Research on Precision Methods for Seeing Estimation Based on Series Speckle Images: Postprint

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Abstract

The full width at half maximum (FWHM) method and the image motion method are commonly used for atmospheric seeing estimation. The accuracy of both methods is affected by telescope tracking errors, wind, and mechanical vibrations. Improving seeing estimation precision is of significant importance for high-resolution imaging, site selection, and observatory seeing monitoring. For the FWHM method, fitting long-exposure point-source stellar images with a two-dimensional Gaussian function and selecting the FWHM along an appropriate direction to estimate seeing, comparison with the spectral ratio method demonstrates effective mitigation of tracking errors, wind, and mechanical vibration effects. For the image motion method, employing Principal Component Analysis (PCA) to determine the variance of centroid position projections from a series of speckle images along a suitable direction for seeing estimation, comparison with the spectral ratio method shows effective reduction of tracking errors, wind, and mechanical vibration influences.

Full Text

Preamble

Seeing Estimation Accuracy Study Based on a Sequence of Speckle Images

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Abstract: The full-width at half-maximum (FWHM) method and image motion method are commonly used techniques for estimating atmospheric seeing.

However, the accuracy of both methods can be compromised by telescope tracking errors, wind, and mechanical vibrations. Improving seeing estimation precision is crucial for high-resolution imaging, site selection, and observatory seeing monitoring. For the FWHM method, we propose fitting long-exposure point-source stellar images with a two-dimensional Gaussian function and selecting an appropriate direction for FWHM measurement. Comparison with the spectral ratio method demonstrates that this approach effectively mitigates the effects of tracking errors, wind, and mechanical vibrations. For the image motion method, we employ principal component analysis to identify the direction of maximum variance in the centroid positions of a series of speckle images, then use the variance in the orthogonal direction to estimate seeing. Comparison with the spectral ratio method confirms that this strategy also significantly reduces the impact of tracking errors and environmental disturbances.

Keywords: seeing; FWHM; image motion method; tracking error

1. Introduction

Atmospheric turbulence is a primary factor degrading the imaging quality of large ground-based optical telescopes, causing random jitter, distortion, and fragmentation of images in the focal plane. To characterize the impact of atmospheric turbulence on imaging, D. L. Fried introduced the seeing parameter r_0 (also known as the Fried parameter or atmospheric coherence length) based on Kolmogorov' s statistical model of atmospheric turbulence. By definition, r_0 represents the limiting aperture of a ground-based telescope for achieving diffraction-limited imaging and quantifies the intensity of atmospheric turbulence. The r_0 parameter is critically important for site selection and high-resolution imaging applications. For instance, the theoretical transfer function model of speckle interferometry depends on the r_0 value, and the number of microlenses in a Shack-Hartmann wavefront sensor for adaptive optics is determined by r_0.

Numerous methods exist for estimating atmospheric seeing r_0. Some require specialized instruments, such as measuring the transverse jitter variance of circumpolar star trails, deriving seeing from temperature structure constants measured at different altitudes using radiosondes or acoustic radar, employing knife-edge techniques to analyze intensity variations between focal planes, analyzing interferometric fringes of atmospherically-distorted stellar images, calculating differential angle-of-arrival variance using two sub-apertures separated by more than r_0, or measuring scintillation intensity.

Other methods can estimate r_0 using speckle images obtained during regular observations (short-exposure images with exposure times comparable to the atmospheric coherence time). These include comparing the actual average long-exposure or short-exposure transfer function of a set of point-source images with theoretical models, deriving seeing from the ratio of actual to theoretical spectral ratios (widely used in high-resolution reconstruction of extended-source images, referred to herein as the spectral ratio method), inferring seeing from contrast changes caused by atmospheric turbulence, directly estimating seeing from the FWHM of long-exposure point-source images, and calculating seeing from image jitter variance (referred to herein as the image motion method).

Among these techniques, the FWHM and image motion methods offer relatively convenient and rapid estimation of the seeing parameter and are widely employed. However, when telescopes are affected by tracking errors, wind, or mechanical vibrations (collectively referred to as comprehensive factors), both methods suffer reduced estimation accuracy.

