The Buckling Landscape of Coke-Can Cylindrical Shells Under the Combined Action of Axial Compression-Torsion-Lateral Poking-Internal Pressure

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Abstract

This study investigates the buckling of cylindrical shells using the nonlinear finite element analysis software ABAQUS, with specific application to the buckling analysis of cola cans. First, the buckling experimental results for cola cans reported by Virot et al. are numerically verified. Subsequently, to obtain universal insights into buckling, the buckling behavior of cylindrical shells is further examined through detailed analysis of cylindrical shell structures under various load combinations and geometric parameters. To facilitate intuitive discussion, this paper constructs a three-dimensional landscape diagram of external force-buckling load-displacement for cylindrical shell structures under combined lateral-axial pressure loading (referred to as landscape). The results demonstrate that under combined lateral-axial pressure-torsional loading, the force-displacement curve of the specimen exhibits a 'cliff' phenomenon. The application of torsional loading compromises the overall stability of the specimen and induces sensitivity to initial imperfections. For specimens subjected to axial pressure-torsion, this paper defines the zero-bearing-capacity plane as 'sea level' to distinguish failure modes. Through analysis of specimens with different boundary conditions, it is found that clamping both ends of the specimen can effectively increase structural load-bearing capacity and enhance stability. Internal pressurization of the cylindrical shell structure can substantially enhance both load-bearing capacity and stability while reducing sensitivity to initial imperfections.

Full Text

The Buckling Landscape of Coke-Can Cylindrical Shells Under the Combined Action of Axial Compression-Torsion-Lateral Poking-Internal Pressure

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This paper uses the nonlinear finite element analysis program ABAQUS to study the buckling of cylindrical shells, and applies it to the buckling analysis of coke-can cylindrical shells. First, the coke-can buckling experimental results of Virot et al. are verified through numerical simulation; then, in order to obtain some general results on buckling, the buckling behavior of cylindrical shells is further examined. A detailed analysis is carried out for cylindrical-shell structures under

different load combinations and different geometric parameters. For an intuitive discussion, this paper plots a three-dimensional buckling landscape of external force-buckling load-displacement for a cylindrical-shell structure subjected to lateral pressure-axial compression loading (referred to as the landscape). The results show that, under the action of lateral pressure-axial compression-torsional loading, the force-displacement curve of the specimen exhibits a "cliff" phenomenon; the application of torsional loading is unfavorable to the overall stability of the specimen and causes the specimen to be sensitive to initial imperfections; for specimens subjected to axial compression-torsion, this paper defines the plane with zero bearing capacity as the "sea level" to distinguish the failure modes of specimens; analysis of specimens with different boundary conditions reveals that fixing both ends of the specimen can effectively improve the load-bearing capacity of the structure and enhance stability. Inflating the interior of the cylindrical-shell structure can greatly improve the load-bearing capacity and stability of the structure and reduce its sensitivity to initial imperfections.

The aim of this work is to investigate the buckling of the cylindrical shells based on the non-linear finite element analysis program ABAQUS and applied to the buckling analysis of cola cans. Buckling analysis of soda cans by means of comparing the numerical results with experimental data[34]. In order to obtain some qualitative results of buckling, the buckling behaviour of the cylindrical shells will be investigated. In this article, we focus on the effect of different load combinations and different geometric parameters of the cylindrical shells. The straightforward, simple analysis of the buckling of the cylindrical shells under the axially compressed-lateral perturbation load is presented. We show that the three-dimensional curve of external force-buckling load-displacement called landscape. The numerical results indicate that: the phenomenon of "cliff" appears in the force-displacement curve of specimens under the action of lateral pressure-axially compressed-torsional load; It will be appreciated that the torsional is not conducive to the stability of the specimen and makes the specimen sensitive to the initial imperfections; For specimen under axially compressed-torsional load, in this paper, the plane with zero bearing capacity is defined as "sea level" to distinguish failure modes of specimens; The results of specimens with different boundary conditions shows that the bearing capacity of the cylindrical shells can be improved with fixed boundaries. The internal pressure can greatly improve the bearing capacity and stability of the structure and reduce the imperfection-sensitivity.

Keywords: cylindrical shells, buckling, buckling landscape, coke can, finite element

cylindrical shells,buckling,landscape,cola cans,numerical simulation

I. Introduction

The instability phenomenon of structures under certain loads and boundary conditions is called structural buckling or the stability problem. Ever since Euler (1744) [1, 2, 28] began studying the stability problem of slender elastic compression rods (which Euler called Elastica), structural stability has become a problem that must be considered in structural design. In engineering practice, people have found that when slender and thin-walled structures lose stability, their load-bearing strength is usually far below the material' s buckling stress. At first this phenomenon disturbed the traditional way of thinking, namely the design principle based on material strength. Later, it was recognized that, under the same loading conditions, structures with different geometric configurations may have completely different stability characteristics; this is especially prominent for slender, thin-walled structures, where structural stability problems are particularly significant [6, 13, 14, 16, 17, 23–28, 41, 42].

With the development of new materials, high-strength, high-toughness metamaterials have gradually begun to be used, making it possible for structures that originally had to be constructed from large-sized materials to instead be made from metamaterials; under the same load-bearing capacity, the components constituting the structure can become increasingly thin, thereby making research on structural stability problems increasingly important. Conversely, research in structural mechanics has also inspired and promoted the design of metamaterial cells; that is, from the perspective of structural mechanics, cells possessing the required mechanical performance are designed to improve the overall properties of the material, while the properties of the matrix material remain unchanged, by designing microstructures to achieve special overall material properties. Thus metamaterials are also called materials with microstructures [45–50]. In this sense, research in structural mechanics has promoted the design of high-performance materials.

Regarding the understanding of structural stability, the large discrepancy between early theoretical predictions and experiments [3–7, 39] led people to begin paying attention to the importance of research on stability theory. For this reason, different stability theories were proposed in an attempt to explain the gap between theoretical predictions and experiments, such as Karman–Tsien (Kármán–Tsien) [[unclear: text continues on next page]]

Sen' s nonlinear large-deflection theory [3–5, 7], Stein' s nonlinear pre-buckling-consistent theory [39], and Koiter' s post-buckling theory considering initial imperfections [6]. Unfortunately, although people have already racked their brains to consider various factors, there is still a certain gap between current theoretical predictions and reality. As a result, in practical engineering one still does not dare to carry out structural design completely according to the predictions of stability theory; instead, relatively large knockdown factors have to be adopted. For example, the National Aeronautics and Space Administration (NASA) of the United States has a dedicated research program on structural knockdown factors [10, 43, 44]. For the buckling problem of cylindrical shells, the empirical knockdown-factor formula fitted by NASA from experiments in 1965 is

Figure 1

Figure 1: Figure 1

$$\kappa = 1 - 0.901\left(1 - e^{-\frac{1}{16}\sqrt{\frac{R}{t}}}\right),$$

where R/t is the ratio of the cylindrical-shell radius to the thickness.

