ChinaRxiv

AI translation · View original & related papers at
chinarxiv.org/items/chinaxiv-201904.00063

DOA Estimation for Coherent Sources Based on Hybrid MUSIC Algorithm (Postprint)

Authors: Zhang Quan, Li Simin, Tang Zhiling

Date: 2019-04-01T00:00:00+00:00

Abstract

In practical communication environments, the complexity of the propagation environment leads to a large number of coherent signals in space, resulting in rank deficiency of the source covariance matrix. To restore the matrix rank to equal the number of signal sources and solve the coherent source Direction of Arrival (DOA) estimation problem, a hybrid MUSIC algorithm is proposed. The algorithm preprocesses the antenna array through forward-backward spatial smoothing techniques, and applies the obtained new covariance vector matrix to an improved IMUSIC algorithm for signal data processing and analysis, yielding DOA angle estimates of coherent signals. Simulation results demonstrate that under conditions of low signal-to-noise ratio, small signal separation, and the presence of correlated signals, the hybrid MUSIC algorithm can accurately estimate the source DOA, thereby validating the algorithm' s high resolution and high performance.

Full Text

Preamble

Vol. 37 No. 5
Application Research of Computers
ChinaXiv Partner Journal

DOA Estimation of Coherent Sources Based on Hybrid MUSIC Algorithm

Zhang Quan[1], Li Simin[2], Tang Zhiling[1]† 1. Institute of Electronic Engineering & Automation, Guilin University of Electronic Technology, Guilin, Guangxi 541004, China 2. Institute of Electrical & Information Engineering, Guangxi University of Science & Technology, Liuzhou, Guangxi 545006, China

Abstract: In practical communication environments, the complexity of propagation leads to numerous coherent signals in space, causing rank deficiency in the source covariance matrix. To restore the matrix rank to equal the number of signal sources and solve the direction-of-arrival (DOA) estimation problem for coherent sources, this paper proposes a hybrid MUSIC algorithm. The algorithm preprocesses the antenna array using forward-backward spatial smoothing techniques and applies the resulting new covariance vector matrix to an improved IMUSIC algorithm for signal data processing and analysis, yielding DOA angle estimates for coherent signals. Simulation results demonstrate that the hybrid MUSIC algorithm can accurately estimate source DOA under low SNR conditions with small signal separation and correlated signals, verifying its high resolution and superior performance.

Keywords: coherent signal; multiple signal classification; spatial smoothing; direction of arrival estimation

0 Introduction

Direction-of-arrival (DOA) estimation of spatial signals is a critical problem in array signal processing, widely applied in radar, sonar, and wireless communications for target localization, tracking, and spatial filtering. Under the research efforts of numerous scholars, various DOA estimation algorithms have been proposed. Reference [1] first introduced the Multiple Signal Classification (MUSIC) algorithm, which constructs a spatial spectrum function based on the orthogonality between signal direction vectors and the noise subspace, performing DOA estimation through spectral peak searching. The MUSIC algorithm offers high resolution, estimation accuracy, and stability under specific conditions, and is applicable to various array configurations. Subsequent evolutions include eigenspace-based DOA estimation algorithms, modified MUSIC, improved MUSIC, Root-MUSIC, and weighted MUSIC algorithms.

Reference [2] proposed a modified MUSIC algorithm that essentially takes the subarray count as 1 in forward-backward spatial smoothing, making the subarray length equal to the number of array elements. This approach does not reduce array aperture and maintains high resolution even at low SNR. References [3–5] presented a Root-MUSIC algorithm based on eigen-decomposition, which converts the spatial spectrum peak search into a polynomial solving process, offering high direction-finding accuracy and superior performance with small sample sizes. Reference [6] proposed an improved MUSIC algorithm that preprocesses the signal covariance matrix to ensure orthogonality between the noise and signal subspaces obtained from decomposition, thereby reducing noise interference. Reference [7] derived a weighted MUSIC (w-MUSIC) algorithm using maximum likelihood methods, obtaining the overall spatial spectrum by combining weighted MUSIC spectra from subarrays. Reference [8] proposed an eigen-space-based DOA estimation algorithm that combines spatial smoothing techniques to quickly obtain the signal subspace from a submatrix of the array covariance matrix without requiring eigenvalue or singular value decomposition, reducing computational complexity and improving DOA estimation efficiency.