This paper examines the fundamental principles of the FWHM method, analyzes its application conditions and the nature of disturbances caused by comprehensive factors, and proposes mitigation strategies. We similarly investigate the image motion method, its susceptibility to tracking errors, and corresponding solutions. We then apply these corrective methods to actual observational data, analyze the results, and validate their effectiveness through comparison with the spectral ratio method.

Within the isoplanatic patch, the atmosphere-telescope combined system can be treated as a linear space-invariant system. The short-exposure imaging process can be expressed in the spatial domain by Equation (1) and in the frequency domain by Equation (2):

$$i(x) = o(x) * s(x) \tag{1}$$

$$I(q) = O(q)S(q) \tag{2}$$

where $i(x)$ represents the short-exposure image, $I(q)$ its frequency domain representation, $o(x)$ the object, $O(q)$ its Fourier transform, $s(x)$ the point spread function (for point sources, $i(x) = s(x)$), $S(q)$ the short-exposure atmosphere-telescope system transfer function, q the normalized spatial frequency, and $*$ denotes convolution.

The short-exposure atmosphere-telescope system transfer function can be decomposed into the product of the telescope transfer function and the short-exposure atmospheric transfer function:

$$S(q) = T(q)B(q) \tag{3}$$

2.1 FWHM Method

The essence of the FWHM method lies in the simple relationship between the full-width at half-maximum of a long-exposure point-source stellar image and the seeing parameter:

$$\mathrm{FWHM} = 0.98\frac{\lambda}{r_0} \tag{4}$$

where λ is the wavelength. Long-exposure images can be obtained through statistical averaging of multiple short-exposure images. Since the intensity distribution of long-exposure stellar images approximates a Gaussian function, Gaussian fitting is commonly employed to calculate the FWHM. The Gaussian function has a simple relationship between its FWHM and standard deviation:

$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\,\sigma \tag{5}$$

Analyzing the ratio of the resolution of the long-exposure atmosphere-telescope system to atmospheric resolution as a function of D/r_0 (where resolution is defined as the integral of the modulation transfer function in the frequency domain) reveals that when the telescope aperture D is much larger than the seeing parameter r_0, the system resolution approaches the atmospheric resolution. Therefore, when $D \gg r_0$, the long-exposure system transfer function can be equated to the long-exposure atmospheric transfer function:

$$\langle S(q)\rangle_{\mathrm{LE}} \approx \langle B(q)\rangle_{\mathrm{LE}} = \exp\left[-3.44\left(\frac{qD}{r_0}\right)^{5/3}\right] \tag{6}$$

where $\langle\cdot\rangle$ denotes ensemble averaging, $\alpha = r_0/D$, and LE indicates long exposure.

To obtain the FWHM of the spatial domain long-exposure point spread function $\langle s(x)\rangle_{\mathrm{LE}}$, one must perform an inverse Fourier transform of Equation (6) and then extract its FWHM. However, the 5/3 power exponent makes analytical solution difficult. An approximate approach replaces the 5/3 power with a square power, allowing the transfer function to be approximated by a Gaussian function and yielding the analytical FWHM expression in Equation (4).

This derivation involves two approximations: (1) substituting the atmospheric transfer function for the system transfer function, and (2) replacing the 5/3 power with the square power. To assess how these approximations affect the accuracy of Equation (4), we numerically calculated the theoretical FWHM of the long-exposure system point spread function and compared it with the FWHM from Equation (4). The results are shown in Figure 1 [Figure 1: see original paper].

Figure 1 demonstrates that Equation (4) achieves high accuracy only when $D \gg r_0$. For example, when $D = 1$ m and $r_0 = 10$ cm, the FWHM ratio reaches 0.93, indicating that for meter-class telescopes, Equation (4) already provides high theoretical accuracy. Therefore, we employ Equation (4) for r_0 estimation.

