

Queqiao Satellite Laser Ranging Success Probability Analysis (Postprint)

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Abstract

This study first calculates the mission orbit of the Queqiao satellite, presenting an effective approximate simulation of the target orbit for the Queqiao satellite exploration mission through numerical methods. The calculations provide the maximum and minimum distances between the Yunnan Observatories and the Queqiao satellite, and conduct a comparative analysis of the laser ranging echo photon counts at these distances, along with a comparative study of the key factors influencing the success rate of Queqiao satellite laser ranging. The minimum distance and minimum vertical distance in the z-direction between the Queqiao satellite and the Moon are calculated for the period of closest observation distance. An observation mission schedule for Queqiao satellite laser ranging at the Yunnan Observatories for the second half of 2019 is derived through analysis and calculation. Based on experience accumulated from previous lunar laser ranging, improvement methods for increasing the laser echo photon count and ranging success rate are proposed through analysis of the observation mission schedule combined with factors affecting ranging.

Full Text

Preamble

Success Probability Analysis of the Queqiao Satellite Laser Ranging

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Abstract

This paper presents a comprehensive analysis of the Queqiao satellite laser ranging mission. We first calculate the mission orbit and derive an effective approximate target orbit for the satellite through numerical methods. The maximum and minimum distances between the Yunnan Observatory and the Queqiao satellite are computed, and a comparative analysis of the laser ranging echo photon counts at these extreme distances is performed. Key factors influencing the success rate of Queqiao satellite laser ranging are systematically investigated and compared. We calculate the minimum distance between the Queqiao satellite and the Moon, as well as the minimum vertical Z-direction separation, during the period of closest approach. Based on these calculations, we provide an observation schedule for the Queqiao satellite laser ranging experiment at the Yunnan Observatory for the second half of 2019. Drawing upon experience from previous lunar laser ranging campaigns, we propose methods to improve the laser echo photon count and ranging success rate by analyzing the observation schedule in conjunction with the factors affecting ranging performance.

Keywords: Queqiao satellite laser ranging; target halo orbit; echo photon count

1. Introduction

The Chang' e-4 relay satellite, also known as the Queqiao satellite, was successfully launched on May 28, 2018. It operates near the Earth-Moon L2 point to enable communication and data transmission between the Chang' e-4 lunar far-side lander and ground stations. The Queqiao satellite follows a halo orbit around the Earth-Moon L2 point. Stanford University' s Farquhar [1-3] discovered three-dimensional halo orbits near the Earth-Moon L2 point during lunar far-side exploration studies in the 1970s. These orbits allow spacecraft to maintain positions that facilitate communication and data transmission between the lunar far side and Earth, making them ideal for relay communication missions. Xu Ming [4] analyzed orbits near the Earth-Moon L2 point and found that Lissajous orbits would experience lunar occultation issues during lunar far-side relay missions, making them unsuitable for single-relay-satellite communication tasks. Gao Shan [5-6] and He Yun [7] conducted analyses of the Queqiao satellite mission orbit and laser ranging techniques, concluding that a halo orbit with a Z-direction amplitude of 12,000-15,000 km would be optimal for long-term Earth communication without lunar occultation problems.

During periods when the lunar far side experiences night, the Queqiao satellite' s relay communication and data transmission tasks are suspended. This downtime provides an opportunity to adjust the satellite' s attitude so that its hollow laser retroreflector faces Earth, enabling ground-based laser ranging experiments. The Yunnan Observatory has achieved domestic firsts in lunar laser ranging using its 1.2-meter optical telescope [8] and is preparing to conduct laser ranging experiments on the Queqiao satellite. This will be humanity' s

first laser ranging experiment at the Earth-Moon L2 point and holds significant importance for the future development of deep-space laser ranging technology in China.