Coherent signals cause rank deficiency in the source covariance matrix, rendering conventional signal subspace algorithms ineffective for DOA estimation. Current solutions primarily focus on effectively restoring the rank of the signal covariance matrix or reconstructing a matrix whose rank equals the number of signal sources. The spatial smoothing algorithm was first proposed by Evans et al. [9,10], with the core principle of dividing a uniform linear array into several overlapping subarrays with identical array manifolds. The covariance matrices of these subarrays can be averaged to achieve decoherence. References [11–13] improved spatial smoothing algorithms for effective preprocessing of coherent sources. Reference [11] used multiple cyclic correlations (MCC) combined with in-phase spatial smoothing for DOA estimation, achieving highest accuracy with two cyclic correlation matrices and corresponding cyclic backward matrices (2MCC-CB), enhancing DOA estimation performance. Reference [12] proposed the forward-backward spatial smoothing (FBSS) algorithm, which forms backward spatial smoothing by conjugate reversal of forward smoothing subarrays and combines them. Spatial smoothing techniques and their improvements offer ideal performance for coherent source DOA estimation with low computational complexity, but at the cost of reduced effective array elements and aperture loss, with poor performance at low SNR. References [14–15] proposed an improved forward/backward spatial smoothing Root-MUSIC algorithm that reduces computational complexity by utilizing eigenvalue decomposition (EVD) of real-valued covariance matrices. Reference [16] proposed an improved weighted spatial smoothing (IWSS) technique for coherent source DOA estimation, obtaining weight matrices through nested spatial smoothing algorithms to achieve high resolution and estimation performance without prior information or preliminary angle estimates. Reference [17] proposed a real-valued MUSIC algorithm using FB techniques, converting the direction-finding process to the real domain to significantly reduce computation. Reference [18] proposed the I-MMUSIC algorithm combining forward-backward spatial smoothing with MMUSIC for automotive collision avoidance radar.

Based on the above analysis, this paper proposes a hybrid MUSIC algorithm building upon classical MUSIC and existing decoherence algorithms. The algorithm integrates forward-backward spatial smoothing with improved IMUSIC, and MATLAB simulations compare it with forward-backward spatial smoothing, improved IMUSIC, and conventional MUSIC algorithms. Simulation results demonstrate that the hybrid MUSIC algorithm offers high direction-finding accuracy and resolution for coherent source DOA estimation.

1.1 Uniform Linear Array Signal Model

[Figure 1: see original paper] illustrates a uniform linear array consisting of M antenna elements with element spacing d, signal wavelength λ, forming an array antenna. Assuming p narrowband signal sources impinge on the array as plane waves with DOAs θ_i, the signal model for the t-th sampling point can be expressed as:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t)$$

where $\mathbf{X}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$ is the M-element data matrix; $\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_p(t)]^T$ is the source matrix with $s_i(t)$ representing the complex amplitude of the i-th plane wave; $\mathbf{N}(t) = [n_1(t), n_2(t), \dots, n_M(t)]^T$ is the noise matrix where $n_i(t)$ is white noise with variance σ^2 and uncorrelated with signal sources; $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_p)]$ is the direction matrix. The array response vector can be expressed as:

$$\mathbf{a}(\theta_i) = \left[1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_i}, \dots, e^{-j\frac{2\pi(M-1)d}{\lambda}\sin\theta_i}\right]^T$$

where θ_i is the DOA of source i $(i = 1, 2, \dots, p)$.

1.2 MUSIC Algorithm Principle

Based on the uniform linear array signal model above, the array signal covariance matrix can be obtained from Equation (1) as:

$$\mathbf{R} = E\{\mathbf{X}(t)\mathbf{X}^H(t)\} = \mathbf{A}E\{\mathbf{S}(t)\mathbf{S}^H(t)\}\mathbf{A}^H + E\{\mathbf{N}(t)\mathbf{N}^H(t)\} = \mathbf{A}\mathbf{R}_S\mathbf{A}^H + \sigma^2\mathbf{I}$$

where $\mathbf{R}_S$ is the signal covariance matrix; $\sigma^2\mathbf{I}$ is the noise covariance matrix; and $\mathbf{I}$ is the $M \times M$ identity matrix. Eigen-decomposition of $\mathbf{R}$ yields:

$$\mathbf{R} = \mathbf{U}_S \;{}_S\mathbf{U}_S^H + \mathbf{U}_N \;{}_N\mathbf{U}_N^H$$

where $\mathbf{U}_S$ is the signal subspace spanned by eigenvectors corresponding to large eigenvalues; $\mathbf{U}_N$ is the noise subspace spanned by eigenvectors corresponding to small eigenvalues; and ${}_S$ is a diagonal matrix of p large eigenvalues.

