Super-Resolution Image Reconstruction Based on Adaptive Semi-Coupled Dictionary Learning (Postprint)

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Abstract

In the field of super-resolution image reconstruction, balancing the sparsity and cooperativity of representation coefficients in dictionary learning is of significant importance to reconstruction quality. To address this issue, building upon semi-coupled dictionary learning-based super-resolution reconstruction, we construct a novel regularization term using the nuclear norm that considers sparsity and cooperativity as an integrated whole. The optimization model is solved using the Alternating Direction Method of Multipliers (ADMM), yielding a super-resolution image reconstruction algorithm based on adaptive semi-coupled dictionary learning. Experimental results demonstrate that this method achieves superior reconstruction performance compared to several existing dictionary learning-based reconstruction approaches. The proposed algorithm can adaptively balance sparsity and correlation according to dictionary variations and generate optimal coefficients through coordination between these two properties, thereby conferring a certain degree of anti-interference capability in noisy environments.

Full Text

Image Super-Resolution Reconstruction Based on Adaptive Semi-Coupled Dictionary Learning

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Abstract: In the field of image super-resolution reconstruction, balancing the sparsity and cooperation of representation coefficients in dictionary learning is crucial for reconstruction quality. Addressing this issue, we propose an adaptive semi-coupled dictionary learning super-resolution method built upon semi-coupled dictionary learning. Our approach employs the nuclear norm to construct a novel regularization term that jointly considers sparsity and cooperation, and solves the optimization model using the Alternating Direction Method of Multipliers (ADMM). Experimental results demonstrate that the proposed method achieves superior reconstruction performance compared to existing dictionary learning-based methods. The algorithm adaptively balances sparsity and correlation according to dictionary variations, generating optimal coefficients through their coordination, thereby exhibiting robustness against noise.

Keywords: super-resolution reconstruction; semi-coupled dictionary learning; adaptivity; nuclear norm

0 Introduction

In real-world scenarios, low-resolution images are typically obtained from original high-resolution images through degradation processes including downsampling, blurring, and noise contamination. Image super-resolution reconstruction represents the inverse of this degradation process, aiming to recover high-resolution images from low-resolution inputs. This is an active research area in computer vision [?] with widespread applications in intelligent surveillance [?], remote sensing [?], medical diagnosis [?], and other domains.

Current super-resolution methods primarily fall into three categories: interpolation-based [?], reconstruction-based [?], and learning-based approaches [?]. Learning-based methods, including dictionary learning [?] and deep learning [?], learn the mapping relationship between high- and low-resolution image patches to acquire prior knowledge for reconstruction. This paper focuses on dictionary learning-based super-resolution methods.

In 2008, Yang et al. [?] first proposed the Sparse Coding Super-Resolution (SCSR) method based on compressed sensing theory. This method assumes that high- and low-resolution image patches share identical sparse representation coefficients under their respective dictionaries. Consequently, sparse coefficients obtained from low-resolution patches can be multiplied with the high-resolution dictionary to reconstruct corresponding high-resolution patches. Building on this work, Zeyde et al. improved dictionary training by using Principal Component Analysis for dimensionality reduction of low-resolution patch features and K-Singular Value Decomposition [?] to train the low-resolution dictionary, obtaining the high-resolution dictionary through pseudo-inverse multiplication with high-resolution patches. This approach achieved better reconstruction quality and faster speed than SCSR [?]. Lu et al. [?] argued that structurally similar patches should have similar representation coefficients, adding a non-local self-similarity regularization term to maintain geometric structure and enhance model stability. Cao et al. [?] modified the ℓ_1 regularization to an ℓ_p term for sparser coefficients, proposing a more effective ℓ_p-based super-resolution method.

Hojjat et al. [?] extended sparse representation to multiple color channels by leveraging cross-channel color information. Xie et al. [?] suppressed sparse coding noise using bidirectional similarity points to construct complementary regularization terms, achieving additional coefficient alignment for improved recovery. Yang et al. [?] employed local sub-dictionaries for patch encoding to better characterize local image structures.

Additionally, Cai et al. [?] considered coefficient cooperation by replacing the ℓ_1 regularization with an ℓ_2 term, proposing the Local Structural Similarity and Collaborative Representation (LCR) method [?] that significantly accelerated reconstruction. Zhao et al. [?] jointly considered sparsity and cooperation by constructing a novel nuclear norm-based regularization term, proposing the Adaptive Sparse Coding based Super-Resolution (ASCSR) method that adaptively balances these properties.

