Postprint: A Novel Discrete Current Regulator Design for AC Motors

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Abstract

This paper proposes a method for designing AC motor current regulators in the discrete domain (z-domain), which belongs to the field of high-performance AC motor control. Due to factors such as cross-coupling terms between loops, errors generated during controller discretization, and delays in digital control systems, the control performance of the regulator will degrade. This paper treats the d- and q-axis current loops in the AC motor control system as an integrated whole, establishes a mathematical model of the AC motor current loop in the discrete domain, and directly designs the discrete-domain current regulator while considering the one-step delay of the digital control system. Finally, based on automatic control theory, the maximum and optimal values for current regulator parameter design are derived. Simulation and experimental results show that after adopting the optimally designed current regulator, the dynamic response of the d- and q-axis currents is fast and without overshoot.

Full Text

Preamble

A Novel Design Method for Discrete-Time Current Regulators in AC Motor Drives

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Abstract

This paper proposes a novel design methodology for AC motor current regulators in the discrete-time domain (z-domain), which belongs to the field of high-performance AC motor control. The control performance of conventional regulators degrades due to cross-coupling terms between control loops, discretization errors introduced during controller implementation, and delays inherent in digital control systems. This paper treats the d- and q-axis current loops as an integrated system and establishes a discrete-time mathematical model of the AC motor current loop while accounting for the one-step delay of digital control systems to directly design a discrete-time current regulator. Based on automatic control theory, the maximum and optimal values of the current regulator parameters are derived analytically. Simulation and experimental results demonstrate that the optimized current regulator achieves fast dynamic response in d- and q-axis currents without overshoot.

Keywords: AC motor, discrete-time domain, current regulator, one-step delay

1 Introduction

With the rapid development of power electronics, microelectronics, and modern motor control theory, AC motors have been widely promoted and applied in industrial systems [1]. Among high-performance AC motor control methods, two approaches have gained widespread acceptance: Field-Oriented Control (FOC) [2-4] and Direct Torque Control (DTC) [2,5-8]. Direct torque control features a simple structure and fast dynamic response. However, due to its use of bang-bang controllers that select appropriate voltage vectors from a pre-designed switching table based on controller error outputs and stator flux position signals, DTC suffers from significant torque and flux ripples and variable switching frequency, limiting its application in certain high-performance control fields.

Vector control, on the other hand, achieves rotor flux orientation to decompose stator currents into d- and q-axis components, which are then regulated by PI controllers to independently control rotor flux and electromagnetic torque. This decoupled control approach enables vector control to achieve superior steady-state performance, meeting the requirements of high-performance AC motor control applications.

As modern industry demands increasingly higher performance and control precision from drive systems, traditional vector control techniques can no longer satisfy these requirements. In conventional vector control, linear PI regulators treat the current inner loops as two independent d- and q-axis control loops. However, this approach suffers from cross-coupling terms between the loops, discretization errors from controller implementation, and digital control system delays. Moreover, the coupling terms are proportional to the synchronous angular frequency, preventing traditional linear PI regulators from being designed truly independently and consequently compromising AC motor control precision. Since most practical implementations employ digital control systems, regulators designed in the continuous domain must ultimately be discretized, inevitably introducing errors. Additionally, the one-step delay inherent in digital control systems further degrades control performance.

To address the cross-coupling issue between d- and q-axis stator currents, several solutions have been proposed, though most are overly complex [9]. Reference [1] introduced nonlinear compensation to cancel coupling terms, but the resulting computational expressions are extremely complicated and the parameter tuning process is cumbersome. To avoid complex decoupling procedures, some methods employ complex-vector-based regulator design [10], while others design current regulators directly in the discrete-time domain to avoid discretization errors [11]. However, these methods fail to provide specific design procedures for the adjustable parameter k, which still requires empirical trial-and-error tuning in practical applications. Currently, no satisfactory method simultaneously addresses: (1) elimination of cross-coupling between d- and q-axis control loops; (2) direct regulator design in the discrete-time domain to avoid discretization errors; (3) consideration of digital control system delays; and (4) provision of accurate design formulas for the adjustable controller parameter k. Therefore, there is a need to propose a simple and practical method that achieves better control performance while improving versatility and practicality.

This paper proposes treating the d- and q-axis current loops as an integrated system and establishing the current loop mathematical model directly in the discrete-time domain while considering the one-step delay of digital control systems. Using an induction motor as the control object, this paper presents a detailed study of its mathematical modeling. Based on the engineering requirement of at least 45° phase margin [11], the analytical expression for the maximum value of current regulator parameter k is derived, though this coefficient introduces some overshoot to the current regulator response. To eliminate overshoot in the current regulator inner loop, this paper employs optimal design methodology from automatic control theory using a damping ratio of 1, deriving the optimal value of parameter k that eliminates overshoot while maintaining favorable dynamic performance [12]. Under step excitation, the current loop exhibits rapid response without overshoot. To validate the proposed discrete-time current regulator design method, this novel approach is combined with induction motor vector control and verified through both simulation and experimental testing on a two-level inverter-fed induction motor platform. Comparative results with conventional linear PI regulators are also presented. The simulation and experimental results demonstrate that the proposed discrete-time current regulator achieves excellent dynamic and static performance across the entire speed range, effectively expanding the industrial application scope of vector control.

