Vehicle Routing Problem with Balanced Loading Constraints: A Postprint

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Abstract

This study addresses the Three-Dimensional Loading Capacitated Vehicle Routing Problem (3L-VRP) by introducing balanced loading constraints for vehicles. A Balanced Loading Vehicle Routing Problem (BL-VRP) model is constructed by comprehensively considering traditional constraints including Last-In-First-Out (LIFO), partial support, and fragility. To handle the balanced loading constraints in the model, a contact area-based loading algorithm is proposed. On this basis, a multi-stage algorithmic framework with Backtracking Genetic Algorithm (B-GA) as its backbone is developed to solve the vehicle routing optimization. The results indicate that the multi-stage algorithm not only outperforms existing algorithms in solving 3L-VRP but also demonstrates excellent performance for BL-VRP. The proposed multi-stage algorithm provides a viable reference approach for solving BL-VRP, though further refinement in computational efficiency is needed.

Full Text

Preamble

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Research on Vehicle Routing Problem Under Balance Loading Constraints

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Abstract: This paper investigates the vehicle routing problem with three-dimensional loading constraints (3L-VRP), introducing vehicle balance loading constraints while simultaneously considering traditional constraints such as last-in-first-out (LIFO), local support, and fragility to construct a vehicle routing problem model under balanced loading constraints (BL-VRP). To address the equilibrium constraints in the model, a contact-area-based loading algorithm is proposed. Building upon this, a multi-stage algorithm framework employing a backtracking genetic algorithm (B-GA) as its backbone is developed to solve the vehicle routing optimization problem. Research results demonstrate that the multi-stage algorithm not only outperforms existing algorithms in solving 3L-VRP but also exhibits excellent performance for BL-VRP. The proposed multi-stage algorithm provides a reference approach for solving BL-VRP, though further improvements in computational efficiency are needed.

Keywords: logistics engineering; balanced loading; vehicle routing optimization; backtracking genetic algorithms; multi-stage algorithms

0 Introduction

The Balanced Loading constrained Vehicle Routing Problem (BL-VRP) represents a significant extension of the Three-Dimensional Loading constrained Vehicle Routing Problem (3L-VRP). Gendreau et al. [1] first proposed 3L-VRP in 2006, which optimized not only vehicle routes and delivery requirements but also considered cargo dimensions, loading characteristics, and the "first-in-last-out" principle. BL-VRP further investigates the impact of cargo weight and center of gravity on vehicle loading stability beyond the 3L-VRP framework. BL-VRP problems are ubiquitous in real-world applications, such as the transportation of household appliances, furniture, and construction materials, where unbalanced loading layouts pose serious threats to driving safety due to heavy cargo weights.

Current research findings can be broadly categorized into two streams. One stream focuses on extensions of 3L-VRP, including time window problems [2–4], vehicle backhaul problems [5], and simultaneous pickup and delivery problems [6–8]. The other stream investigates different solution algorithms for basic 3L-VRP, encompassing exact algorithms [9], heuristic algorithms [10–11], and intelligent optimization algorithms. Since 3L-VRP consists of two NP-hard problems, making it particularly challenging, solution algorithms typically combine heuristic and intelligent algorithms. Tabu search algorithms [1,12,13] dominate this field, alongside fuzzy genetic algorithms [14] and ant colony algorithms [15]. Existing research primarily uses cargo dimensions as the basis for extending vehicle routing problems, with limited consideration of cargo weight and center of gravity effects, thereby failing to ensure safe and efficient delivery. To address this gap, this paper proposes a BL-VRP model and designs a multi-stage algorithm comprising three phases: route optimization strategy, cargo loading sequence generation strategy, and single-vehicle cargo loading strategy, thereby effectively solving the problem.