Figure 1: The knockdown factor published by NASA [10] in 1965:

$$\kappa = 1 - 0.901\left(1 - e^{-\frac{1}{16}\sqrt{\frac{R}{t}}}\right)$$

This formula indicates that typical structures generally buckle at 20–50% of the predicted buckling value; in the lower case, buckling occurs at only 20% of the prediction. Therefore, [44] points out that the buckling load knockdown factor proposed by NASA for calculating cylindrical-shell structures with initial imperfections is conservative. After half a century of development, using refined experiments and extensive computational analysis, although there have been some improvements in buckling loads, this empirical formula is still used at present [11]. It can be seen that any method capable of improving buckling capacity will have major practical engineering significance.

Because cylindrical-shell structures are light in weight and have high load-bearing capacity, they are widely used in aerospace, construction, and other fields, and therefore a particularly large amount of research has been carried out on them [6, 8, 9, 12–16, 16–22, 29, 30, 36–41, 51]. In the history of the development of structural buckling theory, as a milestone advance, Koiter [6] revealed that the initial geometric imperfections of a structure have a very large influence on structural buckling; in other words, the structure is highly sensitive to initial imperfections. Tiny dents on the structural surface can lead to a substantial reduction in structural load-carrying capacity. On this basis, Koiter [6] established the post-buckling theory of structural stability. To verify the influence of initial geometric imperfections on buckling, people have assumed geometric imperfections of different forms, and even proposed the problem of random imperfections.

Because cylindrical-shell structures are widely used, interest in the study of their buckling stability has not waned. V. Krasovsky et al. [31] adopted a combined experimental and finite-element approach and proposed a method of creating initial imperfections by using lateral perturbations. Waddy T. Haynie et al. [32] used a combined experimental and finite-element method to calculate the minimum buckling-load limit of structures when initial imperfections are created by lateral forces [42]. J. Michael [33] and others, on the basis of NASA experiments, proposed an experimental method in which, when a cylindrical shell is subjected to axial compression, a probe is used laterally to apply displacement loading to

Figure 2

Figure 2: Figure 2

the structure; this method was also used to study the impact sensitivity of cylindrical-shell structures. In particular, in 2017, Virot et al. [34] carried out a detailed experimental study of the buckling problem of Coca-Cola-can cylindrical shells under combined axial and lateral compression, attempting to find universal buckling laws from it. By studying empty easy-open cans of different brands, that work used the buckling landscape to investigate buckling intuitively, and pointed out that the stability of multidimensionally complex shells can be simplified into a low-dimensional description. The buckling landscapes obtained by Virot et al. [34] from limited experimental data have important significance. However, the loading condition in that literature included only one type—axial and lateral compression—and did not consider the combined effects of torsion and internal pressure. To further verify the buckling laws summarized by Virot et al. [34] from experiments, we believe it is necessary to study the stability of cylindrical shells under other load combinations.

Specifically, in this paper, on the basis of the work of Virot et al. [34], we do not conduct experiments, but instead attempt to study this problem from the perspective of numerical simulation. First, finite-element numerical calculations are used to verify the axial-lateral compression buckling results of Virot et al. [34]. After completing the verification of Virot et al. [34], we consider more load combinations, further investigating the buckling behavior of cylindrical shells under other load combinations. Through detailed calculations, we obtain the buckling landscapes of cylindrical shells under different combinations of loads such as lateral compression, axial compression, torsion, and internal pressure. According to the new description we propose, detailed stability analyses are carried out for the obtained buckling landscapes. In addition to analyzing the influence of parameters on buckling, we also propose ideas for improving and enhancing buckling capacity.

II. Geometric Schematic of the Buckling Landscape

In order to vividly express the new information we obtain from the buckling landscape, in this paper, in addition to inheriting the "ridge"and "valley"proposed by Virot et al. [34], we also modify their "lake" into the more appropriate "basin," and at the same time introduce new concepts such as "cliff," "sea level," and "hill," so as to better portray the buckling landscape (see Figure 2).

Figure 2: Schematic diagram of the buckling landscape, including "ridge," "valley," "lake," "basin," "cliff," "sea level," and "hill."

III. Finite-Element Simulation Verification of the Experimental Results of VIROT [34]

In this section, our purpose is to verify the experiments of Virot et al. [34] from a numerical perspective, so as to examine the feasibility of our finite-element model. To this end, we used the ABAQUS finite-element software to establish a finite-···

finite-element model. To fully reproduce the experiment, this study measured the dimensions of the pull-tab can used in the experimental study in the literature, and used the measured geometric dimensions to construct the finite-element model. The specific geometric parameters are shown in Fig. 3.

D(mm)	D1(mm)	D2(mm)	L(mm)	t(mm)
57.2	52.6	48.6	104	0.104

Figure 3: Geometric dimensions of the cola model

From Refs. [52]-[53], the pull-tab can is made of 3104 aluminum alloy, with an elastic modulus of $75000N/mm^2$, Poisson' s ratio of 0.3, and yield stress of $294.3Mpa$. In the finite-element model, discrete rigid bodies are used to simulate the upper and lower clamping plates and the lateral probe in the experiment. Plastic contact is defined between the top of the pull-tab can and the upper clamping plate, between the bottom of the pull-tab can and the lower clamping plate, and between the rigid sphere and the can body; the normal direction uses hard contact, the tangential direction uses the penalty method, and the friction coefficient is taken as 0.3. The pull-tab can is modeled using the shell element S4R in ABAQUS. The finite-element model of the above specimen is shown in Fig. 4:

Figure 4: Finite-element model of a cola can

Because, in the finite-element simulation, axial loading is first applied and then lateral loading is applied, when the applied axial force is 1084 N, the specimen undergoes a large overall compressive deformation, making it impossible to continue applying the lateral load. Therefore, finite-element numerical simulations are carried out only for the experimental specimens with axial compression of 136 N, 348 N, 467 N, 675 N, 683 N, and 859 N.

Using the finite-element model established above, nonlinear numerical analysis is performed for the six specimens in the experiment, yielding the lateral force-displacement curves of the pull-tab can under different axial compressive forces. The comparison with the lateral force–displacement curves obtained from the experiment is shown in Fig. 5. It can be seen from Fig. 5 that the overall trends of the finite-element simulation curves and the experimental curves are relatively close.