Since each column vector in the direction matrix $\mathbf{A}$ is orthogonal to the noise subspace, and considering the actual received data matrix has finite length, the maximum likelihood estimate of the data covariance matrix is:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{X}(t)\mathbf{X}^H(t)$$

Eigen-decomposition of $\hat{\mathbf{R}}$ yields the noise subspace eigenvector matrix. Based on the orthogonality between noise eigenvectors and signal vectors, the array spatial spectrum function is:

$$P_{\mathrm{MUSIC}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{U}}_N\hat{\mathbf{U}}_N^H\mathbf{a}(\theta)}$$

From Equation (6), when $\mathbf{a}(\theta)$ is orthogonal to each column of $\hat{\mathbf{U}}_N$, the denominator becomes zero. However, due to noise presence, sharp peaks appear, and DOA estimation is achieved by locating these peaks.

1.3 Impact of Coherent Sources on DOA Estimation

Based on the coherent signal source mathematical model, we study the impact on DOA estimation. For two stationary signals $s_i(t)$ and $s_j(t)$, their correlation coefficient is defined as:

$$\rho_{ij} = \frac{E\{s_i(t)s_j^*(t)\}}{\sqrt{E\{|s_i(t)|^2\}E\{|s_j(t)|^2\}}}$$

By the Schwarz inequality, the correlation coefficient satisfies $|\rho_{ij}| \leq 1$. The correlation between different signals can be expressed as: when $\rho_{ij} = 0$, $s_i(t)$ and $s_j(t)$ are uncorrelated; when $0 < |\rho_{ij}| < 1$, they are partially correlated; when $|\rho_{ij}| = 1$, they are coherent. For coherent sources, the mathematical representation shows they differ only by a complex constant. Assuming n coherent sources, i.e., $s_i(t) = \alpha_i s_0(t)$ for $i = 0, 1, 2, \dots, n$, where $s_0(t)$ is the generating source producing n coherent signals impinging on the array. The coherent signal source model yields:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) = \mathbf{A}\alpha s_0(t) + \mathbf{N}(t)$$

where $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]^T$ is an n-dimensional vector of complex constants. According to this model, with p incident signal sources, the array signal can be expressed as shown in the coherent source mathematical model.

[Figure 2: see original paper] shows the division of subarrays. The signal matrix received by the l-th forward subarray is:

$$\mathbf{X}_l^f(t) = [x_l(t), x_{l+1}(t), \ldots, x_{l+m-1}(t)]^T$$

The source autocorrelation matrix is $\mathbf{R}_S = E\{\mathbf{S}(t)\mathbf{S}^H(t)\}$, and the source correlation matrix is:

$$\mathbf{R}_S = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1p}\sigma_1\sigma_p \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2p}\sigma_2\sigma_p \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p1}\sigma_p\sigma_1 & \rho_{p2}\sigma_p\sigma_2 & \cdots & \sigma_p^2 \end{bmatrix}$$

where $\mathbf{D}$ is a diagonal matrix and σ_i^2 is the power of the i-th source. When p signals are completely coherent, $\text{rank}(\mathbf{R}_S) = 1$, where both p and n are less than or equal to M. The array received data covariance matrix rank reduces to 1, causing the signal subspace dimension to be smaller than the number of signal sources, thus preventing correct source direction estimation. Therefore, the core problem in DOA estimation with coherent signals is restoring the rank of the signal covariance matrix $\mathbf{R}_S$ to equal the number of signal sources p.

---

2 Improved Algorithm

The hybrid MUSIC algorithm essentially combines forward-backward spatial smoothing with improved IMUSIC. First, array elements undergo forward and backward smoothing preprocessing to restore the array covariance matrix rank to the number of signal sources. The resulting new covariance vector matrix is then processed using the improved IMUSIC algorithm to obtain DOA angle estimates for coherent signals. The following sections detail the array preprocessing and implementation steps.

Forward-backward spatial smoothing combines forward spatial smoothing with conjugate backward spatial smoothing to increase the effective number of antennas. The array preprocessing is achieved through forward-backward spatial smoothing. When two coherent signals enter the antenna array simultaneously, they reduce the signal subspace vector count of the array covariance matrix. However, if two coherent signals enter different antenna arrays, the sum of these arrays' covariance matrices yields a signal subspace vector count equal to the target number. Based on this principle and using the translation invariance of uniform linear arrays, the array is divided into several overlapping subarrays with identical array manifolds. The covariance matrices of these subarrays can be summed and averaged to replace the original covariance matrix, thereby achieving decoherence.

