

A Postprint Study on the Measurement Method for Laser Uplink Angle-of-Arrival Fluctuations in Laser Guide Star Adaptive Optics Systems

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Abstract

The laser guide star wavefront tilt measurement problem constitutes one of the key limitations restricting the widespread application of adaptive optics technology in astronomy. Measuring and correcting laser uplink arrival angle fluctuations represents an effective solution to this problem. We propose a measurement method based on statistical averaging algorithms that operates without reliance on natural guide stars or auxiliary telescopes, enabling effective measurement of laser uplink arrival angle fluctuations. This approach employs a Hartmann wavefront sensor with a sub-aperture array to detect the laser beacon, selecting a subset of sub-apertures for statistical averaging of tilt measurements to obtain the laser uplink arrival angle fluctuations. The relationship between the statistical averaging algorithm error and the number of sub-apertures is investigated through simulation. The results indicate that the reduction ratio of the minimum algorithm error relative to the telescope's full-aperture tilt error is independent of the atmospheric Fried parameter, but depends on the telescope aperture diameter. Larger telescope apertures result in greater reduction of the algorithm error relative to the full-aperture tilt error. For a telescope aperture of 10 m, the minimum algorithm error is reduced to 33% of the telescope's full-aperture tilt error.

Full Text

Research on a Measurement Method for Laser Uplink Angle-of-Arrival Fluctuations in Laser Guide Star Adaptive Optics Systems

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Abstract

Tip-tilt indetermination with laser guide stars (LGS) represents one of the key challenges limiting the widespread application of adaptive optics technology in astronomy. Measuring and correcting angle-of-arrival fluctuations of the laser uplink offers an effective solution to this problem. This paper proposes a measurement method based on statistical averaging algorithms that operates without relying on natural guide stars or auxiliary telescopes, enabling effective measurement of laser uplink angle-of-arrival fluctuations. The approach utilizes a Shack-Hartmann wavefront sensor with a subaperture array to detect the laser beacon, selecting a subset of subapertures for statistical averaging of tilt measurements to obtain the laser uplink angle-of-arrival fluctuations. We simulate the relationship between the algorithm error and the number of selected subapertures. The results demonstrate that the reduction ratio of the minimum algorithm error relative to the telescope's full-aperture tilt error is independent of the atmospheric Fried parameter but depends on the telescope aperture. Larger telescope apertures yield greater error reduction. For a 10-meter telescope, the minimum algorithm error decreases to 33% of the telescope's full-aperture tilt error.

Keywords: laser guide star; optimization algorithm; wavefront tilt; subaperture

Adaptive optics systems require sufficiently bright guide stars for wavefront sensing. These can be either the target itself or a natural guide star of appropriate brightness located within the isoplanatic angle of the target—such systems are termed Natural Guide Star Adaptive Optics (NGS-AO). In 1985, Foy and Labeyrie proposed applying artificial laser guide stars to astronomical adaptive optics observations [1]. A laser guide star is projected within the target's isoplanatic region to serve as a beacon for wavefront sensing in what is called a Laser Guide Star Adaptive Optics (LGS-AO) system. An ideal artificial beacon must be sufficiently bright and maintain stable relative positioning with respect to the target. While current laser guide stars can achieve the brightness required for astronomical observations (the Next Generation Laser system at Keck II can produce a sodium guide star equivalent to magnitude $R=7.5$ within a zenith angle of 15° [2]), their position stability remains problematic, necessitating a natural star within the tip-tilt isoplanatic angle to correct wavefront tilt. As shown in [Figure 1: see original paper], atmospheric turbulence along the laser uplink path causes instantaneous offsets between the laser beacon and the transmission optical axis, resulting in distorted wavefront tilt measurements by the adaptive optics wavefront sensor.

Research on LGS wavefront tilt measurement methods began with Foy et al. in

the 1990s [3]. In 1995, Ragazzoni et al. proposed using an auxiliary telescope to project the laser while the main telescope observed the guide star from the side [4], reporting experimental results in 2000: at Calar Alto Observatory, a 3.6m telescope projected the laser while a 2.2m telescope observed it from the side, successfully detecting one-dimensional wavefront tilt using a natural guide star [5]. In 2000, Belenki proposed the opposite configuration, with the main telescope transmitting and an auxiliary telescope detecting the laser projection, applying statistical averaging algorithms to the imaged laser streak to determine uplink wavefront tilt [6]. In 1997, Fan and Song analyzed LGS-AO tip-tilt measurement theoretically and compared various auxiliary telescope configurations [7]. Beginning in 2000, Yunnan Observatories and the Institute of Optics and Electronics collaborated on experimental research for laser Rayleigh guide stars [8] while investigating LGS-based atmospheric tip-tilt measurement [9-10], experimentally validating the effectiveness of statistical averaging algorithms for LGS tip-tilt measurement in 2014 using a 1.2m primary telescope and a 25cm auxiliary telescope at Yunnan Observatories [11-12]. In 2016, Meiland et al. analyzed application conditions for the tilt “reciprocity” assumption and proposed a multi-layer tomography technique for LGS tip-tilt detection under non-reciprocity conditions [13-14]. Also in 2016, Luo et al. proposed a vortex beam-based LGS that forms a more stable artificial beacon compared to Gaussian beams, demonstrating a 31% reduction in laser uplink jitter variance for a topological charge of 10 [15].