Figure 5: Comparison of experimental and finite-element curves, where exp denotes the experimental results of Virot et al. [34], and FE denotes our computational results. The buckling load of the cola pull-tab can is exp4 = 675 N

In the elastic stage, the experimental curves and the finite-element curves are relatively close. This indicates that the finite element can simulate the initial stiffness of the specimen fairly well. As the load displacement increases, the deformation of the specimen gradually increases. When the load carried by the specimen reaches the minimum value, the displacement of the finite-element model is larger than that of the experimental model. This is because, compared with the material constitutive model of the actual specimen, the bilinear constitutive model used in the finite element has a relatively large plastic buckling plateau, which causes the plastic development in the finite element to be greater than that in the experiment. In addition, unavoidable errors and random initial defects exist in the experiment, which causes the load-carrying capacity of the specimen in the experiment to degrade more rapidly. The displacement load corresponding to the minimum carrying capacity of the specimen in the experiment is slightly lower than that in the finite element, but the buckling load of the finite-element model is about 675 N, which is similar to the experiment.

Overall, the obtained buckling landscape of the cola pull-tab can under combined axial-lateral compression is consistent with that obtained by Virot et al. [34], and our nonlinear numerical results support the experimental results of [34]. Next, we further investigate the buckling landscapes for several other loading combinations.

IV. Basic parameters of the cola pull-tab can model and several loading boundary conditions

According to the experimental observations of Virot et al. [34], the buckling deformation of the pull-tab can is mainly concentrated in the can body. Therefore, in the finite-element parametric analysis, a simplified analysis method is adopted, in which the pull-tab can is simplified as a cylindrical shell structure for further analysis. To study the effects of different loading combinations, different boundary conditions, different radius-to-height ratios (R/L), different radius-to-thickness ratios (R/t), and internal pressurization of the can body on the buckling-landscape phenomenon.

This paper designs finite-element models with 4 different radius-to-height ratios (R/L), 3 different radius-to-thickness ratios (R/t), 3 different loading combinations, 1 different boundary condition, and 2 cases of internal inflation of the can body. The boundary conditions of the pull-tab can are defined as follows: when the specimen has no constraints with the upper and lower loading plates before loading, the boundary condition is free; when the upper and lower surfaces of the specimen have exactly the same degrees of freedom as the loading plates, it is regarded as the specimen being fixed. In the finite element, in order to fully simulate inflation inside the specimen, an equivalent method is adopted to equivalently convert the inflation inside the can body into pressure applied inside the can body.

strong, and the numerical value of the pressure is one standard atmosphere. The specific model parameters are shown in Table I.

Table I: Geometric parameters of different specimens

Name	Height (mm)	Radius (mm)	Wall thickness (mm)	Diameter-height ratio (R/L)	Diameter-thickness ratio (R/t)	Loading combination	Boundary condition	Whether the can is inflated
Specimen-1	100	30	0.1	0.3	300	Axial compression–lateral compression	Free	No
Specimen-2	110	30	0.1	0.27	300	Axial compression–lateral compression	Free	No
Specimen-3	120	30	0.1	0.25	300	Axial compression–lateral compression	Free	No

Name	Height (mm)	Radius (mm)	Wall thickness (mm)	Diameter-height ratio (R/L)	Diameter-thickness ratio (R/t)	Loading combination	Boundary condition	Whether the can is inflated
Specimen-4	130	30	0.1	0.23	300	Axial compression–lateral compression	Free	No
Specimen-5	90	20	0.1	0.22	200	Axial compression–lateral compression	Free	No
Specimen-6	90	22	0.1	0.24	200	Axial compression–lateral compression	Free	No
Specimen-7	90	24	0.1	0.27	220	Axial compression–lateral compression	Free	No

Name	Height (mm)	Radius (mm)	Wall thickness (mm)	Diameter-height ratio (R/L)	Diameter-thickness ratio (R/t)	Loading combination	Boundary condition	Whether the can is inflated
Specimen-8	90	20	0.1	0.22	200	Axial compression–lateral compression	Fixed	No
Specimen-9	90	20	0.1	0.22	200	Axial compression–lateral compression–torsion	Fixed	No
Specimen-10	90	20	0.1	0.22	200	Lateral compression–torsion	Fixed	No
Specimen-11	90	20	0.1	0.22	200	Axial compression–torsion	Fixed	No

Figure 6

Figure 3: Figure 6

Figure 7

Figure 4: Figure 7

Name	Height (mm)	Radius (mm)	Wall thickness (mm)	Diameter-height ratio (R/L)	Diameter-thickness ratio (R/t)	Loading combination	Boundary condition	Whether the can is inflated
Specimen-12	90	20	0.1	0.22	200	Axial compression–lateral compression	Fixed	Yes
Specimen-13	90	20	0.1	0.22	200	Axial compression–torsion	Fixed	Yes

V. Effect on buckling of cola cans under axial-lateral compression

Displacement-load curves and the three-dimensional landscape of (external force–buckling load–displacement): Specimen-1 to Specimen-7 all use the same loading method as in the experiments: first, axial pressure is applied in the vertical direction, with the axial force ranging from 100 to 1200 N; after the specimen has stabilized, a lateral displacement load is applied, with the displacement increment in each loading step being 0.16 mm. The lateral displacement, lateral force, vertical force, and vertical displacement of each specimen are denoted respectively as: D_p, F_p, F_A, and D_A. The lateral load-displacement curves of the specimens are shown in the following figures:

Figure 6: Specimen 1: two-dimensional curves and three-dimensional surface landscape of the buckling path

Figure 7: Specimen 2: two-dimensional curves and three-dimensional surface

Figure 8

Figure 5: Figure 8

Figure 9

Figure 6: Figure 9

landscape of the buckling path

Figure 8: Specimen 3: two-dimensional curves and three-dimensional surface landscape of the buckling path

Figure 9: Specimen 4: two-dimensional curves and three-dimensional surface landscape of the buckling path

Figure 10: Specimen 5: two-dimensional curves and three-dimensional surface landscape of the buckling path

Figure 11: Specimen 6: two-dimensional curves and three-dimensional surface landscape of the buckling path

Figure 12: Specimen 7: two-dimensional curves and three-dimensional surface topography of the buckling paths

