Strengthened Change Point Detection Model for Weak Mean-Difference Data

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Abstract

Objective: The lifetime difference among adjacent components in parallel structures diminishes as the number of components within the same parallel structure increases. To infer the system structure, it is necessary to identify the components belonging to the same parallel structure. Methods: A Strengthened Change Point Detection Model (SCPDM) for Weak Mean Difference Data (WMDD) is established. WMDD typically indicates that, due to large variance effects, the mean difference between two subsignals within a single data sequence becomes statistically nonsignificant. For repeatedly retrievable WMDD, we performed two enhancement operations that double the mean difference by utilizing variance information and analyzed the asymptotic properties of the enhanced data. Subsequently, we proposed an SCPDM based on these asymptotic results. Results: We compared the SCPDM with two other major change point detection models and verified through simulation that the SCPDM outperforms other models in WMDD change point detection. Limitations: This paper has several limitations. First, we only discussed cases that are independent with normal distribution and a single change point. Second, the reason why the relationship between and has an important influence on the accuracy of change point detection is not discussed in depth. We only defined the ratio boundary of WMDD through experience and simulation. Conclusions: Traditional change point detection models may become insensitive or ineffective for WMDD. We provided some asymptotic analysis and established an enhanced change point detection model (SCPDM) based on the asymptotic results. Compared with traditional methods, SCPDM can effectively detect the change point.

Full Text

Preamble

Strengthened Change Point Detection Model for Weak Mean Difference Data

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Abstract

Objective: The lifetime difference between components in adjacent parallel structures becomes small as the number of components belonging to the same parallel structure increases. To infer the system structure, we must clarify which components belong to the same parallel structure.

Methods: A Strengthened Change Point Detection Model (SCPDM) for Weak Mean Difference Data (WMDD) is established. WMDD typically indicates that, due to large variance, the mean difference between two subsignals in a data sequence becomes nonsignificant. For repeatedly retrievable WMDD, we performed two enhanced operations that doubled the mean difference by utilizing variance information and analyzed the asymptotic properties of the enhanced data. Then, we proposed an SCPDM based on these asymptotic results.

Results: We compared the SCPDM with two other main change point detection models and verified through simulation that the SCPDM is superior to other models for WMDD change point detection.

Limitations: This paper has several limitations. First, we only discussed cases where y ,···, are independent with normal distribution and a single change point. Second, the reason why the relationship between and has an important influence on the accuracy of change point detection is not discussed in depth. We only defined the ratio boundary of WMDD through experience and simulation.

Conclusions: Traditional change point detection models may become insensitive or ineffective for WMDD. We provided asymptotic analysis and established an enhanced change point detection model (SCPDM) based on the asymptotic results. Compared with traditional methods, SCPDM can effectively detect change points.

Keywords: single change point detection; weak mean difference; large variance; enhanced operations; simulation

Introduction

Weak mean difference data (WMDD) represent a type of data where information about mean differences is obscured by large variance. For example, as shown in Fig 1 [Figure 1: see original paper], when the mean difference and standard deviation are , the location of the change point is easily detected. If the standard deviation is , the accuracy of change point detection may decrease. However, if the standard deviation is , the location of the change point is hardly detected by most current models.

An important example of WMDD comes from the reliability field. Assume that the resistance values of all components in Fig 2 [Figure 2: see original paper] are equal. Jin et al. [?] pointed out that components tested in a laboratory environment differ significantly from those that have experienced operations in fielded systems, where homogeneous components suffer different degrees of damage. As shown below, the voltages of components belonging to adjacent parallel structures become more similar as the number of components belonging to the same parallel structure increases, which means that the lifetime difference between components in adjacent parallel structures also becomes small. To infer the system structure, we must clarify which components belong to the same parallel structure. As we know, components in the same parallel structure have homogeneous life data. Hence, detecting small lifetime differences in components is extremely important for distinguishing whether components belong to the same parallel structure and is beneficial for establishing a topology diagram of the system structure.

The difficulty in detecting change points for WMDD lies in capturing small differences between subsignals. To solve this problem, we performed two enhanced operations that increased the mean difference between subsignals by utilizing variance information. In addition, we analyzed the asymptotic properties of the enhanced data. Next, we proposed a Strengthened Change Point Detection Model (SCPDM) based on these asymptotic results. Finally, we compared the SCPDM with two current main models and verified through simulation that the SCPDM has higher efficiency than those other models for WMDD change point detection.

Literature Review

The so-called change point is the location at which a certain property in a signal sample suddenly changes [?]. The change point often represents a qualitative change in our object of focus. Historically, Page [?, ?] first proposed the study of change points in sample testing. To detect change points in a signal sample, , the process roughly involves the following steps. First, select an associated cost function [?] to measure the homogeneity in each subsignal. Second, according to whether the number of change points is fixed, compute a discrete optimization problem to obtain estimations of the change point locations. In different change point detection models, establishing a suitable cost function for a specific sequence is the first and most important step [?].

