Tensor Representation of Multipartite Multi-Degree-of-Freedom Quantum Teleportation

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Abstract

[Purpose] To find a general representation and algebraic structure for quantum teleportation in the most general case.

[Method] Inductive and deductive reasoning, utilizing the equivalence between polynomial multiplication and tensor products to discover general patterns.

[Results] Generalizing quantum teleportation from many-body single-degree-of-freedom or single-body many-degree-of-freedom to many-body many-degree-of-freedom, as well as to mixed states.

[Limitations] Cannot be generalized to continuous-spectrum quantum states.

[Conclusion] Predicts the existence of single-shot many-body many-degree-of-freedom quantum teleportation and includes all special cases that have been experimentally verified.

Full Text

Tensor Representation of Quantum Teleportation for Many-Body Systems with Multiple Degrees of Freedom

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Abstract:
[**Objective**] To find a general representation and algebraic structure for quantum teleportation in the most universal case.
[**Methods**] Inductive and deductive reasoning, utilizing the equivalence between polynomial multiplication and tensor product to discover general

patterns.
[**Results**] Generalization of quantum teleportation from many-body single-degree-of-freedom or single-body multi-degree-of-freedom cases to many-body multi-degree-of-freedom systems, including mixed states.
[**Limitations**] Cannot be extended to continuous-spectrum quantum states.
[**Conclusions**] Predicts the existence of single-shot quantum teleportation for many-body multi-degree-of-freedom systems, encompassing all special cases that have been experimentally verified.

Keywords: Quantum Teleportation, Bell States, Unitary Transformation

1 Introduction

Quantum teleportation is a fascinating and promising quantum mechanical effect first proposed in 1993 by Charles Bennett et al. [**?**]. Teleportation of two-particle entangled states was subsequently proposed by Li Dachuang and Cao Zhuoliang [**?**]. In 2015, Pan Jianwei, Lu Chaoyang and colleagues introduced and successfully demonstrated quantum teleportation of multiple properties of a single photon [**?**]. In this paper, I derive a more general representation of quantum teleportation by utilizing the tensor product between multiple particle properties and the unitary transformation between Bell states and standard orthonormal bases.

2 Single-Qubit Case

We begin by briefly reviewing the simplest scenario. Alice and Bob each hold one particle from an entangled pair: $|\varphi_{AB}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. Simultaneously, Alice possesses particle X in the state $|\psi_X\rangle = \alpha|0\rangle_X + \beta|1\rangle_X$, where $\alpha, \beta \in \mathbb{C}$, $\mathbb{C}$ is the complex field, and $|\alpha|^2 + |\beta|^2 = 1$. The total state is formed by taking the tensor product of X and AB:

$$|\Psi\rangle = |\psi_X\rangle \otimes |\varphi_{AB}\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle_X|0\rangle_A|0\rangle_B + \alpha|0\rangle_X|1\rangle_A|1\rangle_B + \beta|1\rangle_X|0\rangle_A|0\rangle_B + \beta|1\rangle_X|1\rangle_A|1\rangle_B) \tag{2.1}$$

The four Bell states are defined as:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle), \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle-|11\rangle), \quad |\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle+|10\rangle), \quad |\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle-|10\rangle) \tag{2.2}$$

Clearly, the linear space spanned by the Bell states is isomorphic to the two-dimensional second-order tensor space, making them equivalent to the standard orthonormal basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. The basis transformation yields:

$$|00\rangle = \frac{1}{\sqrt{2}}(|\phi^+\rangle + |\phi^-\rangle), \quad |11\rangle = \frac{1}{\sqrt{2}}(|\phi^+\rangle - |\phi^-\rangle), \quad |01\rangle = \frac{1}{\sqrt{2}}(|\psi^+\rangle + |\psi^-\rangle), \quad |10\rangle = \frac{1}{\sqrt{2}}(|\psi^+\rangle - |\psi^-\rangle) \tag{2.3}$$

