Long-term Spin-down of Two-component Models via Pulsar Magnetic Decay Braking Torque (Theoretical Study) Postprint

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Abstract

The purpose of this paper is to investigate the effect of magnetic field decay on the spin-down of pulsars within a two-component model. Using analytical methods, we examine the temporal evolution of the angular velocities of both components in two-component pulsars under the braking torque due to magnetic field decay. The coupled equations for the two-component model with magnetic field decay and their solutions are presented. Numerical calculations are performed using these analytical solutions for the Crab pulsar (PSR0531+21) and the Vela pulsar (PSR0833-45) within two-component models under magnetic field decay. The numerical results indicate that the angular velocities of both pulsars decrease annually, with the spin-down rate being for Crab and for Vela per year. Finally, the results obtained in this study are discussed, and conclusions regarding the presence of magnetic field decay in the two-component model are drawn.

Full Text

Preamble

The Long-Term Spin-Down of Two-Component Pulsar Models Under Magnetic Decay Braking Torque (Theoretical Study)
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Abstract

This paper investigates the effect of magnetic decay on the spin-down of two-component pulsars using analytical methods. We derive the coupled equations for a two-component model under magnetic decay braking torque and present their analytical solutions. These solutions are applied to the Crab pulsar (PSR0531+21) and the Vela pulsar (PSR0833-45). Numerical results demonstrate that both pulsars experience secular decreases in angular velocity: the Crab pulsar slows down by $-0.1710\,\mathrm{rad/s}$ per year, while the Vela pulsar slows by $-0.0071\,\mathrm{rad/s}$ per year. The implications of these results are discussed, leading to the conclusion that magnetic decay is a significant factor in the long-term spin evolution of two-component pulsar models.

Keywords: pulsars; two-component model; magnetic decay torque; spin-down

Introduction

The Crab pulsar (PSR0531+21) and the Vela pulsar (PSR0833-45) are known to exhibit both starquakes and two-component structure. In this model, the outer layer consists of a charged solid crust, while the inner region comprises neutron superfluid. These two components rotate about the spin axis at different angular velocities under the influence of coupling torques, which include gravitational radiation torque, magnetic radiation torque, and magnetic decay torque [1-3]. In previous work [4], we examined the effect of magnetic radiation braking torque on the spin-down of the Crab pulsar. The present study extends this investigation to analyze the influence of magnetic decay braking torque on the spin evolution of both the Crab and Vela pulsars.

1. Equations of the Two-Component Model Under Magnetic Decay Braking Torque

Based on the fundamental equations for a two-component model under coupling torques presented in [1], we have the following system:

$$I_c \frac{d\Omega_c}{dt} = -\frac{I_c}{\tau}(\Omega_c - \Omega_n) - N \quad (1)$$

$$I_n \frac{d\Omega_n}{dt} = -\frac{I_n}{\tau}(\Omega_n - \Omega_c) \quad (2)$$

where Ω_c and Ω_n are the angular velocities of the outer solid crust and inner neutron superfluid, respectively; I_c and I_n are their moments of inertia; N represents the torque acting on both components; and τ is the microscopic relaxation time. The macroscopic relaxation time τ_c is related to τ through:

$$\frac{1}{\tau_c} = \frac{I}{I_c \tau} \quad (3)$$

where I is the total moment of inertia of the two components:

$$I = I_c + I_n \quad (4)$$

The neutron superfluid abundance Q is defined as:

$$Q = \frac{I_n}{I} \quad (5)$$

which yields:

$$I_n = QI, \quad I_c = (1-Q)I \quad (6)$$

and consequently:

$$\frac{I_n}{I_c} = \frac{Q}{1-Q} \quad (7)$$

We now derive the specific form of the torque under magnetic decay. Based on the pulsar magnetic dipole radiation model, previous work [4] utilized:

$$\frac{d}{dt}(I\Omega) = -\frac{2\mu^2\Omega^3}{3c^3} \quad (8)$$

where μ is the magnetic dipole moment, given by $\mu = \frac{1}{2}BR^3 \sin\alpha$ (with R being the pulsar radius and B the surface magnetic field). Assuming the magnetic dipole is perpendicular to the spin axis ($\alpha = 90^\circ$), we have $\mu = \frac{1}{2}BR^3$.

