Postprint: Thermal Error Compensation Based on QTT Actuators

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Abstract

The actuators to be deployed for the Xinjiang Qitai Radio Telescope are required to meet an accuracy specification of $±$15 m across a temperature range of -40~60°C. As thermal errors can attain approximately 400 m in the absence of compensation, it becomes necessary to establish a thermal error model for predicting the temperature-dependent variation of actuator displacement, and to implement active displacement control compensation based on this model, thereby ensuring that the actuators satisfy positioning accuracy requirements at any given temperature. Initially, by integrating fundamental model requirements with significance testing, the optimal point for model establishment was selected from among 16 temperature measurement locations on the actuator. Subsequently, grey prediction theory was utilized to construct the actuator' s thermal error model. Finally, the model was refined according to practical conditions. The research demonstrates that this work innovatively employs a methodology combining statistical analysis with engineering practice for temperature measurement point selection. The established thermal error model can effectively guide active displacement control compensation for the actuator, enabling it to achieve a positioning accuracy of $±$4 m within the required temperature range of -40~60°C.

Full Text

Thermal Error Compensation Based on QTT Actuator

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Abstract

The actuators to be deployed in the Xinjiang Qitai Radio Telescope must maintain a positioning accuracy of $±$15 m across a temperature range of -40°C to 60°C. Since thermal errors can reach approximately 400 m without compensation, establishing a thermal error model to predict actuator displacement as a function of temperature is essential for implementing active displacement control compensation. This paper first identifies the optimal temperature measurement point from 16 candidate locations by combining model requirements with significance testing. A thermal error model is then developed using grey prediction theory and subsequently refined based on practical considerations. The research demonstrates that the innovative integration of statistical methods with engineering practice for temperature point selection yields a robust model that guides active displacement control compensation, enabling the actuator to achieve a positioning accuracy of $±$4 m within the required -40°C to 60°C temperature range.

Keywords: Actuator; Significance test; Grey prediction; Thermal error

1 Introduction

In practice, radio telescopes require active main reflector surface adjustment technology to correct surface profile errors caused by gravity, temperature variations, wind loading, and other environmental factors [1]. This technology adjusts the positions of segmented reflector panels to modify the surface shape, with high-precision displacement actuators serving as the critical enabling component [2]. Major fully steerable radio telescopes employing active surface technology include the GBT in the United States [3], LMT in Mexico [4], SRT in Italy [5], and Tianma in China [6]. The actuators discussed herein are specifically designed for the upcoming 110-meter Qi Tai radio Telescope (QTT) in Xinjiang.

The climate conditions in Qitai, Xinjiang are exceptionally harsh, characterized by wide temperature variations and rapid fluctuations. QTT actuators must maintain positioning accuracy within $±$15 m across a temperature range of -40°C to 60°C to enable high-precision all-weather reflector adjustment. Since mechanical structure and displacement sensors alone cannot eliminate thermal errors, an effective method to compensate for their impact on positioning accuracy is required.

Developing a predictive thermal error compensation model depends on accurate measurement of the actuator' s temperature field distribution. However, obtaining a precise temperature field necessitates numerous temperature sensors, which increases computational and measurement overhead, complicates model development, and potentially interferes with normal actuator operation.

Grey prediction theory offers a methodology for forecasting grey systems by analyzing developmental trends among influencing factors to construct predictive models for system characteristics. According to Professor Deng Julong' s grey system theory [7, 8], thermal errors in mechanical machining are influenced by complex factors and exhibit distinct grey system properties. Consequently, this paper begins by identifying the temperature points most likely to affect actuator positioning accuracy, then selects key measurement locations to reduce sensor count and optimize thermal sensor placement. This approach avoids multicollinearity issues from excessive temperature variables while enhancing model robustness. Grey prediction theory is employed for modeling, with subsequent adjustments based on actual operating conditions across the -40°C to 60°C range to achieve the required precision.