This paper proposes a wavefront tilt measurement method based on telescope subaperture detection, using an optimization algorithm to isolate laser uplink angle-of-arrival fluctuations from subaperture measurement data. Unlike Reeves et al.’s multi-layer tomographic approach, this scheme requires only a single laser guide star to obtain a stable artificial beacon while effectively avoiding detection difficulties associated with auxiliary telescope tracking and pointing issues.

1 System Layout of the Laser Guide Star Wavefront Tilt Measurement Method

The system configuration for the telescope subaperture-based LGS tip-tilt measurement method is illustrated in [Figure 2: see original paper]. The laser is transmitted from the launch telescope to create an artificial beacon at high altitude, whose light propagates back through atmospheric turbulence to the main telescope. The aberrated beacon wavefront first passes through the wavefront correction system (comprising a tip-tilt mirror TM and a deformable mirror DM), then splits via a beamsplitter into imaging and wavefront sensing channels. Assuming negligible atmospheric turbulence effects between the LGS and the observation target, a sufficiently stable beacon would enable real-time correction of target wavefront aberrations for high-resolution imaging. However, atmospheric turbulence along the laser uplink path causes instantaneous offsets between the laser beacon and transmission axis, introducing an additional tilt

component to the received wavefront aberration. The wavefront signal consists of three components: overall downlink tilt from the laser beacon, high-order wavefront distortion, and additional uplink tilt. Without correction of the uplink tilt, the target image in the camera becomes unstable, preventing long-exposure imaging. This paper proposes an algorithm to extract beacon uplink jitter information from the wavefront sensor data, enabling the controller to generate signals for the fast tip-tilt mirror in the laser launch telescope to stabilize the laser beacon. The wavefront detector outputs three signal channels to control: the deformable mirror DM, the receiving telescope tip-tilt mirror TM, and the transmitting telescope fast tip-tilt mirror.

2 Wavefront Tilt Extraction Method

[Figure 3: see original paper] schematically illustrates the LGS subaperture measurement method. The receiving telescope aperture is divided into a regular array of subapertures, with yellow indicating the downlink path of beacon light received by subapertures (only partial light paths are shown for reference) and green indicating the laser uplink path. The laser beam experiences atmospheric turbulence twice: during uplink transmission and downlink reception. Wavefront tilt within each subaperture causes lateral displacement of the microlens focal spot. By measuring the spot centroid displacement relative to a reference position calibrated with a parallel beam, the average wavefront slope in both directions for each subaperture can be determined.

The wavefront tilt measured by any subaperture i consists of the sum of the laser uplink jitter θ_{up} and the downlink subaperture tilt $\theta_{\text{down},i}$:

$$\theta_i = \theta_{\text{up}} + \theta_{\text{down},i} \quad (1)$$

If N subapertures are randomly selected (marked red in [Figure 3: see original paper]), the statistical average of the wavefront tilts measured by these subapertures yields:

$$\frac{1}{N} \sum_{i=1}^N \theta_i = \theta_{\text{up}} + \frac{1}{N} \sum_{i=1}^N \theta_{\text{down},i} \quad (2)$$

For each subaperture, although the downlink path experiences different turbulence (i.e., $\theta_{\text{down},i}$ varies), the uplink-induced jitter θ_{up} is identical. Therefore, Equation (2) can be rewritten as:

$$\frac{1}{N} \sum_{i=1}^N \theta_i - \frac{1}{N} \sum_{i=1}^N \theta_{\text{down},i} = \theta_{\text{up}} \quad (3)$$

where θ_i is obtained from subaperture measurements and $\theta_{\text{down},i}$ is unknown. Assuming the downlink wavefront tilts $\theta_{\text{down},i}$ of the N selected subapertures

are independent random variables, their statistical average approaches zero: $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \theta_{\text{down},i} = 0$. Thus, the statistical average of wavefront tilts measured by these N subapertures equals the laser uplink jitter θ_{up} . In practice, two factors introduce algorithmic error in determining θ_{up} [16]: (1) the number of selected subapertures is finite, and (2) the downlink wavefront tilts $\theta_{\text{down},i}$ exhibit some correlation between subapertures.