Following [1], we rewrite this as an angular momentum equation:

$$\frac{dJ}{dt} = -\frac{2\mu^2 J^3}{3c^3 I^3} \quad (9)$$

Integrating this equation yields:

$$\int_{J(0)}^{J(t)} \frac{dJ}{J^3} = -\frac{2\mu^2}{3c^3 I^3} \int_0^t dt \quad (10)$$

where $J(0)$ is the angular momentum at $t = 0$. Taking the present angular velocity and angular momentum as initial conditions, the integration limits become:

$$\int_{J(0)}^{J(t)} \frac{dJ}{J^3} = -\frac{2}{3c^3 I^3} \int_0^t \mu^2 dt \quad (10')$$

If the magnetic dipole moment decays exponentially with time as $\mu^2 = \mu_0^2 e^{-2\xi t}$ [5], where $\mu_0 = \frac{1}{2}BR^3$ and ξ is the magnetic decay coefficient, then substituting this into the integral gives:

$$J(t) = \frac{J(0)}{\sqrt{1 + K(1 - e^{-2\xi t})}} \quad (11)$$

where:

$$K = \frac{2\mu_0^2}{3c^3 I^3 \xi} = \frac{2\mu_0^2 \Omega_0^2}{3c^3 I \xi} \quad (12)$$

Equations (11) and (12) differ significantly from the J and K expressions in [4]. The torque N is derived from (11) as:

$$N = -\frac{dJ}{dt} = \frac{J(0) K \xi e^{-2\xi t}}{[1 + K(1 - e^{-2\xi t})]^{3/2}} \quad (13)$$

or equivalently:

$$N = \frac{J(t) K \xi e^{-2\xi t}}{1 + K(1 - e^{-2\xi t})} \quad (14)$$

Assuming constant moments of inertia I_c and I_n, the angular momentum $J(t)$ from (11) becomes:

$$J(t) = I_c \Omega_c(t) + I_n \Omega_n(t) = I_c \Omega_c(t) \left[1 + \frac{I_n \Omega_n(t)}{I_c \Omega_c(t)}\right] \quad (15)$$

Using (4) and (6), this can be rewritten as:

$$J(t) = I_c \Omega_c(t) \left[1 + \frac{Q}{1 - Q} \frac{\Omega_n(t)}{\Omega_c(t)}\right] \quad (16)$$

Substituting (14) for N and (17) for Ω_n into (1) and utilizing (7) yields the equation for the crust’ s angular velocity under magnetic decay braking:

$$\frac{d\Omega_c}{dt} = -\frac{\Omega_c}{\tau_c} + \frac{Q}{1 - Q} \frac{\Omega_n - \Omega_c}{\tau} - \frac{N}{I_c} \quad (18)$$

Similarly, using (7), the equation for the inner neutron superfluid angular velocity becomes:

$$\frac{d\Omega_n}{dt} = -\frac{\Omega_n - \Omega_c}{\tau} \quad (19)$$

2. Analytical Solution of the Coupled Equations Under Magnetic Decay Braking Torque

We first solve equation (18) for the crust angular velocity $\Omega_c(t)$, then substitute this solution into equation (19) to obtain $\Omega_n(t)$. Since Ω_n represents the internal superfluid angular velocity, which is generally unobservable, and its derivation yields lengthy expressions, we focus our analysis on the observable crust angular velocity and omit the explicit solution for Ω_n.

Equation (18) is a first-order linear differential equation with the integral form:

$$\Omega_c(t) = e^{-\int_0^t \frac{dt}{\tau_c}} \left[\Omega_c(0) + \int_0^t \frac{Q}{1-Q} \frac{\Omega_n(t')}{\tau} e^{\int_0^{t'} \frac{dt''}{\tau_c}} dt' - \int_0^t \frac{N(t')}{I_c} e^{\int_0^{t'} \frac{dt''}{\tau_c}} dt' \right] \quad (20)$$

Let the solution take the form:

$$\Omega_c(t) = Ce^{-t/\tau_c} + \text{particular solution} \quad (21)$$

where C is an integration constant. The integrals in (20) can be evaluated using the expressions for $N(t)$ and $\Omega_n(t)$. The second term in the integrand can be expanded using the binomial theorem since $Ke^{-2\xi t} \ll 1$, allowing us to neglect higher-order terms.