It can be seen from the figure that all seven models exhibit the landscape phenomenon described in the literature, and the differences among the "ridge" and "valley" phenomena for the specimens are not large. According to the force–displacement curve of each specimen, the specimens are divided into three stages: the first stage is the elastic stage, in which the lateral load-bearing capacity continuously increases with increasing lateral displacement; the second stage is the buckling stage, in which the lateral load-bearing capacity of the specimen, with increasing lateral displacement, first increases, then decreases, and then increases again; the final stage is called the "lake" stage, in which the lateral load-bearing capacity of the specimen first increases and then decreases to zero. If loading continues, two situations may occur: one is that, as the displacement loading gradually increases, the load-bearing capacity of the structure begins to increase from zero; the other is that the load-bearing capacity remains zero and no longer increases. It can be seen from the figure that the axial-force ranges of the "lake" stage for Specimen-1–Specimen-4 are all "950N 1200N," and the axial-force ranges of the "lake" stage for Specimen-5–Specimen-7 are "1000N 1200N." Through comparison, it is found that, as the radius-to-height ratio (R/L) continuously decreases, the displacement at which the lateral load-bearing capacity of the specimen becomes zero continuously increases. For example, when

Figure 10

Figure 7: Figure 10

Figure 11

Figure 8: Figure 11

$N = 1050N$, for radius-to-height ratios $(R/L) = 0.3, 0.27, 0.25, 0.23$, the displacements at which the lateral load-bearing capacity is zero are respectively: $1.5mm, 1.8mm, 1.9mm, 2.1mm$. Moreover, from the three-dimensional curve plots it can also be seen that the "lake" region of the specimens increases as the radius-to-height ratio (R/L) continuously decreases. Analysis of Specimen-5–Specimen-7 shows that, as the radius-to-thickness ratio (R/t) continuously increases, the displacement at which the lateral load-bearing capacity of the specimen becomes zero continuously decreases. When $N = 1050N$, for radius-to-thickness ratios $(R/t) = 200, 220, 240$, the displacements at which the lateral load-bearing capacity is zero are respectively: $1.95mm$, $1.35mm$, $1.3mm$. In addition, from the three-dimensional curve plots it can also be seen that the red-enclosed region is the "lake" region of the specimen; as the radius-to-thickness ratio (R/t) continuously increases, the enclosed region continuously decreases, and the red region continuously decreases.

From the above analysis it can be concluded that structures with larger radius-to-thickness ratio (R/t) and radius-to-height ratio (R/L) have better stability, and the specimens are relatively insensitive to initial defects.

Discussion of instability modes: the failure modes of each specimen are shown in Figures 13–15. Specimens with different radius-to-height ratios and radius-to-thickness ratios have similar failure modes. When the axial pressure is relatively small, the lateral side of the specimen is squeezed by the rigid sphere, an inward dent occurs in the column wall at the contact point, and the overall stability of the specimen is good. As the axial pressure increases, relatively large deformation occurs in the column wall at the contact point. When the axial pressure reaches a certain value (about $950N$), when the specimen is laterally squeezed by the rigid sphere, local buckling occurs in the specimen, and a relatively large inward concave deformation occurs at the contact position between the rigid sphere and the column wall. Moreover, when the lateral load reaches a certain value, the lateral-load displacement no longer increases, but the deformation at the contact point continues to increase, and separation occurs between the loading point and the column wall (the "lake" phenomenon begins to occur). When the inward concavity of the column wall reaches a certain displacement, the specimen reaches a new equilibrium. Continuing loading causes the small sphere and the column wall to make contact a second time (at this point the "lake" phenomenon ends), and the local deformation of the specimen continues to increase, but overall buckling does not occur. Figure 15 shows the failure mode of the component when subjected to a relatively large axial pressure. Because of the large axial force, separation of the loading sphere from the column wall occurs very early in the specimen. In addition, the local buckling on the cylinder surface at this point gradually increases and spreads toward the cylinder top and cylinder foot; axial compression of the specimen ultimately leads to overall instability of the whole component.

Figure 13: Local buckling (left), separation of the sphere from the column wall (right)

Figure 14: Second contact between the sphere and the column wall (left); separation of the small sphere from the column wall when the axial force is relatively large (right)

Figure 15: Propagation of local buckling (left); overall compressive buckling (right)

VI. Influence of Axial-Lateral-Pressure–Torsion Loading on Buckling of the Cola Can

Displacement-load curves and three-dimensional topography of (external force–buckling load–displacement): On the basis of the specimens under axial–lateral pressure, torsion is considered in this paper, and a finite-element model under axial–lateral pressure–torsion is established. In order to apply the torsional load, in the finite element model the two ends of the specimen are respectively tied to the upper and lower rigid clamping plates, so that the boundary condition of the specimen is changed from a free end to fixed. In order to ensure the single-variable principle, the boundary conditions are changed on the basis of Specimen-5, and Specimen-8 is established for comparative analysis with Specimen-9, to study the landscape phenomenon of the cylindrical-shell structure under this condition. The geometric dimensions of Specimen-9 are the same as those of Specimen-8, and the applied torsional load is $5000N \cdot mm$, denoted as M_A. The obtained lateral force–displacement comparison curves are shown in the following figures:

Figure 16: Specimen 8: two-dimensional curves of the buckling path, and the three-dimensional curved-surface landscape

Figure 17: Specimen 9: two-dimensional curves of the buckling path, and the three-dimensional curved-surface landscape

From the curves it can be seen that, under axial compression–lateral compression–torsional loading, the specimen also exhibits the landscape phenomenon. Compared with the curves for specimens without torsion, under the combined action of axial compression–lateral compression–torsion, the shell structure shows a new phenomenon. In this paper this curve phenomenon is called a "cliff" ; the blank region below the "cliff," where the bearing force is zero, is defined as the "sea," and the "lake" region is redefined in this paper as the "basin." The reason for the occurrence of the "cliff" phenomenon is as follows: when the laterally loaded sphere is loaded to a certain displacement, the column wall jumps, the loading point separates from the column wall, and the lateral bearing force of the specimen becomes zero. This phenomenon forms the "basin" in the curve. As the load continues to increase, the sphere again contacts the column wall, and the lateral bearing force of the specimen begins to increase from zero. However, for the specimen under combined axial compression–lateral compression–torsion, after the specimen reaches the "basin" displacement, the existence of the torsional moment causes torsional failure of the specimen in the later stage of loading; the sphere separates from the column wall for a second time, and the bearing force of the specimen instantaneously drops to zero, thereby forming the "cliff" phenomenon in the curve. Compared with Specimen-8, it is found that the "valley" of Specimen-9 ends earlier and the "ridge" is relatively curved; the main reason is that the application of torsional loading increases the deformation of the specimen and advances the failure time. The analysis also shows that applying torsional loading in the finite element model has little influence on the strength of the specimen, but reduces its stability. Moreover, increasing the torsional load causes the specimen to enter the "basin" earlier; in the later stage of displacement loading, the specimen undergoes overall instability, and the bearing force instantaneously drops to zero.

Comparing Specimen-8 with Specimen-5 shows that, when the boundary conditions at both ends of the specimen change from free to fixed, the lateral force of the specimen is greatly increased; the time at which the specimen forms the "basin" is delayed, the area of the "basin" formed is smaller, and both the stability and the bearing capacity of the specimen are greatly improved. Fixing both ends weakens, to a certain extent, the sensitivity of the structure to initial imperfections.

