

Postprint: Research on Interferometric Baselines Between Chang' e-4 Relay Satellite and Lunar Surface Equipment Using Halo Orbit Simulations

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Abstract

A preliminary Halo numerical orbit model for the Chang' e-4 relay satellite has been established and applied to interferometry simulations for space low-frequency radio astronomy observations. Based on the Halo orbit in the Earth-Moon Lagrange point L2 region, and using the published theoretical orbit of the Chang' e-4 relay satellite as a reference, model parameters were calibrated and adjusted. Through transformations of the time and space reference systems, the orbit data were converted to the L2 point rotating coordinate system. Subsequently, differences between the Halo orbit model and the theoretical orbit data were compared, and the baseline lengths formed with lunar surface equipment from both the simulated and theoretical data were compared and analyzed. During the analyzed operational phase, the difference in baseline length when connected with lunar surface equipment was within 60 km, which basically satisfies the initial baseline accuracy requirements for interferometric fringe search in the HF band.

Full Text

Research on Interferometric Baselines Between the Chang' e-4 Relay Satellite and Lunar Surface Equipment Using Halo Orbit Simulation

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Abstract: This study establishes a numerical Halo orbit model for the Chang' e-4 relay satellite and applies it to simulations of space-based low-frequency radio interferometry. Based on the Halo orbit in the Earth-Moon Lagrange libration point L2 region and referencing the published theoretical orbit of the Chang' e-4 relay satellite, we calibrate and adjust model parameters and transform orbital data to the L2-point rotating coordinate system through temporal and spatial reference frame conversions. We then compare differences between the Halo orbit model and theoretical orbital data, analyze baseline lengths formed with lunar surface equipment using both simulated and theoretical data, and evaluate the results. During the analyzed operational phase, baseline length differences remain within 60 km, which fundamentally meets the baseline initial value precision requirements for interferometric fringe searching in the HF band.

Keywords: Halo orbit; Libration point; Coordinate transformation; Simulation

1. Halo Orbits and Relay Satellites

China' s Chang' e program has executed multiple missions, with Chang' e-4 achieving humanity' s first soft landing and exploration on the lunar far side. Landers on the lunar far side cannot communicate directly with Earth. To address this, the Chang' e-4 lunar landing mission deployed a relay communications satellite named "Queqiao," establishing a link between the lander, rover, and Earth to transmit scientific data back to our planet. This relay satellite operates in a mission orbit near the Earth-Moon Lagrange point L2 [1-4]. The low-frequency radio spectrometer onboard the relay satellite and the radio frequency spectrometer on the lunar lander create opportunities for space-based low-frequency radio interferometry [5].

The lunar far side shields against various artificial signal interferences from Earth, providing a quiet environment for low-frequency radio observations in the 0.1-40 MHz band [6]. Due to the long wavelengths of low-frequency radio waves, achieving high spatial resolution requires consideration of Very Long Baseline Interferometry (VLBI) techniques, potentially employing coordinated observations and even interferometric measurements using low-frequency radio equipment on the lunar surface, relay satellite, and ground stations. To investigate the feasibility of such space-based interferometry, establishing theoretical and simulation models for simulation experiments is meaningful, with a key focus being the simulation of baselines from the relay satellite to lunar surface equipment and calculation of their lengths. This paper discusses the applicability of Halo orbit models in simulations of space-based low-frequency radio interferometry.

The Chang' e-4 mission selected the South Pole-Aitken basin as its landing site, where the lander and rover conduct scientific investigations and have obtained detection data regarding lunar mantle materials [7]. Through comprehensive

exploration of topography, material composition, lunar regolith, and shallow subsurface structure, the mission promises to yield new evidence about lunar formation [8]. On May 21, 2018, the Queqiao relay communications satellite successfully launched and entered its mission orbit near the Earth-Moon L2 point. The L2 point refers to one of five special solutions of the Restricted Three-Body Problem (RTBP), known as the collinear Lagrange libration point located outside the smaller celestial body, systematically summarized by Szebehely [9]. The concept of deploying a relay satellite near the Earth-Moon L2 point for lunar far side exploration was proposed by Farquhar as early as 1970, with research by Howell, Breakwell, and others enriching the theoretical framework of libration point orbital mechanics [10-12]. Richardson provided third-order approximate analytical solutions for Halo orbits, which have been widely applied in three-body problem research [13-15]. Following Farquhar's proposal for establishing space stations at libration points [16], Lo and others proposed relay communication links and the Interplanetary Superhighway concept [17]. The first libration point orbit mission was ISEE-3 at the Sun-Earth L1 point, the American ARTEMIS mission first demonstrated Earth-Moon libration point flight, Chang'e-2 visited a Sun-Earth L2 Lissajous orbit, the Chang'e 5T1 lunar return test mission entered an Earth-Moon L2 Lissajous orbit, and the Chang'e-4 relay satellite utilizes a Halo orbit near Earth-Moon L2 [18,19]. Domestic research has also conducted multiple modeling, simulation, and actual measurement studies related to the relay satellite's mission orbit [20,21].