Substituting these into $|\Psi\rangle$ gives:

$$|\Psi\rangle = \frac{1}{2}[|\phi^+\rangle_{XA}(\alpha|0\rangle_B + \beta|1\rangle_B) + |\phi^-\rangle_{XA}(\alpha|0\rangle_B - \beta|1\rangle_B) + |\psi^+\rangle_{XA}(\alpha|1\rangle_B + \beta|0\rangle_B) + |\psi^-\rangle_{XA}(\alpha|1\rangle_B - \beta|0\rangle_B)] \tag{2.4}$$

Bob must prepare four unitary transformations corresponding to the possible measurement outcomes. Alice performs a Bell state measurement on particles X and A, obtaining $|\phi^+\rangle_{XA}$, $|\psi^+\rangle_{XA}$, $|\phi^-\rangle_{XA}$, and $|\psi^-\rangle_{XA}$ with equal probability, which correspond to the binary codes 00, 01, 10, and 11 respectively. She then transmits the measurement result to Bob via classical communication. Bob applies the corresponding unitary transformation to his particle, yielding the final state $|\psi'_B\rangle = \alpha|0\rangle_B + \beta|1\rangle_B$, which completes the teleportation.

We now describe the entire process using tensor notation. The initial states can be expressed as $|\psi_X\rangle = (\alpha, \beta)\begin{pmatrix}|0\rangle_X \\ |1\rangle_X\end{pmatrix} = K_i|i\rangle_X$ and $|\varphi_{AB}\rangle = \frac{1}{\sqrt{2}}\delta_{ij}|ij\rangle_{AB}$. The basis transformation between Bell states and the computational basis is given by:

$$(|\phi_{00}\rangle, |\phi_{01}\rangle, |\phi_{10}\rangle, |\phi_{11}\rangle) = (|00\rangle, |01\rangle, |10\rangle, |11\rangle)\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1\end{pmatrix} \tag{2.5}$$

Denoting this transformation matrix as U, its explicit representation is:

$$\mathbf{U} = e_{00}e_{00} + e_{00}e_{10} + e_{01}e_{01} - e_{01}e_{11} + e_{10}e_{01} + e_{10}e_{11} + e_{11}e_{00} - e_{11}e_{10}$$

In component form, this is $U_{ij,lm}$, where the first two indices are row indices and the last two are column indices in the matrix representation. Notably, U is unitary with respect to the ij indices (currently all real, so it could also be called symmetric). For example, when $l = 0, m = 0$, we have $\mathbf{U} = e_{00}e_{00} + e_{11}e_{00} =$

$(e_{00}+e_{11})e_{00} = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} e_{00}$. Furthermore, U is unitary between the ij and lm indices, meaning the transformation matrix between bases $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$ and $(|\phi_{00}\rangle, |\phi_{01}\rangle, |\phi_{10}\rangle, |\phi_{11}\rangle)$ is manifestly unitary. However, U is not unitary with respect to the lm indices alone.

Writing Eq. (2.5) in component form gives:

$$|\phi_{lm}\rangle = U_{ij,lm}|ij\rangle \tag{2.5'}$$

Inverting this relation yields:

$$|ij\rangle = U^*_{lm,ij}|\phi_{lm}\rangle \tag{2.6}$$

which is equivalent to $(|00\rangle, |01\rangle, |10\rangle, |11\rangle) = (|\phi_{00}\rangle, |\phi_{01}\rangle, |\phi_{10}\rangle, |\phi_{11}\rangle)\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 1\\0 & 1 & 1 & 0\\0 & 1 & -1 & 0\\1 & 0 & 0 & -1\end{pmatrix}$.

