a minimum energy limit caused by the microstructure of space-time, which reveals that space-time may have a discrete structure at the Planck scale.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Note: Figure translations are in progress. See original paper for figures.