All aforementioned sparse coding-based methods assume identical sparse representation coefficients for high- and low-resolution patches. Wang et al. [?] considered a linear relationship between these coefficients, proposing a Semi-Coupled Dictionary Learning (SCDL) sparse coding algorithm for super-resolution that achieved promising results.

This paper proposes an Adaptive Semi-Coupled Dictionary Learning Super-Resolution (ASCDSR) method built upon SCDL. Our approach assumes that high- and low-resolution patches have related but not identical representation coefficients, with a linear relationship between them. We construct a nuclear norm-based regularization term to balance sparsity and cooperation, solved via ADMM [?]. The high-resolution patches are reconstructed by multiplying the high-resolution dictionary with the corresponding coefficients. By simultaneously considering sparsity and cooperation and adaptively balancing their relationship, our method demonstrates robust performance under noise contamination.

1 Proposed Super-Resolution Reconstruction Method

This section presents our ASCDSR method, which incorporates both sparsity and cooperation into semi-coupled dictionary learning.

For a low-resolution image I_h to be reconstructed, let $Y = [y_1, y_2, \dots, y_n]$ denote column vectors of its image patches, where y_i represents the i-th low-resolution patch. Let $X = [x_1, x_2, \dots, x_n]$ be the corresponding high-resolution patch column vectors to be reconstructed, where x_i corresponds to y_i. We first train high- and low-resolution dictionaries D_X and D_Y and the mapping relationship between their sparse coefficients using external images. Let $X' = [x'_1, x'_2, \dots, x'_m]$ and $Y' = [y'_1, y'_2, \dots, y'_m]$ denote column vectors of high- and low-resolution patches from external images, where x'_i and y'_i are corresponding patches. The training optimization model from SCDL is:

$$\begin{aligned}\min_{D_X,D_Y,W_{XY},W_{YX}} & \|X' - D_X\Lambda_X\|_F^2 + \|Y' - D_Y\Lambda_Y\|_F^2 \\ & + \gamma_1\|\Lambda_X - W_{XY}\Lambda_Y\|_F^2 + \gamma_2\|\Lambda_Y - W_{YX}\Lambda_X\|_F^2 \\ \text{s.t.} \quad & \|D_X(:,k)\|_2 \leq 1, \|D_Y(:,k)\|_2 \leq 1, \quad k = 1,2,\ldots,K\end{aligned}$$

where Λ_X and Λ_Y are coefficient matrices, W_{XY} and W_{YX} are mapping matrices, and γ_1, γ_2 are regularization parameters.

While SCSR and SCDL emphasize sparsity via ℓ_1 regularization and LCR emphasizes cooperation via ℓ_2 regularization, our method simultaneously considers both properties. Using the trained dictionaries D_X, D_Y, we propose the following adaptive semi-coupled dictionary learning model:

$$\begin{aligned}\min_{\Lambda_X,\Lambda_Y} & \|Y - D_Y\Lambda_Y\|_F^2 + \lambda_1\|\Lambda_X - W_{XY}\Lambda_Y\|_F^2 + \lambda_2\|\Lambda_Y - W_{YX}\Lambda_X\|_F^2 \\ & + \lambda_3\|\Lambda_X\|_* + \lambda_4\|\Lambda_Y\|_* \\ \text{s.t.} \quad & \|x_i\|_0 \leq T, \|y_i\|_0 \leq T, \quad i = 1,2,\ldots,n\end{aligned}$$

where $\|\cdot\|_*$ denotes the nuclear norm (sum of singular values), λ_i are non-negative regularization parameters, and x_i, y_i are the i-th columns of Λ_X, Λ_Y representing coefficients for the i-th patch. $\text{Diag}(\alpha)$ creates a diagonal matrix from vector α.

The nuclear norm has the following property [**?**, **?**]: when matrix columns are uncorrelated, $\|\Lambda\|_* \approx \|\Lambda\|_1$; when highly correlated, $\|\Lambda\|_* \approx \|\Lambda\|_2$. Since trained dictionary atoms are neither completely independent nor highly correlated, the nuclear norm enables adaptive balancing between sparsity and cooperation.