2 Selection and Screening of Temperature Measurement Points

For complex mechanical systems, directly determining optimal temperature measurement points through mathematical analysis is impractical. The general approach relies on engineering experience to identify probable influence mechanisms, followed by experimental design of measurement point distribution. Statistical analysis of experimental data then reduces the number of candidate points, simplifying the modeling process while improving accuracy and robustness.

2.1 Selection and Distribution of Temperature Measurement Points

The QTT actuator employs a stepper motor coupled with a worm gear mechanism, where the displacement precision at the worm screw tip represents the overall actuator accuracy. Based on engineering experience, primary temperature influences include ambient temperature, stepper motor temperature, worm gear temperature, worm screw temperature, and spline sleeve temperature. Considering measurable locations on the actual actuator, temperature sensors can be distributed across three surfaces plus one ambient measurement point, totaling 16 locations. The distribution of surfaces and points is illustrated in [Figure 1: see original paper].

Surface 1: Points 1-5 collectively characterize the influence of worm gear temperature distribution on actuator displacement.
Surface 2: Points 1, 2, and 5 collectively characterize stepper motor temperature distribution effects; Point 3 directly measures the motor coupling temperature; Point 4 directly measures the motor housing temperature.
Surface 3: Point 1 directly measures temperature near the worm screw tip; Points 2-5 collectively characterize the influence of spline sleeve temperature distribution.

During accuracy testing, the actuator is mounted on a test bench with a grating scale installed at the worm screw front to measure displacement. Tests are conducted in a constant temperature and humidity chamber, with displacement at room temperature (20°C) defined as zero. The experimental setup and results are shown in [Figure 2: see original paper].

2.2 Preliminary Screening of Temperature Measurement Points

Three criteria guide the screening process:

1. **High correlation between temperature and displacement**, quantified using the Pearson correlation coefficient: $\rho_{T,S} = \mathrm{COV}(S,T)/(\sigma_S \cdot \sigma_T)$. Values approaching 1 indicate stronger linear relationships. Correlation coefficients for each point are listed in .
2. **When displacement reaches 0 m during testing, the measured temperature should be approximately 20°C**. Temperature values recorded at zero displacement are shown in .
3. **Sensors must be conveniently installable without affecting normal actuator operation**.

Excluding the third criterion initially, the correlations between the 15 on-actuator temperature points, ambient temperature, and displacement are presented in . Given the high precision requirements, Pearson coefficients closer to 1 are preferred. The temperature values at zero displacement are also considered, as shown in . During experiments, when actuator displacement returns to zero, test point temperatures may not simultaneously return to room temperature. Since both experiments and modeling assume zero displacement at room temperature, significant deviations would compromise model validity. Based on these criteria and the data in and , five suitable measurement points are identified: Surface 1 Point 4, Surface 1 Point 5, Surface 2 Point 2, Surface 2 Point 3, and Surface 3 Point 2.

2.3 Statistical Screening of Temperature Test Points

Significance testing is commonly used in statistics to detect differences between samples and populations. In this study, significance testing compares the credibility between temperature measurements and corresponding displacement values. Typical significance levels of 5% and 1% are adopted, calculated using Equation (1). Smaller significance values indicate smaller differences and higher credibility.

$$t = |\rho| \cdot \sqrt{(n-2)/(1-\rho^2)} \quad |t| > t_{\alpha/2}(n-2) \tag{1}$$

The five points selected through basic screening—Surface 1 Point 4, Surface 1 Point 5, Surface 2 Point 2, Surface 2 Point 3, and Surface 3 Point 2—are renumbered as P1-P5. Regression analysis with significance testing is performed on P1-P5, using thresholds of 0.05 and 0.01 to screen independent variables, yielding two linear models.

presents the regression results, where B represents unstandardized coefficients and Beta represents standardized coefficients. Using a 0.01 significance threshold yields Model 1, which depends only on temperature at P3. Using a 0.05 threshold yields Model 2, which depends on a linear combination of temperatures at P1 and P3.

shows variables excluded from the regression analysis. Partial correlation values range between -1 and 1, with larger absolute values indicating greater influence on the dependent variable. Collinearity tolerance indicates multicollinearity among independent variables; values below 0.1 suggest strong collinearity. Through comprehensive consideration of significance, partial correlation, and collinearity, the final independent variable is selected.