When the subarray element number $m \geq p + 1$, the forward spatial smoothing data covariance matrix $\mathbf{R}^f$ is full rank [18]. Analyzing the reversed uniform linear array, the backward smoothing subarray received data is similarly obtained as:

$$\mathbf{X}_l^b(t) = [x_{M-l+1}^*(t), x_{M-l}^*(t), \dots, x_{M-l-m+2}^*(t)]^T$$

The covariance matrix for each backward smoothing subarray is:

$$\mathbf{R}_l^b = E\{\mathbf{X}_l^b(t)(\mathbf{X}_l^b(t))^H\} = \mathbf{A}_l\mathbf{D}\mathbf{A}_l^H + \sigma^2\mathbf{I}$$

The backward spatial smoothing covariance matrix is:

$$\mathbf{R}^b = \frac{1}{L}\sum_{l=1}^{L}\mathbf{R}_l^b$$

where $\mathbf{R}^b$ is the conjugate reversed matrix of $\mathbf{R}^f$. Based on conjugate reversal invariance, the forward-backward smoothing covariance matrix is defined as:

$$\mathbf{R}^{fb} = \frac{1}{2}(\mathbf{R}^f + \mathbf{R}^b)$$

The advantage of conjugate reversal invariance lies in increasing the number of subarrays and overcoming limitations of eigen-decomposition-based DOA algorithms.

The forward-backward data covariance $\mathbf{R}^{fb}$ is then applied to the improved IMUSIC algorithm for coherent source DOA estimation. The improved IMUSIC algorithm reconstructs the received signal covariance matrix to restore its rank to p, enabling effective DOA estimation.

Let $\mathbf{J}$ be the $M \times M$ reverse identity matrix:

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

The improved signal correlation matrix is defined as:

$$\mathbf{R}_1 = \frac{1}{2}(\mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J})$$

where $\mathbf{R}^*$ is the conjugate of $\mathbf{R}$. Now $\mathbf{R}_1$ becomes a Hermitian Toeplitz matrix and is an unbiased estimate of $\mathbf{R}$.

Performing singular value decomposition on $\mathbf{R}_1$ yields the noise subspace $\mathbf{U}_N$:

$$[\mathbf{U}_S,\ ,\mathbf{V}] = \mathrm{svd}(\mathbf{R}_1), \quad \mathbf{U}_N = \mathbf{U}(:, p+1 : M)$$

For large-scale matrices, a low-rank matrix can approximate a known matrix through matrix decomposition, i.e., decomposing matrix $\mathbf{R}$ into $\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2$ where both correspond to accurate incident angles. The hybrid MUSIC algorithm shows significantly larger spectral values than other algorithms, indicating all three methods can handle coherent signals but the hybrid MUSIC algorithm demonstrates superior performance.

The matrix low-rank decomposition problem can be converted to the following optimization:

$$\min_{\mathbf{R}_1,\mathbf{R}_2} \mathrm{rank}(\mathbf{R}_1) + \lambda \|\mathbf{R}_2\|_F \quad \text{s.t.} \quad \mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2$$

where $\|\cdot\|_F$ denotes the Frobenius norm and λ controls rank approximation. Performing SVD on the new covariance matrix $\mathbf{R}_2$ and repeating the steps reconstructs the noise subspace $\mathbf{U}_N^u$.

Taking the average of the two noise subspace vectors yields:

$$\mathbf{U}_N^{\mathrm{avg}} = \frac{1}{2}(\mathbf{U}_N + \mathbf{U}_N^u)$$

The spatial spectrum is then constructed as:

$$P_{\text{Hybrid MUSIC}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_N^{\mathrm{avg}}(\mathbf{U}_N^{\mathrm{avg}})^H\mathbf{a}(\theta)}$$

Based on the above discussion, the hybrid MUSIC algorithm steps are summarized as:

a) For an M-element antenna array, the signal model for the t-th sampling point is given by Equation (1).

b) Divide the antenna array using forward-backward spatial smoothing to obtain forward data vector $\mathbf{X}_l^f$ and backward data vector $\mathbf{X}_l^b$. Compute corresponding covariance matrices $\mathbf{R}_l^f$ and $\mathbf{R}_l^b$ using Equations (16) and (20).

c) Compute the average covariance matrices $\mathbf{R}^f$ and $\mathbf{R}^b$ for L subarrays from step b).