To conduct research on space-based low-frequency radio interferometry systems, we construct a Halo orbit near Earth-Moon L2, compare its relationship with the relay satellite's actual theoretical orbit, evaluate deviations between the model and theoretical orbit, analyze the model's impact on baseline lengths formed between satellite and lunar surface equipment, and assess the simulation effectiveness.

2. Libration Point Orbit Theory

The Restricted Three-Body Problem describes the motion of a small mass object under gravitational influence from two large celestial bodies. Under certain special conditions, the gravitational forces from the large bodies can satisfy the centripetal acceleration requirements for stable motion. The five special solutions for such stable motion are called "Lagrange libration points," with L2 being the point on the line connecting the two large bodies, outside the smaller body. At this point, certain orbits can be found that allow the small object to maintain a stable position with low energy consumption over extended periods. However, L2 is not a stable point, so such orbits are not perfectly periodic and still require periodic orbital maintenance maneuvers. Common L2 orbits include Lyapunov orbits, Lissajous orbits, and Halo orbits.

We define a right-handed Cartesian coordinate system in the Earth-Moon space called the "Rotation System," whose origin is always at the instantaneous L2 point of the Earth-Moon line, with the x-axis pointing from Earth to Moon and

the z-axis aligned with the rotation angular velocity direction between Earth and Moon. In this coordinate system, both Lissajous and Halo orbit motions can be described as superpositions of in-plane and out-of-plane vibrations. The general expression for libration point orbits is:

$$\begin{cases} x = A_x \cos(\lambda t + \phi) \\ y = A_y \sin(\lambda t + \phi) \\ z = A_z \sin(\nu t + \psi) \end{cases}$$

where A_x and A_z represent in-plane and out-of-plane amplitudes in the rotating coordinate system, λ is the in-plane vibration frequency, ν is the out-of-plane vibration frequency, and ϕ and ψ are the initial phases of in-plane and out-of-plane vibrations, respectively.

An example of a Lissajous orbit is shown in [Figure 1: see original paper] (left). This orbit has different periods for in-plane and out-of-plane vibrations in the Earth-Moon rotating plane, causing the orbit to trace a Lissajous curve. The satellite may be occulted by the Moon when passing through the central region, preventing Earth communication. In contrast, Halo orbits ([Figure 1: see original paper] right) have larger dimensions and identical in-plane and out-of-plane periods, remaining unocculted and ensuring continuous line-of-sight to both the lunar far side and Earth throughout the orbit, guaranteeing continuous ground-relay satellite-lunar far side communications. The general formula for Halo orbits is:

$$\begin{cases} x = A_x \cos(\lambda t + \phi) \\ y = A_y \sin(\lambda t + \phi) \\ z = A_z \sin(\lambda t + \phi) \end{cases}$$

Here, the in-plane and out-of-plane vibration frequencies are equal at λ . Halo orbit dimensions are typically described by two parameters, A_x and A_z , representing in-plane and out-of-plane amplitudes. Halo orbits actually constitute a family of orbits, requiring further simplification for simulation. Moreover, Halo orbits have limited stability, and spacecraft gradually deviate from the standard orbit over time. Reference [12] presents a numerical construction method using quasi-periodic orbits to describe Halo orbits, which requires less computation and better captures Halo orbit characteristics. Its basic form is:

$$\begin{cases} x = x_1 + x_2 + x_3 + x_4 \\ y = y_1 + y_2 + y_3 + y_4 \\ z = z_1 + z_2 + z_3 + z_4 \end{cases}$$

For amplitude description, we can also use the in-plane amplitude A_y in the y-direction and out-of-plane amplitude A_z . To ensure equal frequencies for

in-plane and out-of-plane motions, A_y and A_z should satisfy the following relationship [12]:

$$A_y^2 = 1.176726A_z^2 + 3.361330$$

Using this relationship yields a minimum A_y value of 46,793 km. This establishes the general relationship between the Halo orbit expression and A_z .