Substituting (2.6) into $|\Psi\rangle$ gives:

$$|\Psi\rangle = K_i\delta_{jk}U^*_{lm,ij}|\phi_{lm}\rangle_{XA}|k\rangle_B = K_iU^*_{lm,ik}|\phi_{lm}\rangle_{XA}|k\rangle_B$$

Following the protocol described earlier, Alice performs a Bell measurement on particles X and A to determine the values of l and m, which she transmits to Bob. Bob then applies the unitary transformation $U_{lm,ij}$ (with the received lm indices) to particle B. The transformation yields:

$$\mathbf{U}_{lm}|\Psi\rangle = K_iU^*_{lm,ik}U_{lm,rk}|\phi_{lm}\rangle_{XA}|k\rangle_B$$

Using the unitarity of U, this simplifies to $K_i\delta_{ir}|\phi_{lm}\rangle_{XA}|r\rangle_B = K_n|\phi_{lm}\rangle_{XA}|r\rangle_B$, resulting in the final teleported state $|\varphi'_B\rangle = K_r|r\rangle_B$.

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3 Multiple Particles, Single Attribute

We now generalize to the scenario of teleporting multiple particles simultaneously, where each particle carries a single attribute in a Hilbert space of dimension D. The quantum state to be teleported is:

$$|\psi_X\rangle = K_{i_1i_2\ldots i_n}|i_1\rangle_{X_1}\otimes|i_2\rangle_{X_2}\otimes\ldots\otimes|i_n\rangle_{X_n} = K_{i_1i_2\ldots i_n}|i_1i_2\ldots i_n\rangle_X \tag{3.1}$$

This representation encompasses all possible states, whether product states, entangled states, or partially entangled states. As shown in [Figure 1: see original paper], each particle between Alice and Bob must be pairwise entangled. We define the maximally entangled state for the i-th pair A_iB_i as $|\phi_i\rangle_{AB} = \delta_{A_iB_i}|A_iB_i\rangle_{AB}$, where A_iB_i represents the index. The hyper-entangled state shared by Alice and Bob is then:

$$|\varphi_{AB}\rangle = |\phi_1\rangle_{AB}\otimes|\phi_2\rangle_{AB}\otimes...\otimes|\phi_n\rangle_{AB} = \frac{1}{D^{n/2}}\delta_{A_1B_1}\delta_{A_2B_2}...\delta_{A_nB_n}|A_1B_1A_2B_2...A_nB_n\rangle_{AB} \tag{3.2}$$

The total quantum state of the XAB system is:

$$|\Psi\rangle = |\psi_X\rangle\otimes|\varphi_{AB}\rangle = \frac{1}{D^{n/2}}K_{i_1i_2...i_n}\delta_{A_1B_1}\delta_{A_2B_2}...\delta_{A_nB_n}|i_1i_2...i_nA_1B_1A_2B_2...A_nB_n\rangle \tag{3.3}$$

Reference [**?**] provides a set of Bell bases and associated unitary transformations for $D \geq 2$:

$$|\phi_{lm}\rangle = \frac{1}{\sqrt{D}}\sum_{j=0}^{D-1} e^{2\pi ijl/D}|j\rangle \otimes |(j+m) \bmod D\rangle \tag{3.4}$$

$$\mathbf{U}_{lm} = \sum_{k=0}^{D-1} e^{2\pi ikl/D}|k\rangle\langle(k+m) \bmod D| \tag{3.5}$$

The unitarity of (3.5) is evident. To obtain its component representation, we multiply (3.4) by $\langle a| \otimes \langle b|$, yielding $\langle a| \otimes \langle b|\phi_{lm}\rangle = \delta_{aj}\delta_{b,(j+m) \bmod D}e^{2\pi ijl/D} = \delta_{b,(a+m) \bmod D}e^{2\pi ial/D}$. Therefore:

$$|\phi_{lm}\rangle = \sum_{a,b=0}^{D-1} \delta_{b,(a+m) \bmod D}e^{2\pi ial/D}|ab\rangle, \quad U_{nm,ab} = \delta_{b,(a+m) \bmod D}e^{2\pi ial/D} \tag{3.6}$$

Analogous to Eq. (2.5), $\mathbf{U}_{lm}$ represents the unitary transformation between the $|\phi_{lm}\rangle$ and $|ab\rangle$ bases.