Discussion of instability modes: Since many specimens were simulated, representative specimens were selected for comparative analysis. When $N = 1200N$, the failure modes of the specimens are compared in the figures. It can be seen from the figures that the maximum-stress cloud diagram of Specimen-8 is axisymmetric, and the maximum stress is distributed around the loading point as the center. Compared with Specimen-8, the maximum-stress cloud diagram of Specimen-9 is antisymmetric about the axis, and overall presents an "N" shape. This indicates that increasing the torsional load changes, to a certain extent, the force state of the specimen, thereby changing its failure state. When $N = 1200N$, Specimen-9 undergoes two separations between the small sphere and the column wall. The first separation forms the "basin" phenomenon in the curve, and the second separation forms the "cliff" phenomenon. In contrast, Specimen-8 does not undergo separation between the loaded sphere and the column wall during the entire loading process. This indicates that applying torsional loading weakens, to a certain extent, the stability of the specimen. In the later stage of loading, Specimen-9 undergoes overall torsional instability, whereas Specimen-8 undergoes only local buckling. This phenomenon also proves that the application of torsional loading has an adverse effect on the stability of the structure.

Figure 18: When $M_A = 1200N \cdot mm$, stress distributions of Specimen-8 (left) and Specimen-9 (right)

Figure 19: In the middle stage of loading, the small sphere contacts the column

wall in Specimen-8 (left), while separation between the small sphere and the column wall occurs in Specimen-9 (right)

Figure 20: In the later stage of loading, local buckling occurs in Specimen-8 (left), while overall torsional failure occurs in Specimen-9 (right)

VII. Influence of Lateral Compression–Torsion on Buckling of Cola Cans

Displacement-load curves and three-dimensional landscapes of (external force-buckling load-displacement): The figure shows the lateral force-displacement curves of Specimen-10. From the curves it can be seen that, when the torsional-load range is $1000N \cdot mm \sim 7500N \cdot mm$, the bearing capacity of the specimen gradually increases with the increase of displacement loading. When the torsional load is greater than $7500N \cdot mm$, the bearing capacity of the specimen first increases and then rapidly decreases. The three-dimensional curve diagram shows that, under different torsional loads, Specimen-10 exhibits the "cliff," described above, ⋯

"ridge" and "sea" phenomena. In contrast, Specimen-9 and Specimen-10 did not exhibit a "lake" phenomenon, and the "valley" phenomenon was also not very pronounced. This is because, when the specimen is subjected to lateral loading, the column wall at the loading point becomes indented under the action of the rigid small ball; however, because there is no vertical loading, the indented region of the column wall does not separate from the small ball and continue to deform. Instead, under the action of the torsional load, torsional deformation occurs at the indented region; as loading continues, the range of torsional deformation continues to expand, ultimately causing the specimen to undergo overall torsional buckling. Before the specimen undergoes overall buckling, the rigid loading ball remains in contact with the column wall, and separates only when the specimen undergoes overall buckling.

Figure 21: Specimen 10: two-dimensional curves of the buckling path and the three-dimensional curved-surface landscape

Discussion of buckling modes: The failure-mode diagrams for Specimen-10 are shown below:

It can be seen from the figures that, when the torsional load is relatively small, the specimen undergoes only slight deformation at the contact point. The maximum stress at the loading point of the specimen is approximately axisymmetric about the axis. As the torsional moment increases, the inward concave deformation of the specimen wall gradually increases, and the maximum stress, affected by the torsional load, gradually changes from symmetry to antisymmetry. When the torsional load increases to $8000N \cdot mm$, the bending deformation of the specimen at the loading point changes into torsional buckling; as the torsional load continues to increase, the torsional deformation increases, ultimately causing the specimen to undergo overall torsional buckling.

From the force–displacement curves and failure-mode diagrams of Specimen-10, it can be concluded that the specimens under combined lateral pressure and torsional loading exhibit different landscape phenomena. Compared with Specimen-9, the specimen lacking axial pressure has an increased load-bearing capacity, but the presence of torsional loading changes the final failure mode of the specimen and also changes the landscape phenomena.

Figure 22: When the torsional load is $1000N \cdot mm$, stress distribution diagrams of Specimen-10

Figure 23: When the torsional load is $7000N \cdot mm$, stress distribution diagrams of Specimen-10

Figure 24: Overall torsional failure of Specimen-10 when the torsional load is $8000N \cdot mm$ (left) and $9500N \cdot mm$ (right)

VIII. Influence of Axial Compression–Torsion on Buckling of Cola Cans

Displacement-load curves and the three-dimensional landscape of (external force-buckling load-displacement): To study the buckling performance of a cylindrical-shell structure under axial compression–torsion, Specimen-11 was established. To facilitate the application of torsional loading, the top and bottom ends of the specimen were fixed to the upper and lower clamping plates, respectively. The torsional load was first applied; after the specimen became stable, a displacement load was applied along the axial direction of the specimen. The specific axial force–displacement curves are shown below:

Figure 25: Specimen 11: two-dimensional curves of the buckling path and the three-dimensional curved-surface landscape

It can be seen from the curve plots that the curves for all specimens show a trend of first increasing and then decreasing, and the maximum load-bearing capacities of the specimens are basically the same. As the displacement load increases, the curves of the various specimens exhibit different trends. When the applied torsional load is $3000N \cdot mm$, the axial load-bearing capacity of the specimen becomes negative; moreover, as the torsional load increases, the moment at which the axial load-bearing capacity becomes negative keeps occurring earlier, and its numerical value keeps increasing. The main reason for this phenomenon is as follows: when the applied torsional moment is in the range $1000N \cdot mm \sim 3000N \cdot mm$, the axial load-bearing capacity of the specimen is basically positive; the torsional load does not play a dominant role, and the deformation of the specimen is mostly axial compression deformation. As the torsional load continuously increases, relative to the axial displacement load, the torsional load gradually begins to play the dominant role. In the initial stage of loading, due to the action of the torsional load, multiple oblique 45° ⋯

Failure modes of Specimen-11 at torsional loads of 1000 N · mm and 2000 N · mm

Figure 9: Failure modes of Specimen-11 at torsional loads of 1000 N · mm and 2000 N · mm

Failure modes of Specimen-11 at torsional loads of 3000 N · mm and 4000 N · mm

Figure 10: Failure modes of Specimen-11 at torsional loads of 3000 N · mm and 4000 N · mm

directional wrinkles; the front and rear wrinkles appear in a crossed state. When torsional deformation has not yet appeared, the specimen mainly relies on the column wall to bear the vertical load. When torsional wrinkles appear, the specimen changes from the previous column-wall load-bearing mode to crossed-wrinkle load-bearing together with the action of the torsional load; at this time the axial bearing capacity of the specimen reverses direction, and the magnitude of the bearing capacity increases to a certain extent. Accordingly, following the definition used in coastal zones, the plane at which the bearing capacity is zero is identified, and this plane is defined as "sea level." The stage in which the bearing capacity rises at the beginning of loading is defined as a "hill," and the stage in which the bearing capacity drops rapidly is also defined as a "cliff." Analysis shows that the peak loads of the specimens are similar, indicating that the "hill" heights are the same. For the portion of the specimen curve above "sea level," the axial bearing capacity is positive, and the axial displacement loading plays the dominant role; for the portion of the specimen curve below "sea level," the axial bearing capacity is negative, and the torsional loading plays the dominant role.