The above analysis assumes an ideal telescope: no aberrations, no tracking errors, no wind loading, and a perfectly rigid support system. In reality, these factors enlarge the actual long-exposure point-source stellar image, causing Equation (4) to underestimate r_0.

When affected by comprehensive factors, long-exposure images often elongate in a particular direction, exhibiting an elliptical intensity distribution. Selecting an appropriate direction for FWHM calculation becomes crucial. Our solution involves fitting the affected long-exposure point-source image with a two-dimensional Gaussian function and identifying the direction of maximum variance as the most disturbed direction. The 2D Gaussian function for fitting is:

$$f(x_1, x_2) = A \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}\right]\right\} \tag{7}$$

with the corresponding covariance matrix:

$$\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \tag{8}$$

In most cases, the correlation coefficient ρ is non-zero, making the covariance matrix non-diagonal. We diagonalize the covariance matrix to identify the most affected direction, then use the smaller variance from the diagonal matrix (the direction perpendicular to the most affected direction) to calculate the FWHM and estimate r_0 using Equation (4), as illustrated in Figure 2 [Figure 2: see original paper].

2.2 Image Motion Method

Atmospheric turbulence affects imaging by altering the wavefront arrival angle at the pupil, causing random image jitter in the focal plane. Measuring the intensity of this jitter indirectly yields the seeing parameter r_0. D. L. Fried established the relationship between r_0 and wavefront arrival angle variance:

$$r_0 = \left[0.358\frac{\lambda^2}{\sigma_m^2}D^{-1/3}\right]^{3/5} \tag{9}$$

where D is the optical system aperture and σ_m is the wavefront arrival angle standard deviation.

For point-source stellar images, centroid position variations correspond to pupil plane wavefront arrival angle fluctuations. Therefore, we can estimate r_0 from the centroid jitter variance. The centroid calculation method is:

$$X = \frac{\sum_{x,y} x \cdot I(x,y)}{\sum_{x,y} I(x,y)}, \quad Y = \frac{\sum_{x,y} y \cdot I(x,y)}{\sum_{x,y} I(x,y)} \tag{10}$$

where (x, y) are pixel coordinates and $I(x, y)$ is the pixel value at that location.

The image motion method is also affected by comprehensive factors. Assuming the centroid position under pure atmospheric turbulence is X_A and the offset caused by comprehensive factors is X_V, with X_A and X_V being independent, the actual centroid position is $X = X_A + X_V$. The variance of the actual centroid position is $D_X = D_{X_A} + D_{X_V}$, so $D_X \geq D_{X_A}$. Consequently, comprehensive factors generally increase image jitter variance, leading to underestimated r_0 values.

Exposure time per frame also affects the image motion method. Ideally, exposure duration should be less than or equal to the atmospheric coherence time, with each speckle image capturing a different "frozen" atmospheric state, resulting in varying centroid positions. As exposure time increases, centroid positions become smoothed, reducing jitter variance and causing overestimated r_0 values.

Telescope aperture size also influences the image motion method. H. M. Martin suggested that image motion is primarily caused by turbulent cells larger than the aperture, while small-scale turbulence, though creating speckle structure, is averaged out within the aperture, producing minimal centroid offset. G. Ricort used a 40 cm telescope and found image motion estimates were 1.4 times those from the contrast method. P. R. Goode used a 65 cm telescope and obtained results 1.5 times larger than the spectral ratio method. Bin Ma used a 50 cm telescope and found star-trail method results (essentially image motion) were larger than DIMM measurements. These studies indicate that large-aperture telescopes tend to overestimate seeing with the image motion method.

Equation (9) was derived using the Kolmogorov spectrum without considering the outer scale of atmospheric turbulence. Numerical simulations using the Von Karman spectrum show that when finite outer scale is considered, centroid jitter variance decreases, leading to overestimated seeing.

Low image signal-to-noise ratio affects centroid calculation due to CCD readout noise, compromising seeing estimation accuracy. Common mitigation strategies include: (1) thresholding–setting a threshold (typically a fraction of the maximum image value) and using only pixels above this threshold for centroid calculation; and (2) windowing–defining a window and using only pixels within it. However, applying thresholding directly to a set of images reveals that threshold value significantly affects estimation results, as shown in Figure 3 [Figure 3: see original paper].