Many classical change point detection models have been proposed for various kinds of signals, mainly including three types. For piecewise independent identically distributed (i.i.d.) signals, the mean shift model was first established for normal random signal samples with piecewise constant mean and constant variance [?, ?]. Second, certain signals may have their means shift along with shifts in their variances. For example, mean shift and scale shift models were established for normal random signal samples with piecewise constant means and variances by certain predecessors [?, ?]. Except for normal random samples, the rate shift model has been studied for Poisson distribution signals with piecewise constant rate parameters [?, ?]. The second type of change point model is appropriate for signals with linear dependency between variables along with changes happening at certain unknown instances, which are also called structural changes [?]. In this situation, several well-known models were established, such as the autoregressive model [?, ?] and multiple regression models [?, ?]. Other commonly used change point detection models include kernel change point detection [?] and the Mahalanobis-type metric [?]. Kernel change point detection can be operated on the high-dimensional mapping of the original signal that is implicitly defined by a kernel function. Certain machine learning techniques may be involved in this kind of method, such as support vector machines or clustering [?, ?]. In addition, in certain clustering methods, the Mahalanobis-type metric is usually used to replace the cost function in the mean shift model [?].

In addition to the models mentioned above, several classical models based on algorithms have been proposed for inferring change points, mainly including four types [?]. The first model is based on the likelihood ratio. Csörgö and Horváth [?] established a change point detection model under the assumption of multivariate Gaussian distribution. This model is mainly used for analyzing change points in time series data. The second model is based on the Bayes model. Many researchers have focused on this method. Kander and Zacks [?] aimed at the exponential family to establish a change point detection model, while Gardner [?] established a model based on normal distribution. Later, the model was extended to large sample distribution theory, multivariate normal distribution, and general linear regression fields [?, ?, ?]. The third model is based on maximum likelihood. This kind of method has mature large sample theory. For example, Fotopoulos et al. [?, ?] established exact computable expressions, bounds, and approximations for certain analysis results. The last model is based on samples. This kind of method focuses on nonparametric methods, which have the advantage of being distribution-free.

Among the subdirections of change point detection, signal samples with mean shifts have always been a research hotspot. Hawkins et al. [?] used sample variance without degrees of freedom to determine the change point. In [?], maximum likelihood estimation was utilized to analyze the change point under the premise of verifying the type of population distribution. Later, prior knowledge was incorporated in establishing the change point detection model in [?]. In [?], the change point location was determined by analyzing local information near a point, which involved complex distribution information usually substituted by certain approximate results. In recent years, certain new methods have also been discussed regarding change point detection. As indicated in [?], an optimal algorithm was introduced to determine the location of a change point. In [?], an adapted algorithm was established by the polynomial maximization method. In [?], partition models were set up for testing the existence of a mean shift and estimating the location of the change point. In addition, lasso methods were established and improved by many authors [?].

3.1 Two Enhanced Operations

Because the standard deviation is far larger than the mean difference, it is unwise and inefficient to perform change point detection on WMDD using traditional models. In fact, compared with the information of the mean difference, the variance is very remarkable and may supply more information. Therefore, we considered utilizing the variance, which belongs to a kind of disturbance information, in analyzing change points by performing the following two enhancements.

We assume that *t (,···,2{) is the only abrupt location in sequence y ,···, . For y ,···, , we first conducted the first operation called enhanced- at t (,···,2{) and obtained the enhanced- sequence 1 ,···, ,···,1 ,···,1 Then, we conducted the second operation called enhanced- at t (,···,2{) and obtained the enhanced- sequence 1 ,···, ,···,1 y ' ' ,···,1 Intuitively, these enhancements utilize variance information directly by taking the larger or smaller value between adjacent samples.

We now provide several symbolic explanations. For a signal sample y ,···, , we performed the above two operations at location t (,···,2{), where indicates the sample mean of y , ; 1 ···, indicates the sample mean of ' ···, y , ; indicates the sample mean of y , ; and 1 ···, indicates the sample mean of ' ' ···, y , ; indicates the sample mean of y , . In addition, ° and represent operations on the homogeneous signal sample 1 ···, and the signal sample containing a change point, respectively.

3.2 Asymptotic Property of the Enhanced Sequence

We considered establishing certain asymptotic properties of the enhanced sequence in the following theorem. First, we explain several symbols. Assume that tY , with (,···,1), are independent random variables with probability distribution (tyf , and is a vector-valued parameter. y ,···, is a set of samples of Y ,···, . For a signal sample that has one change location ,···,2{ 1 indicates the population mean of the first subsignal and 2 indicates the population mean of the second subsignal. 2 indicates the constant population variance. Generally, WMDD indicates that

Theorem 1. Assume that y ,···, is i.i.d., i.e., homogeneous with a constant variance 2 . The above two operations are performed cn times independently at location t (,···,2{). Then, we have the following asymptotic property:

Proof. y ,···, is i.i.d. According to the law of large numbers, we have the following: where min is the smaller population mean of the enhanced sequence.