Model 1 indicates that actuator precision can be fully represented by temperature at P3 alone, while Model 2 suggests representation through a linear combination of P1 and P3. Comparing the models, P3 alone provides greater precision, as evidenced by significance values. Structurally, P3 is located near the motor, worm gear, and worm screw base, enabling it to comprehensively capture temperature effects from all critical components—explaining its effectiveness in characterizing thermally induced displacement. Additionally, P3 installation does not interfere with actuator operation, satisfying the third basic requirement. Therefore, subsequent modeling employs only P3 (Surface 2 Point 2).

3 Model Establishment and Application

Grey system theory constructs predictive models by extending past and present known or uncertain information into the future, thereby identifying developmental trends. Using differential equations as its mathematical tool, grey system theory reflects the essential nature of system evolution without requiring explicit understanding of the predictive system. Its research data can be randomly generated [9], offering advantages for studying small samples, poor information, and arbitrarily distributed data compared to traditional statistical methods.

3.1 Grey Prediction Model Establishment

As shown in [Figure 2: see original paper], linear error compensation models prove inadequate. Grey prediction is appropriate when limited or uncertain data representing system behavioral characteristics must describe continuous internal processes. Thermal error research data exhibit small-sample, poor-information characteristics, and the complex mechanical system precludes direct identification of all thermally relevant factors, making grey system theory particularly suitable.

During experiments, temperature values are recorded each time actuator precision changes by 5 m. Let S represent actuator displacement (m), ΔS the displacement change, and $T^{(0)}(t)$ the observed temperature (K). Starting from $\Delta S = 0$ m at $t = 0$ with temperature $T^{(0)}(0)$, when $\Delta S = 5$ m we denote $t = 1$ and temperature $T^{(0)}(1)$, and so forth. This yields:

$$S = 5t - B_t$$

where constant B_t is determined by the displacement value S at $T^{(0)} = 293.15$ K, ensuring zero displacement at room temperature. Experiments confirm $B_t = 210$ m.

The temperature observation sequence is $T^{(0)} = \{T^{(0)}(0), T^{(0)}(1), T^{(0)}(2), \dots, T^{(0)}(N)\}$. Grey prediction requires one accumulated generating operation to obtain $T^{(1)} = \{T^{(1)}(0), T^{(1)}(1), T^{(1)}(2), \dots, T^{(1)}(N)\}$, where $T^{(1)}(n) = \sum T^{(0)}(n)$.

According to grey prediction theory, we assume $T^{(1)}$ satisfies the first-order ordinary differential equation:

$$\frac{dT^{(1)}}{dt} + aT^{(1)} = u$$

Since experiments use discrete points with $t_0 = 0$, the solution is:

$$T^{(1)}(k) = \left[T^{(1)}(0) - \frac{u}{a}\right] \cdot e^{-aK} + \frac{u}{a} \tag{3}$$

Parameters a and u are estimated via least squares:

$$\begin{bmatrix} a \\ u \end{bmatrix} = (B^{TB})^{-1} B^T \cdot \begin{bmatrix} T^{(0)}(1) \\ \vdots \\ T^{(0)}(N) \end{bmatrix} \tag{4}$$

$$B = \begin{bmatrix} -\frac{1}{2}[T^{(1)}(1) + T^{(1)}(0)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[T^{(1)}(N) + T^{(1)}(N-1)] & 1 \end{bmatrix} \tag{5}$$

The grey prediction yields:

$$\hat{T}(k) = T^{(1)}(k) - T^{(1)}(k-1); \quad \hat{T}(0) = T^{(1)}(0) \tag{6}$$