To facilitate index manipulation, we simplify (3.6). Since indices a and m both range from 0 to $D-1$, the sum $a+m$ ranges from 0 to $2D-2$, meaning $a+m$ can exceed D by at most $D-2$. Thus, $\delta_{b,(a+m) \bmod D} = \delta_{b,a+m} + \delta_{b,a+m-D}$. The unitarity condition gives:

$$\sum_{l,m=0}^{D-1} U^*_{ij,lm} U_{lm,ab} = D\delta_{ia}\delta_{jb}$$

Consequently, we obtain the inverse transformation:

$$|ij\rangle = U^*_{lm,ij}|\phi_{lm}\rangle \tag{3.7}$$

Applying this to Eq. (3.3) yields:

$$|\Psi\rangle = \frac{1}{D^n} K_{i_1 i_2 \dots i_n} U_{l_2 m_2, i_2 A_2} \dots U_{l_n m_n, i_n A_n} \delta_{A_1 B_1} \dots \delta_{A_n B_n} U_{l_1 m_1, i_1 A_1} |\phi_{l_1 m_1}\rangle |\phi_{l_2 m_2}\rangle \dots |\phi_{l_n m_n}\rangle |B_1 B_2 \dots B_n\rangle$$

Alice performs hyper-entangled Bell state measurements (h-BSM) [**?**] on each particle pair $(X_1A_1, X_2A_2, \dots, X_nA_n)$, obtaining the binary sequences $l_1m_1, l_2m_2, \dots, l_nm_n$, which she transmits to Bob. Bob then applies the corresponding sequence of unitary transformations to his particles:

$$\mathbf{U}_{l_1 m_1} \dots \mathbf{U}_{l_n m_n} |\Psi\rangle = \frac{1}{D^n} K_{B'_1 \dots B'_n} |B'_1 \dots B'_n\rangle$$

completing the teleportation. It is worth noting that the Bell bases in (3.4) and unitary transformations in (3.5) are not unique; teleportation can be achieved with any set satisfying certain conditions, as discussed in Theorem 1 of reference [**?**].

Using the notation from [**?**], we can combine multiple scalar indices into a vector index, simplifying the above formulas. Let $|\psi_X\rangle = \sum_{\vec{i}} K_{\vec{i}} |\vec{i}\rangle$ (3.1'), $|\varphi_{AB}\rangle = \sum_{\vec{A},\vec{B}} \delta_{\vec{A}\vec{B}} |\vec{A}\vec{B}\rangle$ (3.2'), and $|\Psi\rangle = \sum_{\vec{i},\vec{A},\vec{B}} \frac{1}{D^n} K_{\vec{i}} \delta_{\vec{A}\vec{B}} U_{\vec{l}\vec{m},\vec{i}\vec{A}} |\phi_{\vec{l}\vec{m}}\rangle |\vec{B}\rangle$, where $\delta_{\vec{A}\vec{B}} = \delta_{A_1B_1} \dots \delta_{A_nB_n}$, $|\phi_{\vec{l}\vec{m}}\rangle = |\phi_{l_1m_1}\rangle \dots |\phi_{l_nm_n}\rangle$, $\vec{i} = (i_1, i_2, \dots, i_n)^T$, and $\vec{i} \in [0, D-1]^n \cap \mathbb{N}^n$ (with $\mathbb{N}$ the set of natural numbers). Similar definitions apply to other vector indices. Hereafter, we omit explicit summation symbols, with vector indices understood as dummy indices summed over all possible values.

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4 Multiple Particles, Multiple Attributes

The quantum state to be teleported is:

$$|\psi_X\rangle = K_{\mathbf{X}}|\mathbf{X}\rangle = K_{X_I^1 X_I^2 \dots X_I^{n_I} X_{II}^1 X_{II}^2 \dots X_{II}^{n_{II}} \dots X_N^2 \dots X_N^{n_N}} |X_I^1 X_I^2 \dots X_I^{n_I} X_{II}^1 X_{II}^2 \dots X_{II}^{n_{II}} \dots X_N^2 \dots X_N^{n_N}\rangle \tag{4}$$