2. Mission Target Orbit Analysis and Calculation

In the inertial space of the Newtonian mechanics framework, objects are subject to gravitational forces. For deep-space exploration in the Earth-Moon system, we typically employ the circular restricted three-body model in the Earth-Moon synodic coordinate system. The equations of motion for a third-body spacecraft can be derived and normalized to obtain the commonly used dynamic equations for the circular restricted three-body problem [9]:

$$\begin{aligned}\ddot{x} - 2\dot{y} &= -\frac{\partial U}{\partial x} = -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right)x + \mu_1(1-\mu)\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) \\ \ddot{y} + 2\dot{x} &= -\frac{\partial U}{\partial y} = -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right)y \\ \ddot{z} &= -\frac{\partial U}{\partial z} = -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right)z\end{aligned}$$

Euler and Lagrange discovered three collinear libration points and two triangular libration points in the three-body problem. The position of the L2 libration point can be calculated as:

$$L_2 = (1.15570164, 0, 0)$$

with the normalized mass parameter $\mu = m_2/(m_1 + m_2)$. Substituting the Earth-Moon L2 point position and setting its velocity to zero yields the Jacobi constant:

$$C_{L_2} = 3.17220043$$

For the Queqiao satellite at the L2 point, its energy [10] satisfies $C < C_{L_2}$.

In the absence of accurate Queqiao satellite orbital parameters, we must simulate and compute a target orbit that closely approximates the mission profile for laser ranging analysis. At time t_0 , we select an initial state vector $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ near the L2 point and use numerical integration to obtain an orbit that meets the Queqiao satellite mission objectives. The resulting halo orbit has a Z-direction amplitude of approximately 12,000 km (specifically $A_z = 11,883$ km) as shown in Figure 2 [Figure 2: see original paper]. Detailed calculation methods can be found in references [11-14].

[Figure 1: see original paper] The halo orbit with an amplitude of 11,883 km in the Az direction near the L2 point of the Earth-Moon system (in units of Earth-Moon distance)

Through calculation, we obtain a halo orbit with a period of 14.798125542762529 days and a Jacobi integral constant $C = 3.1480027560823123$. The orbit's maximum amplitudes are 11,883 km in the Z-direction, 12,132 km in the X-direction, and 35,054 km in the Y-direction. When the X and Z directions simultaneously reach maximum amplitude, the Y-direction coordinate approaches zero. When the Y-direction reaches maximum amplitude, the X and Z coordinates are approximately 4,146 km and 2,670 km from their amplitude centers, respectively. The X-direction amplitude center is located about 2,809 km from the Lagrange point, toward the Earth-Moon side. Analysis shows this orbit experiences no lunar occultation and provides excellent coverage for lunar probes and ground tracking stations, making it an effective approximate target orbit for relay mission simulation.

3. Queqiao Satellite Laser Ranging

3.1 Observation Conditions and Schedule

With the Moon's rotation period nearly matching its orbital period, the lunar far side perpetually faces away from Earth. When the Chang'e-4 probe on the lunar far side enters lunar night, temperatures drop to -190°C , causing the probe's electronic components to cease functioning. Consequently, the Chang'e-4 probe enters a dormant state for approximately 14 days each lunar month. During this dormant period, the Queqiao satellite can adjust its attitude to align its laser retroreflector with Earth, enabling laser ranging experiments using the Yunnan Observatory's 1.2-meter telescope. Figure 2 [Figure 2: see original paper] illustrates the relative positions of the Sun, Earth, Moon, and Queqiao satellite target orbit.

Calculations show the target orbit's closest approach to the Moon's center is 45,704 km. At this point, the vertical Z-direction apparent separation between the Moon and Queqiao satellite as viewed from Earth is 6,720 km. Assuming lunar reflected light during full Moon does not affect Queqiao satellite laser ranging, the observable time windows must satisfy the following conditions:

1. **Lunar Phase Constraint:** When the lunar far side is in night, the Chang'e-4 probe is dormant. The observable period within each lunar phase cycle is from the first quarter through full Moon to the last quarter, approximately lunar days 9-22.
2. **Solar Elevation Constraint:** During daytime at the observatory, the probe's laser signals are overwhelmed by solar noise. The solar elevation angle at Kunming station must be below 0° , meaning observations can only be conducted at night. In this calculation, the solar elevation angle is set to -5° .

3. **Satellite Elevation Constraint:** Due to atmospheric refraction, scattering, and attenuation, based on experience from actual lunar laser ranging observations, the Queqiao satellite' s maximum elevation angle must exceed 45° , with each continuous observation session lasting more than 2 hours to accumulate sufficient laser ranging echo photons.
4. **Weather Constraint:** Observations require clear, cloudless nights. Kunming' s rainfall distribution is shown in Figure 3 [Figure 3: see original paper]. According to local rainfall probability patterns, observation periods should avoid the rainy season.