The algorithm error is given by:

$$\sigma_{\theta}^2 = \langle (\theta - \langle \theta \rangle)^2 \rangle = \frac{\sigma_{\text{down}}^2}{N} + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \rho_{ij} \sigma_{\text{down},i} \sigma_{\text{down},j} \quad (4)$$

where σ_{down}^2 is the subaperture tilt variance, d is the subaperture diameter, r_0 is the atmospheric turbulence Fried parameter, and ρ_{ij} is the normalized tilt correlation coefficient between any subapertures i and j , expressed as [12]:

$$\rho_{ij} = \frac{\langle \theta_{\text{down},i} \theta_{\text{down},j} \rangle}{\sigma_{\text{down},i} \sigma_{\text{down},j}} \quad (5)$$

with

$$\sigma_{\text{down}}^2 = 0.182 \lambda^2 r_0^{-5/3} d^{-1/3} \quad (6)$$

and

$$\langle \theta_{\text{down},i} \theta_{\text{down},j} \rangle = \frac{\int_0^H C_n^2(h) I_0\left(\frac{kd\rho}{h}\right) dh}{\int_0^H C_n^2(h) dh} - \frac{\int_0^H C_n^2(h) I_0\left(\frac{kd}{h}\right) dh}{\int_0^H C_n^2(h) dh} \quad (7)$$

where $C_n^2(h)$ is the atmospheric refractive index structure constant, J_n is the n -th order Bessel function, $k = 2\pi/\lambda$ is the wavenumber, H is the laser guide star altitude, and ρ is the separation between subaperture centers.

For a fixed LGS altitude and constant turbulence conditions, the tilt correlation coefficient ρ_{ij} between any subapertures decreases with increasing separation. Therefore, from Equations (4)-(7), the algorithm error σ_{θ}^2 depends on both the number of selected subapertures N and the distances between subapertures.

3 Numerical Calculation Results and Analysis

Equation (4) is used to calculate the algorithm error for randomly selected subapertures. As shown in [Figure 4: see original paper], N subapertures are selected within the telescope aperture. The normalized tilt correlation coefficient between any two subapertures is calculated using Equation (6), where

inter-subaperture distances are determined from their coordinates, and the atmospheric turbulence Fried parameter is computed from:

$$r_0 = \left[0.423k^2 \sec \beta \int_0^L C_n^2(z) dz \right]^{-3/5} \quad (8)$$

using the Hufnagel turbulence structure constant profile model:

$$C_n^2(h) = 2.72 \times 10^{-16} \left[3 \exp\left(-\frac{h}{1.5}\right) + 10 \exp\left(-\frac{h}{10}\right) \right] h^{-10} v^2 \quad (9)$$

In these equations, β is the zenith angle and v is the wind speed as a function of altitude. For simplified calculations, the zenith angle is set to 0° and wind speed is treated as constant. Equations (6)-(9) relate the normalized tilt correlation coefficient to the Fried parameter r_0 .

[Figure 4: see original paper] Subaperture selection schematic. The large circle represents the telescope aperture, while small solid circles represent selected subapertures.

Substituting the normalized tilt correlation coefficient into Equation (4) yields the algorithm error. Multiple random selections produce the simulation results.

The simulation results are presented in [Figure 5: see original paper], showing the relationship between algorithm error and the number of selected subapertures for the subaperture optimization algorithm. Two reference lines indicate subaperture tilt error and full telescope aperture tilt error, calculated from [17-18]:

$$\sigma_{\text{tilt}}^2 = 0.182\lambda^2 r_0^{-5/3} D^{-1/3} \quad (10)$$

where D is the aperture diameter. The results show that when only one subaperture is selected, statistical averaging is absent and the algorithm error equals the subaperture tilt error. As the number of subapertures increases, statistical averaging reduces the algorithm error. For a 1-meter telescope, the algorithm error is minimized when selecting 10-20 subapertures; for a 3-meter telescope, the minimum occurs with 20-50 subapertures. When too many subapertures are selected, the error approaches the full-aperture tilt error because dense subaperture sampling increasingly approximates the full-aperture wavefront tilt.

For a 1-meter telescope, the minimum algorithm error reduces to 70% of the full-aperture tilt error; for a 3-meter telescope, it reduces to 50%. [Figure 5: see original paper] demonstrates that the reduction in minimum algorithm error relative to full-aperture tilt error does not vary with atmospheric turbulence (characterized by r_0) but depends solely on telescope aperture. Larger apertures produce greater error reduction. The simulation shows that for a 10-meter

telescope, the minimum algorithm error decreases to 33% of the full-aperture tilt error. In this case, if the laser is expanded to 1-meter for transmission, the laser uplink jitter error is reduced by 78% compared to the uncorrected scenario.

[Figure 5: see original paper] Algorithm error of tilt measurement as a function of the number of selected subapertures.

4 Conclusion

In the proposed subaperture optimization algorithm, when randomly selecting a subset of telescope subapertures, the atmospheric turbulence effects on each subaperture are relatively independent. The wavefront tilt within each subaperture causes random displacements of the microlens focal spot. Using a parallel-beam calibrated reference position as zero, the average of multiple random displacements approaches zero, converging closer to zero as more subapertures are included. However, as the number of selected subapertures continues to increase, the independence of turbulence effects between subapertures decreases, causing their average to approach the full-aperture wavefront tilt.

Simulation results demonstrate that selecting an appropriate number of subapertures for statistical averaging effectively reduces the impact of atmospheric turbulence on downlink beacon wavefront tilt. Equation (4) then yields the laser uplink angle-of-arrival fluctuation information for correcting the LGS projection system to achieve a more stable laser guide star. As telescope aperture increases, the relative error introduced by the statistical averaging algorithm decreases. For a 10-meter telescope, the algorithm error is only 33% of the full-aperture tilt error.

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