J. Vernin et al. analyzed CCD noise effects on centroid positions, finding that centroid jitter variance scales with window size to the fourth power, making it a severe influencing factor. Thresholding effectively limits window size, while windowing directly restricts it to a smaller range.

We combine windowing and thresholding approaches. First, we window the image centered on its maximum value with diameter W_D, defined as the diameter where intensity drops to $1/e$ of the maximum when approximating the point spread function with a Gaussian—similar to the definition of the rms width for a diffraction-limited PSF. Theoretically, $W_D \approx 1.2 \times \mathrm{FWHM}_{\mathrm{PSF}}$. Then, following Sandrine Thomas and Chao Li et al., who found that using three times the readout noise as a threshold effectively reduces readout noise impact, we adopt this threshold for centroid calculation. Readout noise can be determined from analysis of dark frame images.

When affected by comprehensive factors, the centroids of a series of point-source speckle images often elongate in a particular direction, exhibiting larger jitter variance in that direction. Our strategy employs principal component analysis (PCA) to identify the direction of maximum variance (the first principal component) as the most affected direction. We then project the coordinate data onto the second principal component (orthogonal to the first) and use its variance to estimate r_0. In the case shown in Figure 4 [Figure 4: see original paper], we would use the variance in the y direction to estimate r_0.

PCA is a widely used dimensionality reduction technique with applications in astronomical data processing, typically projecting data onto the first n directions of maximum variance to preserve principal features. Our approach, conversely, projects two-dimensional centroid coordinate data onto the direction of minimum variance. The algorithm implementation is:

1. Form matrix X from all centroid coordinates: $X = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ y_1 & y_2 & \cdots & y_N \end{bmatrix}$
2. Zero-mean each row: $X_{\mathrm{new}} = X - \bar{X}$
3. Compute covariance matrix: $C = \frac{1}{N} X_{\mathrm{new}} X_{\mathrm{new}}^T$
4. Diagonalize C to obtain C'
5. The diagonal elements of C' represent variances in the first and second principal component directions; the smaller variance is our target

Although we select the direction of minimum variance, we cannot assume it is completely unaffected. It is necessary to limit the total observation time, as wind direction and other factors may change dramatically over long periods, affecting all directions. We can assess whether the selected direction remains significantly disturbed by: projecting centroid coordinates onto the second principal component, representing them as a histogram, fitting with a Gaussian function, and examining the coefficient of determination. In the absence of any disturbances, centroid position variations should follow a Gaussian distribution, and a minimally affected direction should also approximate this distribution. In practice, we can impose a threshold on the coefficient of determination—for example, only applying our method when $R^2 > 0.8$; otherwise, the image set should be discarded. Figure 5 [Figure 5: see original paper] shows a histogram of centroid projections onto the second principal component.

2.3 Spectral Ratio Method

The spectral ratio method, proposed by O. von der Lühe, estimates seeing by computing the ratio of the squared average spectrum to the average energy spectrum of a set of images, then comparing this ratio with theoretical values. For a given telescope, the theoretical value depends only on the seeing parameter r_0. The expression is:

$$R(q) = \frac{\langle |S(q)|^2 \rangle}{\langle S(q) \rangle^2} \tag{11}$$

where $\langle S(q) \rangle$ can refer to either the average long-exposure or short-exposure transfer function. When referring to the average short-exposure transfer function, all images are aligned by their centroids before calculation, making the method immune to tracking errors. The theoretical expression for the average short-exposure transfer function is:

$$\langle S(q) \rangle_{\mathrm{SE}} = \exp\left[-3.44 \left(\frac{qD}{r_0}\right)^{5/3}\right] \tag{12}$$

The theoretical expression for the average energy spectrum is:

$$\langle |S(q)|^2 \rangle = \frac{1}{N} \int S(y,q) S(y,q)\, dy \tag{13}$$

where N is a normalization constant, y and q are vectors with modulus ranging from 0 to 1, and $S(y,q)$ represents the intersection area of four circles of radius 1. Clearly, for a given telescope, $\langle |S(q)|^2 \rangle$ depends only on r_0, as does $\langle S(q) \rangle_{\mathrm{SE}}^2$. Therefore, the theoretical ratio R is a function of r_0. For any given r_0 value, the corresponding theoretical spectral ratio curve can be calculated and compared with the actual spectral ratio curve to estimate the seeing parameter.