Setting $t = 0$ determines the constant C:

$$C = \Omega_c(0) - [\text{terms from integrals evaluated at t=0}] \quad (22)$$

After substituting C back into (21), we obtain the analytical solution:

$$\Omega_c(t) = \Omega_c(0) \left[1 - \frac{K}{2(1+QK)} (1 - e^{-2\xi t}) \right] e^{-t/\tau_c} + \frac{QK^2\xi}{2(1+QK)} \left[\frac{1 - e^{-(2\xi + 1/\tau_c)t}}{2\xi + 1/\tau_c} \right] \quad (23)$$

This expression gives the time evolution of the crust angular velocity under magnetic decay braking.

3. Numerical Results for PSR0531+21 (Crab) and PSR0833-45 (Vela)

We investigate the spin evolution of the crust component for these two pulsars under magnetic decay. Table 1 lists their physical parameters .

Table 1 Physical parameters for PSR0531+21 (Crab) and PSR0833-45 (Vela)

The parameters Ω_0, Q, and τ are taken from references [6,7], surface magnetic fields from [8], and we adopt a typical neutron star radius $R = 10^6$ cm and moment of inertia $I = 10^{45}$ g·cm² from [1,9].

Substituting the values from Table 1 along with R, I, and $\mu_0 = BR^3/2$ into equations (11) and (13) yields the values of K shown in Table 2 .

Table 2 Numerical results for PSR0531+21 and PSR0833-45 (t=1 yr)

The calculations show that for a time interval of $t = 1$ year, the Crab pulsar' s angular velocity decreases by $-0.1710\,\text{rad/s}$, while the Vela pulsar' s decreases by $-0.0071\,\text{rad/s}$.

4. Discussion and Conclusions

(1) The numerical results in Table 2 demonstrate that both pulsars exhibit secular decreases in crust angular velocity under magnetic decay torque. The Crab pulsar' s spin-down rate ($-0.1710\,\text{rad/s/yr}$) exceeds that of the Vela pulsar ($-0.0071\,\text{rad/s/yr}$), primarily due to differences in their K values. Since K depends on both magnetic field strength and angular velocity, and the Crab pulsar has a stronger magnetic field and more than twice the angular velocity of Vela (see Table 1), its K value is substantially larger, resulting in greater braking.

(2) Compared with our previous study [4], the magnetic radiation torque caused a spin-down of $-0.2450\,\text{rad/s/yr}$ for the Crab pulsar, whereas magnetic decay produces $-0.1710\,\text{rad/s/yr}$. Thus, magnetic decay is slightly less effective than magnetic radiation braking. While magnetic radiation arises from conversion of rotational energy, magnetic decay stems from field strength reduction. However, the energy loss due to radiation dominates over that from field decay, making magnetic radiation the more significant spin-down mechanism.

(3) Regarding the integration limits in equation (10), the lower limit $t = 0$ could represent the pulsar' s birth time, but this work does not address early evolution. Alternatively, it could mark the time after a starquake, though post-starquake values of $J(0)$ and $\Omega(0)$ are uncertain. Since we know the present values (Table 1), setting $t = 0$ at the present epoch is most appropriate for our analysis.

(4) The binomial expansion used in deriving equations (21) and (22) requires $Ke^{-2\xi t} \ll 1$. Estimating its magnitude: with $\tau = 10^8$ years and $\xi = 10^{-11.16}\,\text{yr}^{-1}$, for $t = 1$ year we have $e^{-2\xi t} \approx 1$. For the Crab pulsar, $K = 1546.2$, giving $Ke^{-2\xi t} \approx 0.0017 \ll 1$; for Vela, $K = 170.2$, yielding $0.00018 \ll 1$. Thus the expansion is valid and higher-order terms can be neglected.

5. Starquake Behavior in PSR0531+21 and PSR0833-45

Both pulsars exhibit periodic starquakes: the Crab every ~3 years and Vela every ~2 years. These events cause sudden, temporary spin-ups followed by relaxation to pre-glitch values. This periodic glitch activity is superimposed on the secular spin-down trend. Our analysis concerns only the long-term braking effect, which is not affected by these transient spin-up events [10,11].

6. Final Conclusion

This study demonstrates that magnetic decay in two-component pulsar models is a real physical process that contributes to the long-term spin-down of pulsars. The braking effect operates through the decay of the magnetic dipole moment, which reduces the angular momentum and consequently slows the rotation. While less dominant than magnetic radiation braking, magnetic decay represents a measurable component of pulsar spin evolution.

Note: Figure translations are in progress. See original paper for figures.