Postprint of Experimental Study Based on Fourier Transform Dispersive Fringe Method

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Abstract

Optical interference requires the achievement of in-phase superposition across all optical paths, with even more stringent demands for wide-band white-light interference. Consequently, stable white-light interference necessitates real-time detection and control of optical path difference. This paper presents theoretical derivation and algorithmic design for the dispersion fringe method based on Fourier transform, and employs a laboratory-established Fizeau-type interferometric experimental apparatus to conduct both open-loop and closed-loop experimental investigations. In open-loop experiments, multiple dispersion fringe images at various optical path differences were acquired to study the relationship between the secondary peak shift in the image spectrum and the optical path difference, revealing a well-defined linear relationship between the shift and the optical path difference. In closed-loop experiments, real-time calculated optical path differences and an optical path compensation mechanism were utilized to implement closed-loop control of the optical path difference, demonstrating that the system can consistently maintain its initial interference state even under applied perturbations.

Full Text

Experimental Research on Dispersed Fringe Sensing Based on Fourier Transform

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Abstract

Optical interference requires achieving co-phased superposition of all optical paths, with even more stringent demands for broadband white-light interference. Consequently, realizing stable white-light interference necessitates real-time detection and control of optical path differences. This paper presents theoretical derivations and algorithmic designs for dispersed fringe sensing based on Fourier transform, along with open-loop and closed-loop experimental studies conducted using a Fizeau-type interferometric testbed established in our laboratory. In open-loop experiments, multiple dispersed fringe images were acquired at various optical path differences to investigate the relationship between secondary peak shifts in the image spectrum and the optical path difference. The results demonstrate a strong linear relationship between the shift magnitude and the optical path difference. In closed-loop experiments, real-time calculation results of the optical path difference and a piston compensation mechanism were employed for closed-loop control. The results show that the system can maintain its initial interference state even under external perturbations.

Keywords: Piston error detection; Fourier transform; Dispersed fringe sensing; Control

Optical synthetic aperture technology involves precisely arranging multiple small-aperture optical elements or systems in a specific configuration to achieve coherent superposition of the optical fields from each sub-aperture at the focal plane. After processing with image restoration techniques, this approach achieves diffraction-limited resolution comparable to that of a single large-aperture system, thereby overcoming the resolution limitations imposed by the diffraction limit of individual telescopes.

Optical synthetic aperture telescopes are classified into two types based on their segmentation approach: segmented primary mirrors and segmented sub-telescopes. Numerous optical synthetic aperture telescopes of the segmented sub-telescope type have been established worldwide. In 2002, the Massachusetts Institute of Technology developed the white-light adaptive GOLAY-3 ARGOS telescope, which consists of three sub-telescopes with 21 cm apertures, achieving an angular resolution of 0.35 , a spectral range of 400–700 nm, a field of view of 3 ×3 , an equivalent aperture of 0.62 m, and a signal-to-noise ratio of 100. Lockheed Martin, in collaboration with NASA, JPL, Caltech, the University of California, and the University of Rochester, established the Multi-Aperture Diverse Imaging System (MIDAS). This system enables wide-field, diffraction-limited imaging, capable of Earth observation from a 5000 km orbit with ground resolution better than 1 m, or high-resolution imaging from a 100 km orbit achieving 2 cm ground resolution.

For interferometric imaging with optical synthetic aperture telescopes, coherent superposition of light beams is essential to enhance spatial resolution. In white-light broadband imaging systems, the coherence length of the source is extremely short, effectively requiring zero optical path difference interference between beams traveling different paths. Therefore, precise detection of optical path differences becomes a prerequisite for achieving optical interference.

Extensive research on optical path detection has been conducted both domestically and internationally. Pupil-plane detection methods include the pyramid wavefront sensor, which calculates optical path differences by detecting intensity variations among four image points at the pupil plane. Defocus-plane detection methods include phase retrieval, which solves for phase distributions from a pair of intensity measurements at the focal and defocus planes. Focal-plane detection methods include: (1) narrowband and broadband Hartmann-Shack methods, which compute piston errors between segmented mirrors by analyzing correlation coefficients between sampled aperture diffraction patterns and reference templates; and (2) dispersed fringe sensing, which extracts lateral intensity signals from interference fringes between adjacent apertures and solves for optical path differences through nonlinear least-squares fitting.