Likewise, we have the following: where max is the larger population mean of the enhanced sequence. As a result, we have the following:

On the other hand: Consequently: In addition, is a continuous function, so the ..sa converge can be preserved under the transformation of this function, so we have the following:

Thus, because the process is independently performed cn times, ,···,1) can be viewed as independently and identically distributed. According to the law of large numbers, we have the following:

Remark 1. Theorem 1 demonstrates that when y ,···, is homogeneous and the above two operations are performed cn times independently at any location t (,···,2{), the value of fluctuates at approximately 0.

Next, we present the asymptotic property when there is only one change point among the signal samples. We present the asymptotic results at the position of the change point.

Theorem 2. Assume that the independent y ,···, only has one change point *t with a constant variance 2 . At* t , the above two enhanced operations are independently performed cn times. Then, we have the following asymptotic property:

(1) If , we then have:

The meanings of are the same as in Theorem (2) If , we then have:

Proof. (1) First, we prove the first part. The following formula is clear when 1 Because , an incorrect enhanced operation cannot reverse the direction of the mean difference, so we have the following:

Likewise, when 1 , we have the following two relationships:

In summary, we have the following: Thus, because the process is independently performed cn times, ,···,1) can be viewed as being independent and identically distributed. According to the law of large numbers, we have the following:

Proof. (2) The following formula is clear when 1 Due to the following: , the enhanced- operation can reverse the mean difference, so we have the Likewise, when 1 , we have the following two relationships:

Thus, because the process is independently performed cn times, ,···,1) can be viewed as being independently and identically distributed. Therefore, we have the following: tTtnsa

Remark 2. For WMDD, Theorem 2 demonstrates that when y ,···, ,···,2{) and if the above two operations are performed cn times independently at *t (, ···,2{), the value of reaches the maximum value ; if the above two operations are performed cn times independently at a nonchange location, the value will be less than the maximum value . The latter finding is verified in Section 3 in detail.

3.3 SCPDM

For WMDD, the different asymptotic properties in Theorem 1 and Theorem 2 provide important information for judging whether there is a change point and its location. Consequently, we propose a model to detect the change point in WMDD by sampling repeatedly in this section.

Assume that there is only one change point whose location is *t (y ,⋯, . To obtain a better estimate of* t , we should establish a contrast function [?] to measure the goodness-of-fit of the signal. First, at the location of t (,⋯,2), the above two enhanced operations are performed cn times. The following contrast function is established:

According to Theorem 2 and Theorem 3, the value of is large when t and small when *t is not well-estimated. is expected to be Throughout the establishment process of the SCPDM, how to solve this discrete optimization problem becomes clear. We calculate ,⋯,2 and make . Details can be seen in the next section. In the next section, we performed several simulation studies to estimate* t under a normal distribution with various parameters.

4. Results

In this section, we perform simulations in two parts. In the first part, we verify the correctness of Theorem 2 and Remark 2. In the second part, we performed several simulation studies to estimate *t under a normal distribution with different parameters and compared the SCPDM with two current main models, verifying that the SCPDM has higher efficiency than those other models for WMDD.

4.1 Verifying the Correctness of the Theorem

To verify the correctness of Theorem 2, we generated random numbers based on the normal distribution, and the parameter settings are shown in the caption of each figure. For the public parameters, we set , and . As shown in Fig 3 [Figure 3: see original paper], we carried out two sets of simulation analyses.

To verify the correctness of Remark 2, when , at the location of ,⋯,), is calculated and the results are shown in Fig 5 [Figure 5: see original paper]. It can be seen from Fig. that when reaches a maximum value, which is approximately . The simulation result is consistent with Theorem 2.

5. Discussion

To verify that the SCPDM' s estimation of *t is better than those of other models, we present certain simulated results based on traditional models, including the* *least squares model [?] and Bayes method [?]. We generated random samples y , ···, based on the normal distribution, and the parameter settings are shown in the corresponding figure; we set the public parameters, namely, , and . For the three models with the same parameter settings, we repeated the same operation 1000 times, counted the estimation results of* t and regarded the frequency of each as the probability of becoming a real change point, which reflects the accuracy of each model. The results are shown in Fig 6 [Figure 6: see original paper], Fig 7 [Figure 7: see original paper] and Fig 8 [Figure 8: see original paper].