Discussion of instability modes: The following figures show the failure-mode diagrams of Specimen-11 under different torsional loads. It can be seen from the figures that when the torsional load is less than $3000N \cdot mm$, the specimen undergoes axial compressive buckling. This is because the torsional load is relatively small and does not affect the axial deformation of the specimen. When the torsional load is greater than $4000N \cdot mm$, the specimen changes from axial compressive buckling to overall torsional buckling, and the deformation of the specimen gradually increases as the torsional load increases. The change in the specimen failure mode occurs because the torsional load is large and affects the final failure mode of the specimen. It can be judged that when the specimen curve is entirely above "sea level," the specimen mostly undergoes axial compressive buckling; below "sea level," the specimen mostly undergoes overall torsional instability.

Figure 26: Failure-mode diagrams of Specimen-11 when the torsional load is $1000N \cdot mm$ (left) and $2000N \cdot mm$ (right)

Failure modes of Specimen-11 at torsional loads of 5000 N • mm and 6000 N • mm

Figure 11: Failure modes of Specimen-11 at torsional loads of 5000 N • mm and 6000 N • mm

Figure 27: Failure-mode diagrams of Specimen-11 when the torsional load is $3000N \cdot mm$ (left) and $4000N \cdot mm$ (right)

Figure 28: Failure-mode diagrams of Specimen-11 when the torsional load is $5000N \cdot mm$ (left) and $6000N \cdot mm$ (right)

IX. Effect of Internal Gas Filling of the Can Body on the Buckling of Easy-Open Cans

Displacement-load curves and three-dimensional landscape of (external force-buckling load-displacement): To study the "landscape" phenomenon when column-shell structures such as easy-open cans are internally filled with gas, two models, Specimen-12 and Specimen-13, were designed on the basis of the original model. The figure below shows the force–displacement curves of this specimen. It can be seen from the figure that the force–loading displacement curves of Specimen-12 under the action of axial compression, lateral compression, and internal pressure exhibit a monotonically increasing trend; the specimen does not show a relatively obvious landscape phenomenon. The maximum bearing forces in the curves of Specimen-12 and Specimen-8 are extracted to plot the maximum bearing force–axial load curve. From this curve, it can be seen that the degradation trends of the bearing capacities of the two specimens are similar, but the maximum bearing capacity of Specimen-12 is far greater than that of Specimen-8. Calculated by the average value, the average maximum load of Specimen-12 is $68.72N$, while that of Specimen-8 is $28.25N$. Moreover, when the axial load is $1300N$, Specimen-8 has already exhibited the "lake" phenomenon, and the specimen undergoes local buckling; however, the curve of Specimen-12 still maintains a monotonically increasing trend, and the specimen remains in the elastic stage. This shows that filling the column-shell structure with gas can greatly improve the lateral bearing capacity of the specimen, increase the buckling load of the specimen, and reduce the sensitivity of the column-shell structure to initial defects.

Figure 30 is the force–displacement curve diagram of Specimen-13. It can be seen from the figure that the curves of all specimens show a trend of first increasing and then decreasing, and their maximum bearing capacities are similar. Because of the effect of the internal-pressure load, the specimen curves do not exhibit a "sea level" phenomenon. The minimum values of the bearing capacities in the curves of Specimen-13 and Specimen-11 are taken to produce a comparison diagram. It can be seen from Figure 31 that the axial bearing capacity of Specimen-13 is much greater than that of Specimen-11, and as the torsional load

Specimen 12 curves and landscape

Figure 12: Specimen 12 curves and landscape

increases, the degree of degradation of the bearing capacity of Specimen-11 is much greater than that of Specimen-13. When the torsional load is $4000N \cdot mm$, the minimum axial bearing capacity of Specimen-11 has already degraded to a negative value, whereas the minimum bearing capacity of Specimen-13 remains positive throughout. From the above analysis, it can be concluded that applying internal pressure to a specimen subjected to axial-compressive–torsional loading can substantially improve the bearing capacity of the column-shell structure and mitigate the degradation of the bearing capacity.

Figure 29: Specimen 12: two-dimensional curves of the buckling path and the three-dimensional surface landscape

Figure 30: Specimen 13: two-dimensional curves of the buckling path and the three-dimensional surface landscape

Figure 31: Comparison of maximum load-carrying capacity (left), and comparison of minimum load-carrying capacity (right)

Discussion of instability modes: The failure modes of Specimen-12 and Specimen-13 are shown in the figures below. Specimen-12 did not undergo buckling failure. Compared with Specimen-8, the application of internal pressure significantly improved the stability of the cylindrical-shell structure and increased the specimen' s resistance to lateral disturbance. Specimen-13 underwent local buckling under different torsional loads, and the buckling form gradually changed into local torsional buckling as the torsional load increased; the overall stability of the specimen was good. By comparison with Specimen-11, it can be seen that, when the torsional-load range was $1000N \cdot mm \sim 6000N \cdot mm$, Specimen-13 did not undergo overall torsional instability. When the torsional load was $6000N \cdot mm$, Specimen-13 developed local buckling only at a location $30mm$ away from the upper loading plate; whereas Specimen-11 underwent a similar failure mode when the torsional load was $2000N \cdot mm$. This indicates that inflation of the cylindrical-shell structure can improve the structure' s resistance to torsional loading, delay the occurrence of local instability, and, to a certain extent, enhance the stability of the cylindrical-shell structure.