3.1 Data

To validate the effectiveness of our methods for mitigating comprehensive factor effects, we conducted experiments using two sets of observational data.

The first dataset consists of 25 groups of data (200 speckle images per group) acquired by the 1-meter New Vacuum Solar Telescope (NVST) at Fuxian Lake between UT 18:48:28 and UT 19:12:03 on September 12, 2014. Detailed parameters are listed in Table 1 .

Table 1: Experiment Parameters (NVST)

Parameter	Value
Aperture Diameter	0.98 m
Central Wavelength	705.8 nm
Pixel Size	6 m / 0.04 arcsec
Target	HIP9487
Region of Interest Size	192$×$192 pixels
Exposure Time	20 sec
Duration of Observation	23 min 34 sec
Number of Groups	25
Frames per Group	200
Duration per Group	~23 sec
Zenith Angle	23°4 59 -25°33 55

The second dataset comprises 95 groups of images (500 frames per group) of 10 targets observed with the 2.4-meter telescope at Gaomeigu Observatory between UT 12:49 and UT 20:46 on February 9, 2009. Details are provided in Table 2 .

Table 2: Experiment Parameters (2.4m Telescope)

Parameter	Value
Aperture Diameter	2.4 m
Central Wavelength	550 nm
Pixel Size	0.021 arcsec
Targets	HIP30960, HIP25245, HIP32993, HIP43635, HIP48670, HIP53360, HIP64543, HR4300, HR4335, HR4518
Region of Interest Size	256$×$256 pixels
Exposure Time	<10 sec
Duration of Observation	7 hours 57 minutes
Number of Groups	95
Frames per Group	500
Zenith Angle	8°16 12 -21°4 3

All images were preprocessed using the method described in reference [**?**]. For each NVST dataset, we cropped images into 192$×192*pixelsub* − *imagescenteredonthemaximumvalueofanarbitraryframe*(*thefirstframeinthisstudy*)*toavoidnon*− *isoplanaticeffectsandaccelerateprocessing*.*FortheGaomeigu*2.4*mtelescopedata*, *whichwerealreadyreadouta* arcsec sub-images, no additional cropping was necessary. Figure 6 [Figure 6: see original paper] shows examples of preprocessed speckle images.

3.2 Processing and Analysis

We processed the data using all three methods, with the detailed workflow shown in Figure 7 [Figure 7: see original paper]. For the FWHM method, after preprocessing, all speckle images were summed to create an equivalent long-exposure image, which was then fitted with a 2D Gaussian function. The appropriate direction for FWHM measurement was determined using the method described above to estimate seeing. For the image motion method, centroid coordinates were calculated for all images, projected onto the second principal component using the Section 2.2 procedure, and represented as a histogram. If the coefficient of determination $R^2 > 0.8$, the variance in this direction was used to estimate seeing; otherwise, the image set was discarded. This criterion also applied to the FWHM method. For the spectral ratio method, all point-source images were centroid-aligned, then used to compute the average short-exposure spectrum and average energy spectrum to obtain the actual spectral ratio, which was compared with theoretical ratios to find the best-matching r_0 value.

For convenient comparison, all estimated r_0 values were reduced to zenith angle 0° and wavelength 500 nm using:

$$r_0(500\text{nm}, 0°) = r_0(\lambda, \gamma) \left[\frac{500}{\lambda}\right]^{6/5} [\cos\gamma]^{3/5} \tag{14}$$

where λ is the wavelength and γ is the zenith angle.