1.1 Problem Description

The BL-VRP problem can be described as follows: Given a distribution network $G = (V, E)$, where $V = \{0, 1, 2, ..., n\}$ represents n customers and E represents the paths connecting customers. Node 0 denotes the distribution center, and C_{ij} represents the travel distance from customer i to customer j. A fleet of delivery trucks is available, each with compartment dimensions $L \times W \times H$, rated load capacity D, and empty weight D_0. For each customer i, the total number of cargo items is m_i, with the k-th cargo item I_{ik} having length l_{ik}, width w_{ik}, height h_{ik}, volume $v_{ik} = l_{ik} \times w_{ik} \times h_{ik}$, weight d_{ik}, fragility f_{ik}, and bottom area $a_{ik} = l_{ik} \times w_{ik}$. The total volume of customer i' s cargo is vol_i and total weight is wei_i.

The compartment is placed in a coordinate system with its length, width, and height parallel to the x, y, and z axes, respectively. The coordinates of the rearmost, leftmost, and lowest vertex of cargo I_{ik} are (x_{ik}, y_{ik}, z_{ik}), representing the geometric center of the cargo.

BL-VRP constraints are divided into two categories: vehicle routing constraints and loading constraints.

a) Vehicle Routing Constraints: - (R1) Trucks complete loading at the distribution center, deliver to all customers, and return to the distribution center. - (R2) Each customer is served by exactly one truck. - (R3) The total weight and total volume of cargo on each route must be less than the truck' s load capacity and compartment volume. - (R4) The loading plan for each route must be feasible.

b) Three-Dimensional Loading Constraints: - (C1) Cargo items cannot overlap and cannot be positioned outside the compartment. - (C2) When placed, cargo edges must be parallel to the compartment edges. - (C3) Cargo must be placed vertically upright but can be rotated in the horizontal plane. Assume the original length, width, and height of cargo are l^0_{ik}, w^0_{ik}, and h^0_{ik}, respectively. - (C4) LIFO constraint: During unloading, cargo to be removed cannot be blocked by cargo that will be unloaded later. - (C5) Cargo requires a certain support area at its bottom for stability. The support area consists of the compartment floor or the top surfaces of other cargo and must reach a proportion θ of the cargo' s bottom area. - (C6) Fragile cargo can only have fragile cargo placed on top; non-fragile cargo has no restrictions. - (C7) Balance constraint: Balance includes longitudinal balance and transverse balance. Longitudinal balance refers to axle load constraints; transverse balance has been less studied, and this paper adopts railway standards where the vehicle's center of gravity cannot deviate more than 0.1 m from the median plane laterally.

Due to the numerous constraints in BL-VRP, a feasible route solution requires a corresponding cargo loading layout plan to determine vehicle stability while minimizing the total travel distance.

1.2 Model Construction

Given the extensive notation used in the model, parameter descriptions are provided in Table 1.

Table 1 Description of Model Parameters

Based on the above constraints, the BL-VRP mathematical model is constructed with the objective of minimizing total travel distance:

The objective function (1) minimizes total travel distance. Constraints (2)-(4) ensure each vehicle serves at least one customer and each customer is served exactly once. Constraint (5) ensures the total number of customers served equals the total number of customers. Constraints (6)-(9) guarantee that the total mass and volume of customer cargo remain within vehicle limits. Constraints (10)-(11) prevent cargo overlap and ensure items remain within compartment boundaries. Constraint (12) permits 90° rotation in the horizontal plane. Constraint (13) enforces the LIFO principle. Constraint (14) represents stability constraints, with the support proportion θ selected as 0.75. Constraint (15) describes cargo fragility constraints. Constraints (16)-(17) are axle load constraints. Constraint (18) ensures the vehicle' s center of gravity does not deviate more than 0.1 m from the median plane during loading.

2 Multi-Stage Solution Algorithm Framework for BL-VRP

For the BL-VRP problem, this paper proposes a multi-stage optimization algorithm. The basic structure is illustrated in Figure 1 [Figure 1: see original paper]. The algorithm consists of three stages: route optimization decision, cargo loading sequence generation decision, and single-vehicle cargo loading decision.

2.1 Route Optimization Decision

Randomly generated initial solutions serve as input for the Backtracking Genetic Algorithm (B-GA). Based on individual vehicle load and volume constraints, the algorithm assigns customers to each truck, forming multiple small-scale TSP problems. Backtracking is then used to traverse and find the optimal travel route and distance for each vehicle.