To achieve a piston error detection range greater than ±50 m with precision better than 20 nm, and leveraging the optical synthetic aperture prototype system established at the Space Astronomy and Technology Laboratory of the National Astronomical Observatories, this paper presents theoretical derivations and algorithmic designs for dispersed fringe sensing based on Fourier transform, along with open-loop and closed-loop experiments. In open-loop experiments, a series of dispersed fringe images at different piston errors were acquired. The relationship between secondary peak shifts in the image spectrum and the piston error was investigated, and the results were compared with set values to verify the linear relationship. In closed-loop experiments, real-time calculation and closed-loop control of artificially introduced piston errors were implemented to evaluate the system’ s correction capability.

1 Theoretical Foundation

The intensity distribution expression at the focal plane based on the dispersed fringe method is given by:

$$I(x,y) = I_0\left\{1+\gamma\cos\left[2\pi\left(\frac{\phi}{\lambda_0}+\frac{C_0x}{\lambda_0^2}\right)+\varphi_0(y)\right]\right\}$$

where I_0 is the average intensity, γ is the fringe contrast, ϕ is the piston error, $\lambda(x)=\lambda_0+C_0x$ with λ_0 being the central wavelength, x the dispersion direction, C_0 the linear dispersion rate, and $\varphi_0(y)$ the initial phase. Since the wavenumber expression is $k=2\pi/\lambda$, performing a first-order Taylor approximation yields:

$$k \approx \frac{2\pi}{\lambda_0}-\frac{2\pi C_0}{\lambda_0^2}x$$

Substituting this first-order Taylor expansion into the expression above gives:

$$I(x,y) = I_0\left\{1+\gamma\cos\left[\left(\frac{2\pi}{\lambda_0}-\frac{2\pi C_0}{\lambda_0^2}x\right)\phi+\varphi_0(y)\right]\right\}$$

Performing a two-dimensional Fourier transform on this expression yields:

$$\mathcal{F}\{I(x,y)\}(f_x,f_y) = \iint I(x,y)e^{-j2\pi(xf_x+yf_y)}dxdy$$

Since the cosine term in the expression can be represented using Euler' s formula, the cosine term in the spectrum can be expressed as:

$$\cos\left[\left(\frac{2\pi}{\lambda_0}-\frac{2\pi C_0}{\lambda_0^2}x\right)\phi+\varphi_0(y)\right] = \frac{1}{2}\left\{e^{j\left[\left(\frac{2\pi}{\lambda_0}-\frac{2\pi C_0}{\lambda_0^2}x\right)\phi+\varphi_0(y)\right]}+e^{-j\left[\left(\frac{2\pi}{\lambda_0}-\frac{2\pi C_0}{\lambda_0^2}x\right)\phi+\varphi_0(y)\right]}\right\}$$

Using this representation for the cosine term, the two-dimensional spectral expression becomes:

$$\mathcal{F}\{I(x,y)\} = I_0\delta(f_x,f_y)+\frac{\gamma I_0}{2}\left[e^{j\varphi_0}\delta\left(f_x-\frac{C_0\phi}{\lambda_0^2},f_y\right)+e^{-j\varphi_0}\delta\left(f_x+\frac{C_0\phi}{\lambda_0^2},f_y\right)\right]*A(f_x,f_y)$$

where $A(f_x,f_y)$ represents the aperture function. The spectrum contains three peaks: a main peak at $(0,0)$ and two secondary peaks at $\pm\left(\frac{C_0\phi}{\lambda_0^2},0\right)$. Therefore, the offset of one secondary peak relative to the y-axis is:

$$\Delta x = \frac{C_0}{\lambda_0^2}\phi$$

Since C_0 and λ_0 are constants, the offset of the secondary peak relative to the y-axis exhibits a good linear relationship with the piston error ϕ. Thus, the Fourier transform-based method can be used to process dispersed fringe images and calculate the piston error value.