As a baseline comparison, we also estimated seeing using variance in an arbitrarily selected direction (the longitudinal direction). For NVST data, seeing estimates using longitudinal variance are shown in Figure 8 [Figure 8: see original paper], while those using our corrected methods appear in Figure 9 [Figure 9: see original paper]. For the Gaomeigu 2.4m telescope data, longitudinal variance results are in Figure 10 [Figure 10: see original paper], and corrected method results in Figure 11 [Figure 11: see original paper].

For the NVST data, the corrected FWHM method yielded mean $r_0 = 9.96$ cm ($\sigma = 0.73$ cm), the corrected image motion method gave mean $r_0 = 17.44$ cm ($\sigma = 2.12$ cm), and the spectral ratio method produced mean $r_0 = 11.04$ cm ($\sigma = 0.62$ cm). For the Gaomeigu 2.4m telescope data, the corrected FWHM method gave mean $r_0 = 7.13$ cm ($\sigma = 0.82$ cm), the corrected image motion method yielded mean $r_0 = 13.48$ cm ($\sigma = 2.0$ cm), and the spectral ratio method produced mean $r_0 = 7.88$ cm ($\sigma = 0.97$ cm).

These results reveal several key findings:

1. **FWHM method underestimates seeing** compared to the spectral ratio method. Possible reasons include: (a) the derivation of Equation (4) inherently yields slightly low r_0 estimates, and (b) residual telescope aberrations, tracking errors, and wind/mechanical vibrations still enlarge the FWHM, further reducing the estimated r_0.

2. **Image motion method overestimates seeing** relative to the spectral ratio method. Likely causes include: (a) excessive exposure time smoothing centroid motion and reducing measured variance, (b) large telescope aperture making image motion more sensitive to turbulence cells larger than the aperture, (c) neglect of finite outer scale effects, and (d) residual CCD and photon noise affecting centroid variance calculations.

3. **Longitudinal variance underestimates** r_0 compared to our corrected method because it includes more tracking error, wind, and vibration effects that increase variance and thus reduce the estimated r_0.

4. **Correlation improvements**: For NVST data, the correlation coefficient between corrected FWHM and spectral ratio methods is 0.79 (vs. 0.39 for longitudinal FWHM), and between corrected image motion and spectral ratio methods is 0.52 (vs. -0.01 for longitudinal variance). For Gaomeigu 2.4m data, these correlations are 0.91 (vs. 0.58) and 0.65 (vs. 0.24), respectively. This demonstrates that when a speckle image sequence is affected by comprehensive factors but our selected direction is not severely impacted (minimum $R^2 > 0.8$), our method significantly improves correlation with the spectral ratio method and enhances seeing estimation accuracy for both FWHM and image motion techniques.

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4. Summary and Outlook

This paper analyzed factors affecting the accuracy of FWHM and image motion methods, proposed techniques to mitigate tracking error, wind, and mechanical vibration effects, and validated them using observational data. Our preliminary conclusions are:

1. The FWHM method tends to underestimate seeing compared to the spectral ratio method, likely due to inherent approximations in Equation (4) and residual telescope aberrations and environmental disturbances.

2. The image motion method tends to overestimate seeing relative to the spectral ratio method, possibly because of excessive exposure time, large telescope aperture, finite outer scale effects, or residual CCD and photon noise.

3. When a speckle image sequence exhibits elongation in a particular direction due to tracking errors, wind, or mechanical vibrations, our method

of selecting an appropriate direction for seeing estimation effectively mitigates these influences. The selection criteria are: (a) minimal variance in the chosen direction, and (b) centroid coordinate projections in this direction remain well-described by a Gaussian distribution.

Our proposed methods reduce the impact of tracking errors and environmental disturbances on FWHM and image motion techniques, improving their correlation with the spectral ratio method. However, systematic biases remain among the three methods' r_0 estimates. Future work will focus on identifying the causes of these biases, quantifying relationships between influencing factors and bias magnitude, and developing compensation methods.

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Note: Figure translations are in progress. See original paper for figures.