d) Obtain the forward-backward data covariance matrix $\mathbf{R}^{fb}$ from the mean of $\mathbf{R}^f$ and $\mathbf{R}^b$.

e) Let $\mathbf{J}$ be the $M \times M$ reverse identity matrix. Reconstruct the received signal covariance matrix to make $\mathbf{R}_1$ a Hermitian Toeplitz matrix using Equation (24).

f) Perform SVD on $\mathbf{R}_1$ to obtain noise subspace $\mathbf{U}_N$.

g) Replace full-rank matrix $\mathbf{R}_1$ with a low-rank matrix to construct new covariance matrix $\mathbf{R}_2$. Repeat steps to reconstruct noise subspace $\mathbf{U}_N^u$.

h) Compute the average of noise subspaces to obtain $\mathbf{U}_N^{\text{avg}}$.

i) Construct the spatial spectrum function.

---

3 Simulation and Analysis

To visually demonstrate algorithm performance, MATLAB simulations were conducted. Signals satisfy far-field and narrowband conditions, with array element noise being stationary zero-mean Gaussian white noise with variance σ^2 and uncorrelated with target sources.

Experiment 1

The simulation model uses an 8-element uniform linear array with element spacing $d = \lambda/2$. Three signal sources impinge on the array from directions -45°, 0°, and 30°, with 0° and 30° signals fully coherent. SNR is 10 dB for all signals, with 1,024 snapshots. Results are shown in [Figure 3: see original paper].

[Figure 3: see original paper] clearly shows that for coherent signals, classical MUSIC performance degrades severely and cannot resolve source DOAs. The other three algorithms exhibit three distinct spectral peaks, all corresponding to accurate incident angles, with the hybrid MUSIC algorithm showing significantly larger spectral values, demonstrating superior performance in handling coherent signals.

Experiment 2

The simulation model uses an 8-element uniform linear array with $d = \lambda/2$. Three signal sources from directions $[-2^\circ, 0^\circ, 2^\circ]$ are fully coherent, with SNR = 30 dB and 1,024 snapshots. Results are shown in [Figure 4: see original paper].

[Figure 4: see original paper] demonstrates that with small angular separation and coherence, classical MUSIC and improved IMUSIC completely fail. Under the same conditions, the hybrid MUSIC algorithm more accurately resolves adjacent coherent sources compared to forward-backward spatial smoothing alone.

Experiment 3

The simulation model uses an 8-element uniform linear array with $d = \lambda/2$. Three fully coherent sources from $-5°$, $0°$, and $5°$ are simulated at SNR values of 30 dB, 20 dB, 10 dB, and 5 dB, with 1,024 snapshots. Results are shown in [Figure 5: see original paper]-[Figure 8: see original paper].

[Figure 5: see original paper]-[Figure 8: see original paper] show that as SNR decreases, all algorithms' resolution degrades to varying degrees. At SNR = 30 dB, improved IMUSIC, forward-backward spatial smoothing, and hybrid MUSIC can all estimate DOA. At SNR = 20 dB or 10 dB, improved IMUSIC fails to resolve signals and forward-backward spatial smoothing shows estimation bias, while hybrid MUSIC maintains correct resolution. At SNR = 5 dB, with small separation, low SNR, and full coherence, hybrid MUSIC still accurately estimates signal DOA, proving its high performance and stability across different SNR conditions.

Experiment 4

The simulation model uses an 8-element uniform linear array with $d = \lambda/2$. Two fully coherent sources from $0°$ and $5°$ are simulated with 100 snapshots. Subarray element count is 6 (3 smoothing operations for forward smoothing, 6 for double smoothing). With SNR varying from -10 dB to 30 dB, the estimation performance curves are shown in [Figure 9: see original paper]-[Figure 11: see original paper].

[Figure 9: see original paper] shows that as SNR increases, all four algorithms' success probabilities improve significantly. Overall, hybrid MUSIC achieves the highest success probability at low SNR, followed by forward-backward spatial smoothing, while classical MUSIC performs worst. [Figure 10: see original paper] shows that at high SNR, all algorithms have small estimation bias (high accuracy), while at low SNR, hybrid MUSIC maintains low bias. [Figure 11: see original paper] shows hybrid MUSIC has the smallest estimation standard deviation, indicating better stability.

In summary, the hybrid MUSIC algorithm has a lower SNR threshold, lower estimation bias and standard deviation, and higher estimation accuracy and stability, particularly when spatial sources are closely spaced and SNR is low.