Combining Equations (1), (3), and (5) yields:

$$S = -\frac{K}{a} \cdot \ln(T) + B + \frac{K}{a} \cdot \ln\left[(x^{(1)}(1) - u/a) \cdot (e^a - e^{2a}) \cdot e^{-at}\right] \tag{7}$$

where T is temperature (K). For simplicity, define:

$$K_y = -\frac{K}{a} \tag{8}$$

$$B_y = B + \frac{K}{a} \cdot \ln\left[(x^{(1)}(1) - u/a) \cdot (e^a - e^{2a}) \cdot e^{-at}\right] \tag{9}$$

where K_y and B_y have units of m. Substituting all experimental data into the grey prediction yields:

$$S = K_y \cdot \ln(T) + B_y = 1063.8298 \cdot \ln(T) - 6043.28137 \tag{10}$$

[Figure 3: see original paper] compares the grey prediction model (red line) with experimental results (black line). The model shows good agreement in the 0-40°C range but exhibits errors at extreme temperatures. Since model errors cause accumulation during real-time control, it is necessary to implement the theoretical model in the actuator for real-time adjustment and subsequently refine it based on actual performance.

3.2 Model Verification and Correction

After model development, the algorithm is programmed for real-time actuator adjustment. The first experiment uses Equation (10) directly for error compensation, with results shown in (Test 1) and [Figure 4: see original paper] (Test 1). This test reveals that direct application of the grey model leads to error accumulation due to incomplete error correction, with larger deviations occurring farther from room temperature.

The most direct and effective modification involves piecewise processing of the formula. Since error values vary approximately linearly within different temperature ranges, and given that $dS/dT = K_y/T$, we can approximate within consistent temperature trends. Let T_y and T_{y+1} represent the lower and upper bounds of temperature range y, and ΔS_y the required displacement adjustment within that range:

$$\Delta S_y = \int \frac{\Delta K_y}{T} dT = \left[\frac{2\Delta K_y}{T_{y1} + T_{y2}} + \cdots + \frac{2\Delta K_y}{T_{y(n-1)} + T_{yn}}\right] \cdot (T_{y+1} - T_y) \approx \frac{\Delta K_y \cdot (T_{y+1} - T_y)}{T_y} \tag{11}$$

Equation (11) enables calculation of corrected parameters $\bar{K_y}$ and $\bar{B_y}$:

$$\bar{K_y} = K_y - \Delta K_y \tag{12}$$

$$\bar{B_y} = \begin{cases} -\bar{K_y} \cdot \ln(293.15) & \text{if range } y \text{ includes room temperature} \\ \bar{K_{y-1}} \cdot \ln(T_y) + \bar{B_{y-1}} = \bar{K_y} \cdot \ln(T_{y+1}) + \bar{B_y} & \text{if range } y \text{ excludes room temperature} \end{cases}$$

Subsequent experiments target different temperature intervals. Test 2 divides the range into -40°C to -20°C, -20°C to 0°C, 0°C to 40°C, and 40°C to 60°C, with results in and [Figure 4: see original paper]. Test 3 refines the model for -40°C to 0°C, Test 4 for 0°C to 40°C, with all results summarized in and [Figure 4: see original paper].

Through multiple verification tests, Test 4 successfully reduces thermally induced errors to $±$4 m, meeting QTT' s actuator precision requirements.

4 Conclusion

This paper addresses thermal error compensation for actuators in the Xinjiang Qitai Radio Telescope, which demands $±$15 m accuracy across -40°C to 60°C. By compensating for temperature-induced precision variations, the thermal error magnitude has been reduced to $±$4 m. Given the actuator' s complex structure and numerous thermal influence factors, purely theoretical determination of optimal measurement points or models is infeasible.

This study innovatively combines engineering practice with statistical methods for temperature point selection and derives model correction formulas through experimental validation. This approach provides a valuable foundation for future research on radio telescope actuators.

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Note: Figure translations are in progress. See original paper for figures.