3. Verification Experiment Using Relay Satellite Ephemeris

To validate the Halo orbit model, we selected the theoretical orbit actually used by the Chang' e-4 Queqiao relay satellite from 00:00:00 on December 22, 2019, to 23:59:59 on December 23, 2019 [19] (hereinafter referred to as “actual orbit” or “actual data”), with data point intervals of one second. The actual orbit data includes Beijing time, position coordinates (km) of the relay satellite in the geocentric coordinate system, and its velocity vector (km/s). Based on relevant parameters of the Chang' e-4 relay satellite' s mission orbit, we adjusted the out-of-plane amplitude A_z and initial phase θ in the model to match the model with the actual orbit [20,21]. The Halo orbit simulation essentially models the standard orbit of the mission orbit; the satellite performs multiple orbital maneuvers to maintain its position near the standard orbit, but considering both orbital stability and propellant conservation for extended mission life, the actual orbit still differs from the standard orbit.

To clearly demonstrate orbital characteristics, we selected the rotating coordinate system for comparing actual and model data. First, we transformed the time system and spatial coordinate system of the actual data. Time was unified to Terrestrial Time (TT) to meet precision requirements for both Earth-based timing and planetary ephemerides [22]. For spatial coordinate system conversion, we used JPL' s DE430 planetary ephemeris.

First, time was unified to TT. Since the relay satellite orbit operates in the Earth-Moon space, to obtain corresponding planetary ephemeris data at specific moments, TT should be used instead of ephemeris time when reading ephemerides, unifying the time baseline for model and actual orbit data while ensuring precision. Beijing Standard Time (BST) was first converted to UTC; then using Earth Orientation Parameters (EOP) published by the International Earth Rotation Service (IERS), UTC was converted to UT1 time; after accounting for leap seconds, time was converted to TT.

[Figure 2: see original paper] Time system transformation

The relay satellite' s actual orbit is estimated and predicted using ranging, velocity measurements from ground deep space stations, and VLBI measurement data. The reference frame adopted for this information should be the Earth-centered Earth-fixed coordinate system. To utilize actual measurement data,

this coordinate system should be transformed to an appropriate space coordinate system. The Earth-centered Earth-fixed coordinate system incorporates Earth's rotation, nutation, precession, and other factors that are independent of both satellite motion near the Moon and lunar libration. Describing the motion of the relay satellite platform and lunar lander platform in the Earth-centered Earth-fixed coordinate system is complex and fails to reveal characteristic features, necessitating transformation to a suitable coordinate system. To present the main characteristics of Halo orbits (approximately vertical circular rings), orbital data should be transformed into the rotating coordinate system described above for intuitive comparison between orbital data and model.

[Figure 3: see original paper] Spatial coordinate system transformation

The Earth-fixed coordinate system is a coordinate system with its origin at Earth's center, fixed to Earth and moving with it. First, the geocentric coordinate system (DX-2) coordinates are transformed to the International Terrestrial Reference System (ITRS); then using EOP data published by IERS, accounting for polar motion, Earth rotation angle, and nutation-precession, coordinates are transformed to the Geocentric Celestial Reference System (GCRS) and International Celestial Reference System (ICRS).

The next step transforms coordinates to the rotating coordinate system. Since the rotating coordinate system is related to the Earth-Moon system, we first obtain the positions of Earth's center and Moon's center at a given moment. The line connecting them defines the x-axis of the rotating coordinate system, the angular velocity direction of the Moon's orbit around Earth defines the z-axis, and the right-hand rule determines the y-axis direction. This allows transformation of the relay satellite's position from the ICRS coordinate system to the rotating coordinate system. The transformation result is shown below.