We solve Eq. (2) by decomposing it into two subproblems:

$$\begin{aligned}\Lambda_Y^{k+1} = \arg\min_{\Lambda_Y} & \|Y - D_Y\Lambda_Y\|_F^2 + \lambda_1\|\Lambda_X^k - W_{XY}\Lambda_Y\|_F^2 \\ & + \lambda_2\|\Lambda_Y - W_{YX}\Lambda_X^k\|_F^2 + \lambda_4\|\Lambda_Y\|_*\end{aligned}$$

$$\begin{aligned}\Lambda_X^{k+1} = \arg\min_{\Lambda_X} & \lambda_1\|\Lambda_X - W_{XY}\Lambda_Y^{k+1}\|_F^2 + \lambda_2\|\Lambda_Y^{k+1} - W_{YX}\Lambda_X\|_F^2 \\ & + \lambda_3\|\Lambda_X\|_*\end{aligned}$$

We solve these using ADMM. Due to structural similarity, we detail only Eq. (4). For convenience, we present the vector form of Eq. (4). Let $\Lambda_Y = [\alpha_{y1}, \alpha_{y2}, \ldots, \alpha_{yn}]$ and $\Lambda_X = [\alpha_{x1}, \alpha_{x2}, \ldots, \alpha_{xn}]$ be coefficient matrices for high- and low-resolution patches respectively, with W_{XY} and W_{YX} as mapping matrices. The vector form becomes:

$$\alpha_{yi}^{k+1} = \arg\min_{\alpha_{yi}} \|y_i - D_Y\alpha_{yi}\|_2^2 + \lambda_1\|\alpha_{xi}^k - W_{XY}\alpha_{yi}\|_2^2 + \lambda_2\|\alpha_{yi} - W_{YX}\alpha_{xi}^k\|_2^2 + \lambda_4\|\alpha_{yi}\|_2^2$$

The augmented Lagrangian function is:

$$\mathcal{L}(\alpha_{yi}, e_i, v_i, H_i) = \|y_i - D_Y\alpha_{yi}\|_2^2 + \lambda_1\|\alpha_{xi}^k - W_{XY}\alpha_{yi}\|_2^2 + \lambda_2\|\alpha_{yi} - W_{YX}\alpha_{xi}^k\|_2^2 + \lambda_4\|e_i\|_* + \langle v_i, \alpha_{yi} - e_i\rangle + \frac{\rho}{2}\|\alpha_{yi} - e_i\|_2^2$$

where v_i and H_i are Lagrange multipliers, and $\langle \cdot, \cdot \rangle$ denotes inner product. The ADMM iterations are:

$$\begin{cases} \alpha_{yi}^{k+1} = \arg\min_{\alpha_{yi}} \mathcal{L}(\alpha_{yi}, e_i^k, v_i^k, H_i^k) \\ e_i^{k+1} = \arg\min_{e_i} \mathcal{L}(\alpha_{yi}^{k+1}, e_i, v_i^k, H_i^k) \\ v_i^{k+1} = v_i^k + \rho(\alpha_{yi}^{k+1} - e_i^{k+1}) \\ H_i^{k+1} = H_i^k + \rho(\mathcal{D}(e_i^{k+1}) - \mathcal{Z}(D_Y \mathrm{diag}(\alpha_{yi}^{k+1}))) \end{cases}$$

The specific solutions are:

$$\alpha_{yi}^{k+1} = (D_Y^T D_Y + \lambda_1 W_{XY}^T W_{XY} + \lambda_2 I + \rho I)^{-1}(D_Y^T y_i + \lambda_1 W_{XY}^T \alpha_{xi}^k + \lambda_2 W_{YX}\alpha_{xi}^k + \rho e_i^k - v_i^k)$$

For the nuclear norm term, let $e_i = U\mathrm{diag}(\sigma)V^T$ be the SVD. Then:

$$e_i^{k+1} = U\mathrm{diag}(\mathcal{S}_{\lambda_4/\rho}(\sigma))V^T$$

where $\mathcal{S}_\tau(\sigma) = \max(\sigma - \tau, 0)$ is the singular value thresholding operator. The iteration terminates when $\max(\|e_i^{k+1} - e_i^k\|_\infty, \|\alpha_{yi}^{k+1} - \alpha_{yi}^k\|_\infty) \leq \varepsilon$.