---

4 Conclusion

This paper proposes a hybrid MUSIC algorithm for coherent source DOA estimation. The algorithm integrates forward-backward spatial smoothing with improved IMUSIC, preprocessing the antenna array through forward-backward smoothing and applying the resulting covariance vector matrix to improved IMUSIC for signal processing. Numerical simulations compare the hybrid MUSIC algorithm with forward-backward spatial smoothing and improved IMUSIC algorithms. Results show that the hybrid MUSIC algorithm offers high direction-finding resolution, effectively estimating DOAs of both independent and coherent sources. Even under low SNR, small signal separation, and strong correlation, the hybrid MUSIC algorithm accurately estimates signal DOA, verifying its high resolution and superior performance.

References

[1] Schmidt R, Schmidt R O. Multiple emitter location and signal parameters estimation [J]. IEEE Trans on Antennas & Propagation, 1986, 34(3): 276-280.
[2] Zhang Xiaofei. Array signal processing and MATLAB implementation [M]. Beijing: Electronics Industry Press, 2015.
[3] Gupta P, Kar S P. MUSIC and improved MUSIC algorithm to estimate direction of arrival [C]//Proc of International Conference on Communications and Signal Processing. [S.l.]: IEEE Press, 2015.
[4] Kundu D. Modified MUSIC algorithm for estimation DOA of signals [J]. Signal Processing, 1996, (48): 85-891.
[5] Qian Cheng, Huang Lei, So H C. Improved unitary root-MUSIC for DOA estimation based on pseudo-noise resampling [J]. IEEE Signal Processing Letters, 2013, 21(2): 140-144.
[6] Gupta P, Kar S P. MUSIC and improved MUSIC algorithm to estimate direction of arrival [C]//Proc of International Conference on Communications and Signal Processing. [S.l.]: IEEE Press, 2015.
[7] Yan Fenggang, Shen Yi, Jin Ming, et al. Computationally efficient direction finding using polynomial rooting with reduced-order and real-valued computations [J]. Journal of Systems Engineering and Electronics, 2016, 27(4): 739-745.
[8] Xiong Wei. Research on direction of arrival (DOA) estimation algorithm based on feature subspace [D]. Wuhan: Huazhong University of Science and Technology, 2007.
[9] Evans J E, Johnson J R, Sun Fuding. High resolution angular spectrum estimation techniques for terrain scattering analysis and angle of arrival estimation [C]// Proc of Assp Workshop Spectral Estimation. 1982.
[10] Evans J E, Sun Fuding, Johnson J R. Application of advanced signal processing techniques to angle of arrival estimation in atc navigation and surveillance systems [J]. Calculation, 1982.
[11] Kamiya Y, Kikuma N, Sakakibara K. DOA estimation of desired signals using in-phase combining of multiple cyclic correlations and spatial smoothing processing [C]// Proc of International Symposium on Antennas & Propagation. [S.l.]: IEEE Press, 2017.
[12] Iwai T, Hirose N, Kikuma N, et al. DOA estimation by MUSIC algorithm us-

ing forward-backward spatial smoothing with overlapped and augmented arrays [C]// Proc of International Symposium on Antennas and Propagation. [S.l.]: IEEE Press, 2015: 375-376.
[13] Mei Feng, Kammeyer K D. Modified subspace smoothing technique for multipath direction finding [C]// Proc of Signal Processing Conference. [S.l.]: IEEE Press, 2015.
[14] Choi Yangho. On conditions for forward/backward spatial smoothing [J]. IEEE Trans on Signal Processing, 2002, 50(11): 2900-2901.
[15] Shu Changgan, Liu Yumin. An improved forward/backward spatial smoothing Root-MUSIC algorithm based on signal decorrelation [C]// Advanced Research and Technology Industry Applications. [S.l.]: IEEE Press, 2014: 1252-1255.
[16] Wu Xiangdong, Ma Lun, Liang Zhonghua. Improved weighted spatial smoothing algorithm [J]. Journal of Data Acquisition & Processing, 2015.
[17] Wang Yan, Si Weijian, Wang Kun. A real-valued MUSIC algorithm with forward/backward technique [C]// Proc of Applied Computational Electromagnetics Society Symposium. [S.l.]: IEEE Press, 2017: 1-2.
[18] Jia Jiemin. Research on DOA estimation of coherent signals based on MUSIC algorithm [D]. Xi' an: Xidian University, 2015.
[19] Shan Tiejun, Wax M, Kailath T. On spatial smoothing for estimation of coherent signals [J]. IEEE Trans on Acoustics Speech & Signal Processing, 1985, 33(4): 806-811.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.