2 Piston Error Detection Algorithm Design

The piston error detection algorithm flow based on dispersed fringe images is shown in Figure 1 [Figure 1: see original paper]. First, an interference fringe image at a certain piston error is read and subjected to a two-dimensional Fourier transform to obtain its amplitude spectrum. After obtaining the spectral image, the region containing the main peak is set to zero, and the position of the maximum amplitude is determined, which corresponds to the location of the two secondary peaks. One of the secondary peaks is selected, and its offset relative to the y-axis, denoted as $\Delta x'$, is determined. Sub-pixel precision fitting is then performed centered at this secondary peak location. If the peak is located at (x_0, y_0) with amplitude $F(x_0, y_0)$, the fitting equation is:

$$\Delta x' = \frac{0.5\left[F(x_0+1, y_0) - F(x_0-1, y_0)\right]}{F(x_0+1, y_0) + F(x_0-1, y_0) - 2F(x_0, y_0)}$$

The piston error value can then be obtained by multiplying this result by a scaling coefficient, which can be either calculated theoretically or calibrated experimentally. This completes the piston error solving process for a single interference fringe image.

3 Optical Synthetic Aperture Experimental System

The Fizeau-type optical synthetic aperture testbed established at the National Astronomical Observatories consists of five subsystems: the light source system, sub-telescope system, pointing detection and compensation system, piston detection and compensation system, and beam-combining imaging and processing system, as shown in Figure 2 [Figure 2: see original paper]. The electronic system comprises eight circuit boards: an image distribution board, three computation and control boards, a digital-to-analog output board, analog and digital power supply boards, and an interface backplane, as shown in Figure 3 [Figure 3: see original paper].

In open-loop experiments, series of dispersed fringe images were acquired and processed using MATLAB to solve for piston errors. In closed-loop experiments, the electronic system was used to develop TS201 programs based on Visual DSP++ 5.0 for real-time piston error detection and control.

The light source system employs an NKT Photonics SuperK EXTREME series high-power supercontinuum white-light source with a filter output spectral range of 400–700 nm, featuring adjustable central wavelength, bandwidth, and output power. The source telescope is a classic Cassegrain design with a 500 mm clear aperture, 20 m focal length, field of view greater than 2 arcmin, and F/# of 40. The sub-telescope system comprises three sub-telescopes in a reflective afocal configuration with 5× magnification, 200 mm baseline length, 100 mm entrance beam diameter, and 20 mm exit beam diameter, yielding a combined system focal length of 4500 mm.

To enable coherent imaging of the three beams from sub-telescopes #1, #2, and #3, the piston detection module incorporates two piston detection telescopes. Using sub-telescope #1 as a reference, piston errors between #1-#2 and #1-#3 are detected and calculated to control the optical path differences among the three beams. The beams from #1-#2 and #1-#3 pass through Amici dispersion prisms in the piston detection module and the piston detection telescopes to form dispersed interference fringes on the focal plane.

The piston compensation module consists of coarse and fine adjustment stages, as shown in Figure 4 [Figure 4: see original paper]. In open-loop experiments, the coarse adjustment stage (PI M-122 micro-positioning stage) in the piston compensation assembly is used to adjust the position of a moving mirror to vary the optical delay line, thereby obtaining interference fringe images at different piston errors. The M-122 stage uses a linear encoder as an integrated sensor with 25 mm travel range, 0.1 m resolution, 0.2 m minimum incremental motion, and 20 mm/s maximum velocity. In closed-loop experiments, the fine adjustment stage (PI P-753 actuator) is used for precise piston compensation. The P-753 actuator features a capacitive integrated sensor with 12 m travel range, 0.05 nm resolution, and ±1 nm repeatability. The beam-combining imaging system consists of a single Cassegrain telescope with 110 mm aperture and 1680 mm focal length, which coherently images the three beams after correction of tilt errors and piston errors.

4 Open-Loop Experiment

In the open-loop experiment, the NKT source was configured with a spectral range of 550–650 nm and central wavelength of 600 nm. By moving the M-122 stage, 181 dispersed interference fringe images were acquired at piston error intervals of 1 m across a range of -90 m to +90 m. Figures 5 [Figure 5: see original paper] and 6 [Figure 6: see original paper] show the dispersed interference fringe images and their corresponding spectra at piston errors of -60 m, 0 m, and +60 m (from top to bottom). When the piston error is zero, the interference fringes show no tilt. For positive and negative piston errors, the fringes tilt downward and upward, respectively. Since the tilt direction of the interference fringes is perpendicular to the connecting line between the two secondary peaks, the two secondary peaks are symmetrically distributed on the positive and negative y-axes when the piston error is zero, and symmetrically distributed in the first/third and second/fourth quadrants for positive and negative piston errors, respectively.