Similarly, the solution for Eq. (5) follows the same pattern:

$$\alpha_{xi}^{k+1} = \arg\min_{\alpha_{xi}} \lambda_1\|\alpha_{xi} - W_{XY}\alpha_{yi}^{k+1}\|_2^2 + \lambda_2\|\alpha_{yi}^{k+1} - W_{YX}\alpha_{xi}\|_2^2 + \lambda_3\|e_i\|_* + \langle v_i, \alpha_{xi} - e_i\rangle + \frac{\rho}{2}\|\alpha_{xi} - e_i\|_2^2$$

Using the trained high-resolution dictionary D_X and coefficient matrix Λ_X from Eq. (4)-(5), we reconstruct high-resolution patches via $X = D_X\Lambda_X$.

Algorithm 1 summarizes the ASCDSR algorithm:

Algorithm 1: Adaptive Semi-Coupled Dictionary Learning for Super-Resolution

Input: Low-resolution image I_h to reconstruct; matrices X' and Y' of high- and low-resolution patch vectors from external training images.

1. Train high-resolution dictionary D_X, low-resolution dictionary D_Y, and mapping matrices W_{XY}, W_{YX} via Eq. (1).

2. Upsample I_h using bicubic interpolation and extract low-resolution patches Y.
3. For each patch y_i, solve Eq. (4)-(5) to obtain high-resolution coefficients α_{xi}, then reconstruct patch $x_i = D_X \alpha_{xi}$.
4. Fuse all reconstructed patches x_i into the final image I_{sr}.

Output: Reconstructed high-resolution image I_{sr}.

2 Experiments and Analysis

We compare ASCDSR against Bicubic, SCSR [?], LCR [?], ASCSR [?], and SCDL [?] methods. Experiments evaluate performance under noiseless conditions and with Gaussian and salt-and-pepper noise. Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index (SSIM) serve as evaluation metrics. Test images are shown in [Figure 1: see original paper].

To ensure fair comparison, we use a patch extraction stride of 4 and patch size 5×5. Dictionary sizes are 512 for both high- and low-resolution. Regularization parameters are set as $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.1$. Experiments run on Intel® Xeon® CPU, Windows 10, MATLAB R2016b.

2.1 Noiseless Image Reconstruction

Table 1 compares PSNR (dB) and SSIM for 2× upscaling, with best values in bold and second-best underlined. Figure 2 [Figure 2: see original paper] shows visual results for "Bird."

ASCDSR achieves the best performance on all seven test images. Even when matching SCDL's PSNR on "Flower,"ASCDSR produces higher SSIM, indicating better structural preservation.

2.2 Salt-and-Pepper Noise

We evaluate robustness by adding salt-and-pepper noise with density 10^{-4} to 10^{-3} to "elephant,""foreman,"and "pepper"images. Table 2 shows that ASCDSR consistently achieves the highest PSNR and SSIM across all noise levels.

2.3 Gaussian Noise

We test Gaussian noise with mean 0 and standard deviations $\sigma = 0.01, 0.02$. Table 3 demonstrates ASCDSR maintains the highest PSNR. Notably, at $\sigma = 0.02$, all dictionary-based methods except ASCDSR perform worse than Bicubic on "elephant," while ASCDSR remains superior. For SSIM, ASCDSR ranks second or third in most cases, though all dictionary methods underperform Bicubic on "elephant" at $\sigma = 0.01$.

2.4 Reconstruction Time

Table 4 compares average reconstruction times across seven test images. LCR, SCSR, and SCDL are fastest due to analytical solutions or simple ℓ_1 regularization solvable via compressed sensing. ASCSR and ASCDSR are slower because nuclear norm regularization requires ADMM with SVD computations. ASCDSR is slower than ASCSR as it additionally models the linear mapping between high- and low-resolution coefficients.

3 Conclusion

We propose an adaptive semi-coupled dictionary learning super-resolution algorithm that balances sparsity and cooperation through a nuclear norm regularization term, solved via ADMM. By assuming a mapping relationship between high- and low-resolution coefficients, we relax model constraints while adaptively balancing sparsity and cooperation according to dictionary characteristics. Experiments demonstrate superior PSNR and SSIM compared to existing methods, with robustness to noise.

Future work will incorporate additional prior knowledge such as local similarity to further improve reconstruction quality.

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Note: Figure translations are in progress. See original paper for figures.