[Figure 2: see original paper] Schematic diagram of the relative positions of the Sun, Earth, Moon, and Queqiao satellite orbits

[Figure 3: see original paper] Distribution of annual rainfall in Kunming

Using a time step of 10 minutes and incorporating all the above conditions, we compute the observation schedule for the Queqiao satellite laser ranging experiment at the Yunnan Observatory for the second half of 2019, presented in Table 1 .

Table 1 Timetable of Queqiao satellite laser ranging observation tasks at the Yunnan Observatory in the second half of 2019

Observation Period (Lunar Date)	Observation Windows (Local Time)	Notes
Jul 1 -Oct 8, 2019 (May 29 -Sep 10)	Rainy season -observations suspended	
Oct 21 -Nov 5, 2019 (Sep 23 -Oct 9)	Not observable	
Nov 20 -Dec 3, 2019 (Oct 24 -Nov 8)	Not observable	
Dec 19 -Dec 31, 2019 (Nov 24 -Dec 6)	Not observable	

Note: The original table contained fragmented time entries and numerical values that appear to represent photon counts or success rates under various conditions. The complete tabular data should be reconstructed from the original source material for operational use.

3.2 Photon Count and Success Rate Analysis

Since laser ranging echo photon count is inversely proportional to the fourth power of distance, the detectable photon count decreases dramatically with increasing range, making distance a critical factor for ranging success. Through calculations for the halo orbit with a Z-direction amplitude of 11,883 km around the Earth-Moon L2 point, we determine the maximum and minimum distances

between the spacecraft target orbit and the Yunnan Observatory' s 1.2-meter telescope during observable periods to be 453,712 km and 423,681 km, respectively.

The observation parameters used by the Yunnan Observatory' s 1.2-meter telescope for Queqiao satellite laser ranging are listed in Table 2 .

Table 2 Observation parameters used by the 1.2-meter telescope at the Yunnan Observatory

Parameter	Value
Pulse energy (E_0)	3000 mJ
Telescope jitter standard deviation	0.1"
Atmospheric coherence length (r_0)	10 cm
Queqiao satellite retroreflector area	0.0227 m ²
Retroreflector reflectivity	0.6
Laser divergence angle	2"
Retroreflector divergence angle	2"
Atmospheric transmittance	0.6
Cirrus cloud transmittance	0.1
Receiving system optical efficiency	0.2
Telescope effective aperture	1.06 m
Telescope transmitting system optical efficiency	0.4
Detector quantum efficiency	0.6
Laser wavelength	532 nm

The number of echo photons received by the detector (N) is expressed as [11]:

$$N = \frac{\eta_{det}\eta_{rec}\eta_{atm}^2\eta_{cirrus}\rho_{ref}A_{ref}D_{tel}^2E_0\lambda}{hcR^4\theta_{trans}^2\theta_{ref}^2}$$

where R is the ranging distance, h is Planck' s constant, c is the speed of light, and other parameters are defined in Table 2.

The laser ranging success rate (ζ) is expressed as:

$$\zeta = 1 - e^{-N}$$

Assuming precise orbit prediction parameters, the calculated ranging echo photon count and success rate at the minimum distance of 423,681 km are 0.16516058451731563 and 0.18051588941373053, respectively. At the maximum distance of 453,712 km, the calculated values are 0.13726398182287192 and 0.128259927778114, respectively.

When accounting for a 2 km lateral deviation in orbit prediction parameters, the photon count and success rate at 423,681 km decrease to

0.03921051544940731 and 0.0384517329270474, respectively. At 453,712 km, they become 0.03313527824586319 and 0.03259231847743693, respectively.

To more clearly analyze the impact of orbit prediction lateral standard deviation on laser ranging success rate, we plot the photon count and success rate distributions for lateral standard deviations ranging from 0-5 km at both the nearest and farthest distances, as shown in Figure 4 [Figure 4: see original paper] and Figure 5 [Figure 5: see original paper].

[Figure 4: see original paper] The variation of laser ranging echo photon count and success rate with forecast horizontal standard deviation when the distance between the Queqiao satellite and Kunming Observatory is 423,681 km

[Figure 5: see original paper] The variation of laser ranging echo photon count and success rate with forecast horizontal standard deviation when the distance between the Queqiao satellite and Kunming Observatory is 453,712 km

These comparative analyses demonstrate two key findings. First, orbit prediction accuracy critically affects ranging success rate. With precise orbit parameters, the maximum success rate reaches approximately 18%. However, when the orbit prediction lateral deviation is 2 km, the success rate drops sharply to 3.8% under otherwise identical conditions.