[Figure 4: see original paper] Halo orbit model and partial relay satellite trajectory

In [Figure 4: see original paper], the thick blue line represents actual orbital data—the trajectory of the relay satellite in the rotating coordinate system from 00:00 on December 22 to 24:00 on December 23, also called observational data (marked "O" in the figure); the thin red line represents the trajectory of the corresponding Halo orbit model over a complete period matched to the actual orbit during this time, also called computational data (marked "C" in the figure). The actual orbit matches the Halo orbit model well during this period. If we difference the overlapping portions of actual and Halo orbit model data and take the modulus of the difference, we obtain the following:

[Figure 5: see original paper] Differences between observation and model orbits

[Figure 5: see original paper] shows that the data generally match well, with total error not exceeding 700 km, indicating model parameters are close to actual conditions. However, the trend of differences between observational and model data changes in the middle section, suggesting the actual orbit has more

complex motion forms not fully described by the model.

This Halo orbit model will be applied to space radio interferometry research, so we examine differences between the model and ephemeris in interferometric measurements. First, we examine baselines formed during interferometry between the relay satellite and lunar surface equipment. A baseline refers to the distance between two antenna units in interferometry. As the baseline changes, observation signals from the same target by the two antenna units can form interference. To achieve interferometry, both baseline length and the relationship between baseline direction and target source must be known.

If a low-frequency detection device exists on the lunar far side at a point on the Earth-Moon line, and it performs interferometric measurements with a low-frequency detection device on the relay satellite, then the baseline between them is the vector from the lunar surface device to the relay satellite, with length equal to the vector's modulus. Using observational and model data, we can respectively calculate baselines and their lengths during this period. Baseline length variation over time is shown below:

[Figure 6: see original paper] Baseline lengths of observation and calculation

[Figure 6: see original paper] shows baseline length variations over time calculated from observational and model data starting December 22, 2019. The horizontal axis represents elapsed time from 00:00 on December 22 (in hours), and the vertical axis represents baseline length at that moment. The blue line represents baseline length from observational data, and the red line represents baseline length from model data. Their variation trends are generally consistent. To compare their differences, we can difference them to obtain [Figure 7: see original paper].

[Figure 7: see original paper] Differences of baseline lengths

[Figure 7: see original paper] shows the difference between the two baseline lengths over time. The difference between baseline lengths in the two cases is small, generally within several tens of kilometers. VLBI correlation processing requires delay errors within certain limits. A major component of delay error is geometric delay error, meaning baseline length calculations should remain as consistent as possible with true values. Correlation processing requirements for delay error can be described as [23]:

$$\Delta\tau \leq \frac{1}{2NB}$$

where N is the number of spectral channels and B is bandwidth. Using performance parameters of the Chang'e-4 lander's low-frequency radio spectrometer as a reference, the delay error required for VLBI processing should be less than 0.819 ms, corresponding to a baseline length error below 245 km. Although geometric delay is the main component of total delay, other factors (such as propagation medium delay) also affect final delay estimation. Therefore, delay

estimation error caused by baseline length estimation error should be smaller than the calculated value. As shown in [Figure 7: see original paper], as the satellite continues operating, its orbit gradually returns near the standard orbit, with maximum difference in the latter half only 50.94 km and average difference 30.00 km. We can conclude that during this phase, the model matches the actual orbit well.

4. Discussion and Outlook

Data show certain differences exist between the model and relay satellite ephemeris, possibly for two reasons: (1) For computational simplicity, this model actually uses a periodic Halo orbit to approximate the actual mission orbit for specific purposes. Given the problem's requirements, small differences between the Halo orbit model and theoretical orbit are acceptable. (2) The relay satellite's actual mission orbit is not a Halo orbit family but an operational orbit designed with the theoretical orbit as its standard, seeking to minimize propellant consumption and extend mission life while maintaining position stability and meeting functional requirements, with easy orbital maintenance control. Both factors significantly affect agreement between model and actual orbit.

This study employs a quasi-periodic Halo orbit model and compares it with theoretical orbit data from the Chang' e-4 Queqiao relay satellite. Differences in baseline lengths formed with the lunar lander are small, with an average difference of 30.00 km.

In the HF band, interferometry will employ fringe search methods requiring baseline precision on the order of thousands of wavelengths, i.e., approximately hundreds of kilometers. This model fundamentally meets requirements for space low-frequency radio interferometry simulation experiments. Before implementing space low-frequency radio interferometry, we can use model establishment and numerical simulation methods to calculate and predict interferometry system composition, methods, and performance. These results will provide preliminary references for implementing space low-frequency radio interferometry.

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