By setting different parameters, the detection accuracies of the change point interval, i.e., ˆPr(* t 503,499 , and change point location, i.e., ˆPr(* t , of the three models were compared in Table 1 and Table 2 , respectively. When the parameters were set to 5.0 for 1000 repeated tests, the Bayes model resulted in ˆPr(* 503,499 ˆPr(* and the least squares model resulted in ˆPr(* 503,499 ˆPr(* . Both methods had high accuracy for change point detection. When the type of data is WMDD and we consider ˆPr(* t 503,499 , when the parameters are set to 1 and , the accuracy of the SCPDM is 31% higher than the accuracy of the least squares model and 41.1% higher than the accuracy of the Bayes model.

When the parameters are set to 1 and , the accuracy of the SCPDM is 54% higher than that of the least squares model and 60.7% higher than that of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 62.6% higher than the accuracy of the least squares model and 70.1% higher than the accuracy of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 42% higher than that of the least squares model and 43% higher than that of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 31% higher than the accuracy of the least squares model and 41.1% higher than the accuracy of the Bayes model. When we consider ˆPr(* t , when the parameters are set to , the accuracy of the SCPDM is 25% higher than the accuracy of the least squares model and 33.6% higher than the accuracy of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 27.7% higher than that of the least squares model and 32.1% higher than that of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 23.2% higher than the accuracy of the least squares model and 28.8% higher than the accuracy of the Bayes model. When the parameters are set to 1 and , the accuracy of the SCPDM is 9.1% higher than that of the least squares model and 9.5% higher than that of the Bayes model. When the parameters are set to , the accuracy of the SCPDM is 9.2% higher than the accuracy of the least squares model and 9.5% higher than the accuracy of the Bayes model.

6. Conclusion

This paper focused on detecting the change points of WMDD. We conducted asymptotic analysis and established an enhanced change point detection model (SCPDM) based on the asymptotic results. According to Theorem 2 (2), the enhanced sequence uses significant variance information so that the weak mean difference increases from 1 , which makes the change point easier to detect and increases the accuracy of change point detection. In addition, for WMDD, traditional methods may be improved by purely adding sample capacity to the sequence. While under the premise of having the same amount of data, the SCPDM greatly increases the efficiency of change point detection by repeatedly detecting sequences with the same data structure. In addition, repeated measurements are possible for the life data of components at the same location. Hence, compared with traditional methods, the SCPDM can effectively detect change points.

Although the accuracy of change point detection has been improved, this paper also has several limitations. First, we only discussed cases where y ,···, is independent with a normal distribution and a single change point. Second, the reason why the relationship between and has an important influence on the accuracy of change point detection is not discussed in depth. We defined the ratio boundary of WMDD only by experience and simulations. In future studies, we will extend the SCPDM to other distribution types and multiple point detection. In addition, for Theorem 2, we will reprove the theorem by introducing the relationship between and .

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SUPPORTING INFORMATION

[Insert supporting information here]

Figure Legends

Fig 1. Diagram of the accuracy of change point detection influenced by the variance. (A) 1.0 . (B) 1 , . (C)

Fig 2. Variation in the difference between adjacent parallel structural components.

Fig 3. The value of for 500 . (B) 1 , . (C) 1 , . (D) repetitions. (A) 1 , at location . (A) 1 , . (B) 1 , . (C) 1 , . (D) . (E) 1 ,

Fig 6. Probability of each location becoming a change point in the least squares model. (A) 5.0 . (B) 1 , . (C) . (D) 1 , . (E) 1 , . (F) 1 ,

Fig 7. Probability of each location becoming a change point in the Bayes model. (A) 5.0 . (B) 1 , . (C) . (D) 1 , . (E) 1 , . (F) 1 ,

Fig 8. Probability of each location becoming a change point in the SCPDM. (A) 1 , . (D) . (B) 1 , . (C) 1 , 1 ,

Figures 1.0 Fig.1 Diagram of the accuracy of change point detection influenced by variance. 1 , Voltage difference 2 times 1.5 times 1.3 times 1 times Fig. 2. Varieties in difference between adjacent parallel structural components Under 1.0 1 , repetitions, value 1 , 1 , 1 , Fig. 4 [Figure 4: see original paper]. Under 500 repetitions, value of 1 , 1 , 1 , 1 , Fig. 5. The value of location. 1 , 5.0 1 , Fig.6. Probability of the each location becoming a change in Least Square Model 1 , 1 , 5.0 1 , 1 , 1 , 1 , Fig.7. Probability of the each location becoming a change in Bayes Model 1 , 1 , 1 , 1 , Fig. Probability of the each location becoming a change in SCPDM

Tables

Table 1. Estimate probability of a change in certain intervals of three models.

Model	Least Squared Method	Bayes Method	SCPDM Method
Change point location			

Table 2. Estimate probability of t of three models

Model	MD	SD
Least Squared Method		
Bayes Method		
SCPAM		

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.