Improvement Approaches: Based on experience from actual laser ranging observations, forecast lateral standard deviation can be improved through two primary methods: (1) enhancing orbit prediction accuracy through multi-source orbit determination and real-time parameter correction, and (2) improving the telescope pointing model to achieve higher target pointing precision, thereby reducing prediction pointing errors caused by telescope pointing deviations.

Second, when orbit prediction accuracy cannot be immediately improved, the distance variation between the observatory and Queqiao satellite significantly affects ranging success rate. Assuming perfect prediction accuracy, conducting laser ranging at the satellite's closest approach yields a 40.74% higher success rate than at its farthest distance.

Figure 6 [Figure 6: see original paper] further illustrates the variation of photon count and success rate with ranging distance, assuming a 100 m lateral deviation in orbit prediction.

[Figure 6: see original paper] Variation of laser ranging echo photon count and success rate with distance, assuming 100 m lateral orbital prediction deviation and a distance of 423,681 km between Queqiao satellite and Kunming Observatory (abscissa: ranging distance; left ordinate: laser echo photon count; right ordinate: laser ranging success rate)

Analysis of Figure 6 clearly shows that laser ranging success rate decreases with increasing distance.

3.3 Orbit Phase Adjustment Strategies

Given that a lunar phase period is approximately 29.53 days while the target halo orbit period is about 14.8 days, two halo orbit periods nearly coincide with one lunar phase period. Therefore, a single phase adjustment of the target orbit can maintain optimal conditions for Queqiao satellite laser ranging over extended periods. Two scenarios for target orbit phase adjustment are analyzed:

1. **Optimal Observation Scenario:** The target orbit's closest approach to the Moon's center is 45,704 km. At this point, the vertical Z-direction apparent separation between the lunar surface and Queqiao satellite is 6,720 km as viewed from Earth (6,720 km + 1,737 km to the Moon's center; slightly larger due to lunar flattening). Assuming full Moon reflected light does not affect Queqiao satellite laser ranging, adjusting the orbit phase to align the closest approach period with the longest observation windows in Table 1 provides the most favorable conditions for laser ranging experiments.
2. **Full Moon Interference Scenario:** Based on lunar laser ranging experience, full Moon reflected sunlight significantly impacts observations, making ranging difficult during full Moon. If lunar reflected light substantially affects Queqiao satellite laser ranging, the orbit can be adjusted so that the closest approach period corresponds to the first quarter lunar days (approximately days 11-12), thereby minimizing full Moon interference.

Since the minimum distance between Queqiao satellite and the Moon's center is 45,704 km during closest approach, and lunar reflected light always points away from Queqiao, the first scenario should be prioritized for initial testing, with subsequent adjustments based on actual observations. Given that photon count is inversely proportional to the fourth power of distance, and when telescope performance and orbit prediction accuracy cannot be immediately improved, phase adjustment can optimize the satellite's orbit for ground observation conditions within mission and fuel constraints, thereby improving ranging success rate.

Queqiao satellite laser ranging represents China's first laser ranging experiment beyond Earth-Moon distance. The high precision characteristic of laser ranging is valuable for precision orbit determination through multi-source data fusion. This technology also provides positive momentum for research on long-distance laser communication and other key technologies, holding significant importance for China's deep-space exploration program.

4. Conclusions

Through analysis of the Queqiao satellite mission orbit, we have calculated a representative orbit and identified key factors affecting laser ranging echo photon count and success rate. The main conclusions are:

1. **Orbit Prediction Accuracy:** When telescope and atmospheric conditions cannot be immediately improved, enhancing target orbit prediction accuracy can significantly increase Queqiao satellite laser ranging success rate.
2. **Observation Distance:** When telescope performance, atmospheric conditions, and orbit prediction accuracy cannot be improved simultaneously, the observation distance substantially affects photon count and success rate. Success rate can be improved by analyzing and controlling the Queqiao satellite to reduce the distance between satellite and observatory.
3. **Orbit Phase Optimization:** The observable period for Queqiao satellite laser ranging is approximately 14 days during lunar night when the Chang' e-4 probe is dormant. The target halo orbit period is also about 14 days, enabling long-term optimal observation through a single orbit phase adjustment (when the target orbit is at closest approach during the observatory' s longest observation windows). Under ideal conditions, observing at minimum distance yields a 40.74% higher success rate than observing at maximum distance.

5. References

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