The calculated peak displacements $\Delta x'$ from the 181 images and their linear fitting results are shown in Figure 7 [Figure 7: see original paper]. The horizontal axis represents the set piston error values in m, while the vertical axis shows the displacement of the selected secondary peak above the x-axis relative to the y-axis, in pixels. The red dots correspond to the 181 displacement values, and the blue line represents the linear fit: $\Delta x' = 0.2536\phi + 0.9104$. Using this expression, piston error values can be calculated for all 181 measurements. Figure 8 [Figure 8: see original paper] compares the solved piston errors with the set values, with the horizontal axis showing set piston errors in m and the vertical axis showing calculated piston errors in m. The green dots represent the distribution of the 181 solved piston errors, while the blue line indicates the set values. Partial displacement $\Delta x'$ and actual piston error ϕ results are listed in Tables 1 and 2 .

After obtaining the piston error calculations across the -90 m to +90 m range, the root-mean-square error was computed. Let ϕ_i be the set piston error value and ϕ_d the calculated value; the measurement error ε is:

$$\varepsilon = \phi_d - \phi_i$$

The root-mean-square error ε_{rms} for the 181 piston error measurements was calculated to be 1.564 m.

5 Closed-Loop Experiment

The block diagram for piston error detection and closed-loop control in the optical synthetic aperture system is shown in Figure 9 [Figure 9: see original paper], where ϕ represents the measured piston error. The Fourier transform-based piston calculation method is implemented on a digital signal processor for real-time computation, driving the actuator (P-753) to compensate for piston errors by adjusting mirror positions. The control method employs discrete proportional-integral-derivative (PID) control, where proportional action is proportional to the error magnitude and eliminates steady-state errors, integral action is proportional to the time integral of the error and eliminates residual errors at each sampling instant, and derivative action is proportional to the error change rate and provides anticipatory regulation to prevent parameter variations. The incremental PID expression is:

$$u(k) = u(k-1) + K_p[e(k) - e(k-1)] + K_i e(k) + K_d[e(k) - 2e(k-1) + e(k-2)]$$

where $u(k)$ is the output at the k-th sampling, $e(k) = \phi_{\text{set}} - \phi$ is the error, and K_p, K_i, and K_d are the proportional, integral, and derivative coefficients, respectively. This can be rewritten as:

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

where $q_0 = K_p + K_i + K_d$, $q_1 = -K_p - 2K_d$, and $q_2 = K_d$.

In the closed-loop experiment, the same NKT source was used with a central wavelength of 600 nm and spectral range of 550–650 nm. The PID parameters were set as $K_p = 0$, $K_i = 0.04$, and $K_d = 0$. Figures 10 [Figure 10: see original paper], 11 [Figure 11: see original paper], and 12 [Figure 12: see original paper] show the open-loop and closed-loop results for piston errors of 0 m, 2.5 m, and 5 m, respectively. In open-loop operation, the interference fringe images shift as piston error increases. In closed-loop operation, the interference fringes remain stable at the zero-piston position regardless of piston error variations, demonstrating effective closed-loop control.

6 Summary

This paper presents our research on piston error detection using Fourier transform-based dispersed fringe sensing, including both open-loop and closed-loop experimental results. Theoretical derivations established the linear relationship between secondary peak displacement relative to the y-axis and piston error. Algorithmic procedures were designed and implemented. Using our laboratory' s optical synthetic aperture testbed, open-loop and closed-loop experiments were conducted. Open-loop experiments involved acquiring multiple dispersed fringe images at various piston errors, revealing a strong linear relationship between peak displacement and piston error. Closed-loop experiments employed real-time piston error calculations and compensation mechanisms, demonstrating that the system can maintain its initial interference state even under external perturbations.

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Note: Figure translations are in progress. See original paper for figures.