The genetic framework and concepts employed in this paper can be found in reference [16], which explains fitness calculation. This paper uses travel distance as the fitness measure, with each vehicle' s route calculated via backtracking. After determining the customers served by each vehicle, the backtracking algorithm finds the optimal route and distance. Backtracking is a depth-first search method that performs deep searches according to optimization objectives. When a step is found to be suboptimal or fails to meet three-dimensional loading verification requirements, the algorithm backtracks to make a new selection. Specific algorithmic ideas and procedures are detailed in reference [17].

2.2 Cargo Loading Sequence Generation Decision

After obtaining vehicle routes, a greedy algorithm changes the internal order of customer cargo to form different loading sequences, which are then evaluated by the loading algorithm. If all cargo can be loaded, the route is legal; otherwise, it is illegal.

Algorithm 1: Greedy Algorithm

The algorithm proceeds as follows: First, input the original cargo loading sequence with iteration count equal to the number of cargo items. Calculate the total cargo volume. Then, for each customer, sort cargo by volume in descending order to create a new sequence. Call Algorithm 2 and update the packed volume. If the packed volume exceeds the total volume, output loading success; otherwise, modify the internal order of customer cargo to generate a new sequence and repeat the process. The algorithm continues until either loading succeeds or all iterations are exhausted, at which point it outputs loading failure.

2.3 Single-Vehicle Cargo Loading Decision

Remaining space refers to unused space left after cargo loading, represented by a list $S = \{s_1, s_2, ..., s_n\}$. This paper employs the maximum space method to represent remaining space, as shown in Figure 2 [Figure 2: see original paper]. Maximum space refers to the largest rectangular space formed between cargo surfaces or compartment walls and the compartment door, with the critical requirement that one face must be the compartment door. When updating list S, spaces are sorted by minimum x, minimum y, and minimum z coordinates.

1) Contact Area Definition

Since cargo can rotate 90° in the horizontal plane, there are two placement methods. To obtain better placement, this paper uses cargo-space fitness to determine the orientation of cargo to be loaded and the corresponding loading space. The contact area $C(S, b, s)$ represents the sum of contact areas between cargo b and surrounding cargo or the compartment after placing cargo b into space s. A larger $C(S, b, s)$ value indicates higher fitness between cargo and space, and the space with highest fitness is selected for loading. If cargo b cannot be placed in space s or the support area after placement is less than $0.75a_b$, the fitness is $-\infty$.

2) Vehicle Balance Constraints

This paper divides vehicle balance constraints into longitudinal axle load constraints and transverse center-of-gravity offset constraints.

- **Longitudinal Axle Load Constraint:** Based on longitudinal force analysis of the compartment, a coordinate system is established as shown in Figure 3 [Figure 3: see original paper]. The analysis considers three boundary conditions for the combined center of gravity position to determine maximum allowable loading weight.

- **Transverse Center-of-Gravity Offset Constraint:** This is relatively simpler than longitudinal constraints. Based on transverse force analysis, the vehicle' s combined center-of-gravity lateral coordinate is calculated, which must satisfy $y_{wm} - y_m < 0.12$.

3) Contact Area Algorithm

Using the cargo sequence generated by Algorithm 1 as input, this algorithm outputs the total volume of loadable cargo. The specific procedure is detailed in Algorithm 2.

Algorithm 2: Contact Area Algorithm

The algorithm begins by inputting the cargo loading sequence and initializing the compartment volume. For each customer in the route and each cargo item of that customer, it traverses the space list to calculate fitness values for both placement orientations. The algorithm selects the space with maximum fitness and places the cargo in the rearmost, leftmost, and lowest corner, updating the space list accordingly. It then checks balance constraints for each customer' s cargo. If loading fails at any point, it unloads the current customer' s cargo and continues. The algorithm outputs the total volume of successfully loaded cargo.