Here the index $\mathbf{X}$ is itself a tensor, distinguished by uppercase letters with Roman numerals. Subscripts denote particle number, while superscripts denote property number. The structure is illustrated in [Figure 2: see original paper]. For example, P_{II}^1 represents the first property of the second particle, living in a Hilbert space of dimension D_{II}^1, with other properties defined similarly. The index $\mathbf{X}$ belongs to the set $([0, D_I^1 - 1] \times ... [0, D_I^{n_I} - 1]) \times ... ([0, D_N^1 - 1] \times ... [0, D_N^{n_N} - 1]) \cap \mathbb{N}^{n_I + n_{II} + ... + n_N}$, and other tensor indices follow the same convention. The structure can be summarized as: "particles in different locations are entangled with each other, while identical properties are entangled with each other."

Analogous to Eq. (3.3), we write the total state vector as:

$$|\Psi\rangle = \mathcal{N} K_{\mathbf{X}} \delta_{\mathbf{A},\mathbf{B}} |\mathbf{XAB}\rangle = \mathcal{N} K_{X_I^1 ... X_I^{n_I} X_{II}^1 ... X_{II}^{n_{II}} ... X_N^{n_N}} \delta_{A_I^1 B_I^1} ... \delta_{A_I^{n_I} B_I^{n_I}} ... \delta_{A_N^1 B_N^1} ... \delta_{A_N^{n_N} B_N^{n_N}} |X_I^1 ... X_I^{n_I} X_{II}^1 ... X_I^{n} \tag{4.1}$$

where $\mathcal{N}$ is a normalization coefficient. Substituting Eq. (3.7) yields:

$$|\Psi\rangle = \mathcal{N} K_{\mathbf{X}} \delta_{\mathbf{A},\mathbf{B}} |\mathbf{XAB}\rangle = \mathcal{N}' K_{\mathbf{X}} \delta_{\mathbf{A},\mathbf{B}} U_{\mathbf{lm},\mathbf{XA}} |\phi_{\mathbf{lm}}\rangle |\mathbf{B}\rangle \tag{4.2}$$

In component form:

$$|\Psi\rangle = \mathcal{N}' K_{X_I^1 ... X_N^{n_N}} \delta_{A_I^1 B_I^1} ... \delta_{A_N^{n_N} B_N^{n_N}} U_{l_I^1 m_I^1, X_I^1 A_I^1} ... U_{l_N^{n_N} m_N^{n_N}, X_N^{n_N} A_N^{n_N}} |\phi_{l_I^1 m_I^1}\rangle ... |\phi_{l_N^{n_N} m_N^{n_N}}\rangle |B_I^1 ... B_N^{n_N}\rangle \tag{4.2'}$$

Alice performs hyper-entangled Bell state measurements on the X and A particle groups according to the pattern shown in [Figure 2: see original paper]: $X_I^1 A_I^1, ..., X_I^{n_I} A_I^{n_I}, X_{II}^1 A_{II}^1, ..., X_N^{n_N} A_N^{n_N}$. She obtains $n_I \cdot n_{II} \cdot ... \cdot n_N$ integer pairs: $l_I^1 m_I^1, ..., l_I^{n_I} m_I^{n_I}, l_{II}^1 m_{II}^1, ..., l_N^{n_N} m_N^{n_N}$. As before, Alice transmits these ordered pairs—represented as two tensors $\mathbf{l}$ and $\mathbf{m}$—to Bob via classical communication. Bob applies the corresponding unitary transformations to his particles:

$$\mathbf{U}_{\mathbf{lm}} |\Psi\rangle = \mathcal{N}' K_{\mathbf{X}} \delta_{\mathbf{A},\mathbf{B}} U_{\mathbf{lm},\mathbf{B}'\mathbf{B}} U_{\mathbf{lm},\mathbf{XA}} |\phi_{\mathbf{lm}}\rangle |\mathbf{B}\rangle = \mathcal{N}' K_{\mathbf{X}} \delta_{\mathbf{X},\mathbf{B}'} |\phi_{\mathbf{lm}}\rangle |\mathbf{B}'\rangle = \mathcal{N}' K_{\mathbf{B}'} |\phi_{\mathbf{lm}}\rangle |\mathbf{B}'\rangle$$