Figure 32: Specimen-12: left figure $N = 100N$, right figure $N = 1300N$

Figure 33: Specimen-13: left figure $M = 1000N \cdot mm$, right figure $M = 6000N \cdot mm$

X. Conclusion

In this paper, the finite-element nonlinear numerical simulation method was adopted. First, the trend of the numerically computed results was verified and supported, from a numerical-calculation perspective, by the experiments of Virot et al. [34] and by the buckling landscape. Then, detailed numerical simulation studies were further carried out on the buckling of cylindrical shells under different loading combinations. The buckling landscape of cylindrical shells was obtained, and on this basis the effects of different loading combinations, different boundary conditions, different radius-to-height ratios (R/L), and radius-to-thickness ratios (R/t) on the buckling landscape of cylindrical-shell structures were discussed and analyzed in detail. From the present numerical analysis and the interpretation using the buckling landscape, the following conclusions can be drawn:

(1) Specimens with different radius-to-height ratios and radius-to-thickness ratios have similar buckling landscapes. The trends of the "ridge" and "valley" of the specimens differ little, while the "lake" changes considerably. As the radius-to-height ratio and radius-to-thickness ratio increase, the area over which the specimen forms a "lake" continuously decreases. This indicates that specimens with larger radius-to-height ratios and radius-to-thickness ratios have better stability and are not highly sensitive to initial imperfections.

(2) When subjected to lateral compression–axial compression–torsion, the specimens not only exhibited phenomena similar to those under lateral compression–axial compression, but also developed new buckling landscapes. Owing to the presence of the torsional load, the load–displacement curve of the specimen showed a sudden drop in load-carrying capacity in the later stage of loading. To describe this mechanical behavior more vividly, this paper defines this phenomenon as a "cliff," defines the blank region below it where the load-carrying capacity is zero as the "sea," and changes "lake" to "basin." These phenomena and definitions had not appeared previously.

(3) Compared with specimens under lateral-compression–axial-compression loading, the presence of torsional loading not only changes the failure mode of the specimen from overall compressive buckling to torsional buckling, but also reduces the stability of the specimen, making it more sensitive to initial imperfections.

(4) Specimens subjected to lateral-compression–torsional loading did not exhibit the "basin" phenomenon. This is because, without the action of vertical axial pressure, the concave deformation at the contact point of the specimen is not further intensified. In addition, under the action of the torsional load, the concave shape inside the specimen changes, and the specimen ultimately undergoes overall torsional instability.

(5) Finite-element analysis was conducted in this paper for specimens under axial-torsional loading. It was found that the presence of torsional loading changes the force-bearing mechanism of the cylindrical-shell structure, and the axial force of the specimen becomes negative. To better describe this phenomenon, this paper defines the plane on which the load-carrying capacity is zero as the "sea level," and defines the region enclosed by the curve at which the load-carrying capacity is maximum as the "hill." Analysis shows that above "sea level," the axial load of the specimen plays the dominant role, and the specimen mostly undergoes axial compressive deformation; below it, deformation of the specimen is mainly controlled by the torsional load, and the specimen ultimately undergoes overall torsional instability.

(6) Under axial-compression–torsion–internal-pressure loading, the axial load-carrying capacity of the specimen increases, local instability of the specimen is delayed, and no overall instability occurs. According to the buckling landscape and failure modes under axial-compression–lateral-compression–internal-pressure loading, it can be seen that, due to

Because of the existence of internal pressure, the specimen does not undergo buckling deformation, and the lateral load-carrying capacity of the structure is greatly improved. This indicates that internal pressurization of the cylindrical shell structure increases the specimen' s lateral load-carrying capacity, axial load-carrying capacity, torsional resistance, and overall stability, and reduces the specimen' s sensitivity to initial imperfections.

(7) When the boundary condition of the specimen changes from free to fixed, the load-carrying capacity of the specimen increases, its stability improves, and its sensitivity to initial imperfections decreases.

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XI. References

[1] Euler, L., De curvis elasticis, Leonhard Euler' s Elastic Curves, translated and annotated by W.A. Oldfather, C.A. Ellis, and D.M. Brown, reprinted from Isis, 20, (58), 1933, The St. Catherine Press, Bruges, Belgium.

[2] Timoshenko, S., History of Strength of Materials, McGraw-Hill Book Company, New York/Toronto/London, 1953

[3] von Karman, T. V. and Tsien, H. S. [1939] The buckling of spherical shells by external pressure, J. Aero. Sci., 7, 43-50.

[4] von Karman, T. V. and Tsien, H. S. [1941] The buckling of thin cylindrical shells under axial compression. J. Aero. Sci., 8, 303-312.

[5] Tsien, H. S. [1942] Theory for the buckling of thin shells, J. Aero. Sci., 9, 373-384.

[6] Koiter, W. T. [1945] On the stability of elastic equilibrium, Dissertation, Delft, Holland. (An English translation is available as NASA, Tech. Trans., F 10, 833, 1967 (Koiter, W. T., 1945, "de Stabiliteit van het Elastich Evenwicht," Ph.D. thesis, Delft University of Technology, Delft, The Netherlands.)

[7] Tsien, H. S. [1947] Lower buckling load in the non-linear buckling theory for thin shells, Quart. Appl. Math. 5, 236-7.

[8] Seide, P. and Weingarten, V.I., On the Buckling of Circular Cylindrical Shells under Pure Bending, ASME Journal of Applied Mechanics, 28, (1), March 1961, 112-116.

[9] Hutchinson, J.W., Axial Buckling of Pressurized Imperfect Cylindrical Shells, AIAA Journal, 3, (8), August 1965, 1461-1466.

[10] NASA, 1965, "Buckling of Thin-Walled Circular Cylinders," NASA Space Vehicle Design Criteria, National Aeronautics and Space Administration, Washington, DC, Technical Report No. NASA SP-8007.

[11] NASA, 1969, "Buckling of Thin-Walled Doubly Curved Shells," NASA Space Vehicle Design Criteria, National Aeronautics and Space Administration, Washington, DC, Technical Report No. NASA SP-8032

[12] Babcock Jr., C. D. [1967] The influence of the testing machine on the buckling of cylindrical shells under axial compression, Int. J. Solids Struct. 3, 809-17.

[13] J. W. Hutchinson and W. T. Koiter. Postbuckling theory, Applied Mechanics Reviews, pp. 1353-1366, 1970.

[14] Singer, J., Arbocz, J., and Babcock, C.D., Buckling of Imperfect Stiffened Cylindrical Shells under Axial Compression, AIAA Journal, 9, (1), 1971, 68-75.

[15] Riks, E., The Application of Newton' s Method to the Problem of Elastic Stability, ASME Journal of Applied Mechanics, 39, 1972, 1060-1066.

[16] Thompson, J. M. T. and Hunt, G. W. [1973] A general theory of elastic stability, Wiley, London.

[17] Arbocz, J., The Effect of Initial Imperfections on Shell Stability, in: Thin-Shell Structures, Theory, Experiment and Design Y.C. Fung and E.E. Sechler eds., Prentice Hall, Englewood Cliffs, N.J., 1974, 205-245.