3 Experimental Data and Analysis

Since no benchmark instances currently exist for BL-VRP, this paper conducts two experiments using standard 3L-VRP benchmarks and a real logistics distribution case. The program is developed in Java and runs on the IntelliJ IDEA platform. For the 3L-VRP standard benchmarks, the genetic algorithm is tested with maximum iteration counts of 500, 800, 1,000, and 1,500, population sizes of 30, 50, 80, and 100, crossover probabilities of 0.7, 0.8, and 0.9, and mutation probabilities of 0.05, 0.08, and 0.1. Through extensive experimentation, the best results are achieved with maximum iterations of 1,000, population size of 50, crossover probability of 0.8, and mutation probability of 0.05. These parameter settings are used in subsequent experiments.

3.1 L-VRP Standard Benchmark Experiments

The multi-stage algorithm is applied to 27 standard 3L-VRP benchmark instances, with each instance run 10 times. The average results are compared against the existing VRLH1 [12] and TS-ILA [13] algorithms, as shown in Table 2 , where dv represents the deviation of the multi-stage algorithm' s results from the best results of the other two algorithms.

Table 2 Experimental Results of 3L-VRP Standard Benchmarks

The results indicate that compared with VRLH1, the multi-stage algorithm produces results no worse than VRLH1 for every case. When compared with TS-ILA, the multi-stage algorithm performs slightly worse for instances with fewer than 50 customers. However, as the number of customers increases, the multi-stage algorithm outperforms TS-ILA, demonstrating its superiority. Overall, the multi-stage algorithm' s average performance exceeds that of both comparison algorithms, proving its significant effectiveness in solving 3L-VRP.

3.2 BL-VRP Logistics Distribution Instance

Due to the lack of standard BL-VRP benchmarks, this paper analyzes a delivery task from a regional distribution center in Changsha, conducting multiple runs and comparing results with the widely-used tabu search algorithm. The task involves 15 customers, with location information shown in Figure 4 [Figure 4: see original paper]. Distances between customers and between customers and the distribution center are based on actual shortest routes, with details provided in Table 3 (units: km). Cargo dimension information is given in Table 4, where each customer has no more than three cargo types (Var), with quantities (Num) between 1 and 3 (length unit: m, weight unit: t).

Table 3 Route Data Information

Table 4 Size Information of Goods

The delivery vehicle dimensions are $6.0 \times 2.5 \times 3.0$ m, with an empty weight of 6 t and rated load capacity of 9 t. The maximum load capacity for front and rear axle positions is 12,450 kg. Based on previous experiments, the population size is set to 50, genetic iterations to 1,000, crossover probability to 0.8, and mutation probability to 0.05. After multiple runs, the best result is shown in Figure 4.

Figure 4 [Figure 4: see original paper] Vehicle Traffic Path

The solution comprises four delivery routes: Route 1: 0-11-6-13-0; Route 2: 0-1-12-15-9-0; Route 3: 0-3-8-5-10-0; Route 4: 0-4-7-14-2-0. The total travel distance is 311.3 km.

Under the same constraints, multiple experiments using the tabu algorithm are conducted, with the best result recorded. Experiments are also performed with relaxed balance constraints. The data is recorded in Table 5 , where P represents the best result, T represents runtime, and dv represents result deviation.

Table 5 Comparisons with Tabu Search Algorithms

Horizontal comparison in Table 5 reveals that balance constraints are not merely safety factors but also significantly impact experimental results, making them non-negligible. Vertical comparison shows that due to the deep traversal using backtracking in the route optimization decision, the multi-stage algorithm' s runtime substantially exceeds that of tabu search. However, in terms of solution quality, the multi-stage algorithm outperforms tabu search, demonstrating its effectiveness for BL-VRP.

4 Conclusion

This paper extends 3L-VRP by first considering cargo weight and center of gravity, constructing a BL-VRP model incorporating longitudinal axle load constraints and transverse center-of-gravity offset constraints. A multi-stage algorithm framework based on B-GA is developed for the BL-VRP model. Two sets of experiments demonstrate that the proposed multi-stage algorithm is competitive with existing algorithms for 3L-VRP and exhibits excellent performance for BL-VRP. However, due to the use of deep search, the algorithm's computational efficiency requires improvement, representing a direction for future research.

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