Thus, all properties of the X particle group are teleported to the B particle group in a specific order. The component expansion is:

$$\mathbf{U}_{l_I^1 m_I^1} ... \mathbf{U}_{l_N^{n_N} m_N^{n_N}} |\Psi\rangle = \mathcal{N}' K_{B_I'^1 ... B_N'^{n_N}} |B_I'^1 ... B_N'^{n_N}\rangle$$

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5 Mixed States

A density matrix $\hat{\rho} = \sum_i |\psi_i\rangle\langle\psi_i|$ can be represented in tensor form in a given basis as:

$$\hat{\rho} = \rho_{i_1 i_2 \dots i_n, i'_1 \dots i'_n} |i_1 i_2 \dots i_n\rangle\langle i'_1 \dots i'_n| = \rho_{\vec{i},\vec{i}'} |\vec{i}\rangle\langle\vec{i}'| \tag{6.1}$$

A valid density matrix must satisfy: (i) Hermiticity: $\rho_{i_1 \dots i_n, i'_1 \dots i'_n} = \rho^*_{i'_1 \dots i'_n, i_1 \dots i_n}$; (ii) Unit trace: $\sum_i \rho_{ii \dots i, ii \dots i} = 1$; (iii) Semi-positivity: $C_{i_1 \dots i_n} \rho_{i_1 \dots i_n, i'_1 \dots i'_n} C^*_{i'_1 \dots i'_n} \geq 0$ for all $C_{i_1 \dots i_n} \in \mathbb{C}$.

For the entangled resource, we have $|\phi^+\rangle\langle\phi^+| = \frac{1}{D}\delta_{ij}\delta_{i'j'}|ij\rangle\langle i'j'|$. Taking the tensor product gives:

$$\hat{\rho}_{\text{total}} = \frac{1}{D^n} \rho_{\vec{i},\vec{i}'} \delta_{\vec{j}\vec{k}} \delta_{\vec{j}'\vec{k}'} |\vec{i}\vec{j}\vec{k}\rangle\langle\vec{i}'\vec{j}'\vec{k}'| \tag{6.2}$$

Substituting Eq. (3.7) and using the unitarity of U, we obtain:

$$\hat{\rho}_{\text{total}} = \frac{1}{D^{3n}} \rho_{\vec{i},\vec{i}'} \delta_{\vec{j}\vec{k}} \delta_{\vec{j}'\vec{k}'} U^*_{\vec{l}\vec{m},\vec{i}\vec{j}} U_{\vec{l}\vec{m},\vec{i}'\vec{j}'} (|\phi_{\vec{l}\vec{m}}\rangle\langle\phi_{\vec{l}\vec{m}}|) \otimes |\vec{k}\rangle\langle\vec{k}'|$$

Alice performs hyper-entangled Bell state measurements on the AB subsystem and transmits the results $\vec{l}\vec{m}$ to Bob, who applies the corresponding unitary transformation:

$$\mathbf{U}_{\vec{l}\vec{m}} \hat{\rho}_{\text{total}} \mathbf{U}^\dagger_{\vec{l}\vec{m}} = \frac{1}{D^{3n}} \rho_{\vec{r},\vec{r}'} (|\phi_{\vec{l}\vec{m}}\rangle\langle\phi_{\vec{l}\vec{m}}|) \otimes |\vec{r}\rangle\langle\vec{r}'|$$

The mixed state $\hat{\rho}$ is thus teleported to Bob. As established in Sections 3 and 4, the distinction between multi-particle multi-attribute, single-particle multi-attribute, and multi-particle single-attribute cases lies only in the Hilbert space structure of the total state vector. Without loss of generality, we have used vector indices here; generalization to tensor indices is straightforward.

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Note: Figure translations are in progress. See original paper for figures.