[18] Brush, D. O., Almroth, B .O. [1975] Buckling of bars, plates and shells. McGraw-Hill, New York.

[19] Arbocz, J. and Sechler, E.E., On the Buckling of Stiffened Imperfect Shells, AIAA Journal, 14, (11), 1611-1617 (1976).

[20] Arbocz, J., Past, Present and Future of Shell Stability Analysis, Zeitschrift für Flugwissenschaften und Weltraumforschung, 5,(6), 335-348(1981).

[21] Arbocz, J., Potier-Ferry, M., Singer, J. and Tvergaard, V., Buckling and Post- Buckling, Lecture Notes in Physics No. 288, Springer-Verlag, Berlin, Heidelberg, 1987.

[22] J. Singer, J. Arbocz, T. Weller, Buckling Experiments: Experimental Methods in Buckling of Thin-Walled Structures, John Wiley & Sons, Inc., New York (1998).

[23] Bo-Hua Sun, Buckling Problems of Sandwich Shells, Delft University of Technology, pp.1-99., Report LR-690, 1992.

[24] K.Y. Yeh, Bo-Hua Sun and F.P.J. Rimrott, Buckling of imperfect sandwich cone under axial compression - equivalent-cylinder approach, part 1. Technische Mechanik, Band 14, Heft 3/4: 239-248, 1994.

[25] K.Y. Yeh, Bo-Hua Sun and F.P.J. Rimrott, Buckling of imperfect sandwich cone under axial compression - equivalent-cylinder approach, part 2. Technische Mechanik, Band 15, Heft 1: 1-12, 1994.

[26] Bo-Hua Sun, K.Y. Yeh, and F.P.J. Rimrott, On the buckling of structures, Technische Mechanik, Band 15, Heft 2: 129-140, 1995.

[27] Bo-Hua Sun, Modified Koiter' s buckling theory based on the generalized variational principles, CERECAM Report No. 244, University of Cape Town, 1994.

[28] Timshenko, S.P. and Gere, J.M., Theory of Elastic Stability, 2nd ed., Dover Publications, Inc. New York, 2009.

[29] Singer, J. Buckling experiments : experimental methods in buckling of thin-walled structures[J]. Applied Mechanics Reviews, 2002, 56(1):B5.

[30] Bushnell, D. Shell buckling, website, http://shellbuckling.com/index.php (2004).

[31] Krasovsky V, Marchenko V, Schmidt R. Deformation and buckling of axially compressed cylindrical shells with local loads in numerical simulation and experiments[J]. Thin-Walled Structures, 2011, 49(5):576-580.

[32] Haynie, W., Hilburger, M., Bogge, M., Maspoli, M., and Kriegesmann, B. Validation of Lower-Bound Estimates for Compression-Loaded Cylindrical Shells, AIAA Paper No.2012-1864.

[33] Thompson, J. M T . Advances in Shell Buckling: Theory and Experiments[J]. International Journal of Bifurcation Chaos, 2015, 25(01):1530001.

[34] Virot E, Kreilos T, Schneider T M, et al. Stability Landscape of Shell Buckling[J]. Phys.Rev.Lett, 2017, 119.

[35] Gerasimidis S, Virot E, Hutchinson J W, et al. On Establishing Buckling Knockdowns for Imperfection-Sensitive Shell Structures[J]. Journal of Applied Mechanics, 2018,85(9):091010.

[36] Von Slooten, R. A. and Soong, T. T., Buckling of a long, axially compressed thin cylindrical shell with random initial imperfections, J. Applied Mechanics, Vol.39, No.4, Dec. 1972, p.1066.

[37] Narasimhan, K. Y. and Hoff, N. J., Snapping of imperfect thin-walled circular cylindrical shells of finite length. J. Applied Mechanics, Vol. 38, No.1, Mar. 1971,pp. 162-171.

[38] Tennyson, R. C., Muggeridge, D. B. and Caswell. R. D. New design criteria for predicting buckling of cylindrical shells under axial compression. Journal of Spacecraft and Rockets,Vol. 8, No.10, Oct, 1971, p.1062

[39] Stein, M., The effect on the buckling of perfect cylinders of prebuckling deformations and stresses induced by edge support. Collected papers on instability od shell structures. NASA TN D-1510. Dec. 1962, p.217

[40] Almroth, B. O., Holmes, A. M. C. and Brush, D. O., An experimental study of the buckling of cylinders under axial compression. Experimental Mech. 4, 263-270, 1964.

[41] Budiansky, B., Theory of buckling and post-buckling, Advance in Applied Mech, Vol.14, 1974.

[42] Elishakoff, I., 2014, Resolution of the Twentieth Century Conundrum in Elastic Stability, World Scientific Publishing, Singapore.

[43] Mark W. Hilburger, Developing the Next Generation Shell Buckling Design Factors and Technologies [R], American Institute of Aeronautics and Astronautics, 2007

[44] Gerasimidis, S., Virot, E., Hutchinson, J. W. and Rubinstein, S. M.. On Establishing Buckling Knockdowns for Imperfection- Sensitive Shell Structures, Journal of Applied Mechanics, 85, 091010-1(2018)

[45] Yan H., et al, Sterically controlled mechanochemistry under hydrostatic pressure, 554, Nature, 505(2018)

[46] W. Yang, Z.-M. Li, W. Shi, B.-H. Xie, and M.-B. Yang, Review on Auxetic Materials, J. Mater. Sci. 39, 3269 (2004).

[47] A. Alderson and K. L. Alderson, Auxetic Materials, P I Mech Eng G-J Aer 221, 565 (2007).

[48] Xiaoyu Zheng, et al., Ultralight, Ultrastiff Mechanical Metamaterials, Science, VOL 344 ISSUE 6190, 2014.

[49] Banerjee et al., Ultralarge elastic deformation of nanoscale diamond, Science 360, 300 - 302 (2018)

[50] Bin Liu, et al., Topological kinematics of origami metamaterials, Nature Physics, 14:811 - 815(2018)

[51] Zhou Chengxun. *Theory of Elastic-Plastic Stability of Thin Shells*[M]. National Defense Industry Press, 1979.

[52] Xu Chunjie, Zhang Hao, Xu Shuai, et al. Comparative study on microstructure and mechanical properties of aluminum-based and iron-based materials for easy-open cans[J]. Foundry Technology, 2018, v.39No.314(05):14-17.

[53] Li Weixiang, Song Xuan Yi, Yang Kai, Ma Xinling, Yang Gang. Experimental study and finite element analysis of axial compression buckling of easy-open cans[J]. Manufacturing Automation, 2014,36(15):83-85+101.

Note: Figure translations are in progress. See original paper for figures.