2 Mathematical Model of Induction Motor

The vector-form mathematical model of an induction motor in the stationary reference frame [13] can be expressed as:

$$\mathbf{u}_s = R_s\mathbf{i}_s + p\psi_s \tag{1}$$

$$0 = R_r\mathbf{i}_r + p\psi_r - j\omega_r\psi_r \tag{2}$$

$$\psi_s = L_s\mathbf{i}_s + L_m\mathbf{i}_r \tag{3}$$

$$\psi_r = L_m\mathbf{i}_s + L_r\mathbf{i}_r \tag{4}$$

$$T_e = \frac{N_p L_m}{L_r}\psi_r \times \mathbf{i}_s \tag{5}$$

where $\sigma = L_s L_r - L_m^2$, $T_r = \frac{L_r}{R_r}$, $T_s = \frac{L_s}{R_s}$, and $p = \frac{d}{dt}$ represents the differential operator. In these equations, R_s is the stator resistance, R_r is the rotor resistance, L_s is the stator inductance, L_r is the rotor inductance, L_m is the mutual inductance, N_p is the number of pole pairs, ω_r is the rotor speed, $\mathbf{u}_s$ is the stator voltage, ψ_s is the stator flux vector, and ψ_r is the rotor flux vector.

Using stator current $\mathbf{i}_s$ and rotor flux ψ_r as state variables, the mathematical model of the induction motor in the stationary reference frame [14-15] can be expressed as:

$$p\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

where $\mathbf{x} = [\mathbf{i}_s \quad \psi_r]^T$.

Based on equations (7) and (8), the open-loop transfer function of the current inner loop is:

$$G_{OPI}(s) = \frac{k_p s + k_i}{\sigma L_s s + R_s}$$

Assuming the desired current loop bandwidth is k, we obtain: $k_p = k\sigma L_s$, $k_i = kR_s$. Substituting k_p and k_i into equation (9) yields:

$$G_{OPI}(s) = \frac{k}{s}$$

In practical applications, the motor mathematical model in equation (4) must be discretized to predict state variables at time $(k+1)$. To improve prediction accuracy, this paper adopts the second-order Euler method [16-17] to discretize equation (4):

$$\mathbf{x}_{k+1}^{p} = \mathbf{x}_k + T_{sc}\mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{p} + \frac{T_{sc}}{2}\mathbf{A}(\mathbf{x}_{k+1}^{p} - \mathbf{x}_k)$$

where $\mathbf{x}_{k+1}^{p}$ is the predicted variable, T_{sc} is the control period, and $\mathbf{x}_{k+1} = \begin{bmatrix}\mathbf{i}_s & \psi_r\end{bmatrix}_{k+1}^{T}$ represents the predicted state variables for the next time instant. Consequently, the electromagnetic torque at the next time instant $T_{e,k+1}$ can be predicted.

Under ideal conditions, using linear PI regulators can correct the current inner loop to a first-order inertial element:

$$G_{cl}(s) = \frac{k}{s+k}$$

This shows that the system is always stable without overshoot. According to engineering practice, the system control bandwidth ω_c is defined where the output lags the reference by 45° [11], giving $\omega_c = k$.

However, due to the presence of digital processing delay G_d (which is not unity), the above modeling does not fully reflect practical conditions, as shown in Figure 1 [Figure 1: see original paper]. This paper focuses on the design methodology for discrete-time current regulators and the theoretical design of PI coefficients, while considering the impact of one-step delay in digital systems.

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3 Linear PI Regulator

The conventional linear PI regulator is shown in Figure 1 [Figure 1: see original paper].

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4 Discrete-Time Current Regulator Design

The traditional linear PI regulator separates the current inner loop into independent d- and q-axis control loops. Due to cross-coupling terms between these loops, complete independent design cannot be achieved. In contrast, complex-vector-based regulators treat the d- and q-axis current loops as an integrated system, offering superior control performance and parameter robustness compared to conventional methods [18-20].

Based on the first row of equation (4), transformed to the d-q coordinate system and neglecting the back-EMF term, the transfer function from stator current to voltage after Laplace transformation is:

$$U_{dq}(s) = (\sigma L_s s + R_s + j\omega_e \sigma L_s) I_{dq}(s)$$

Since most practical implementations employ digital control systems, regulators designed in the continuous domain must ultimately be discretized. To avoid discretization errors, this paper adopts a more direct design approach by designing the current regulator in the discrete-time domain. The discrete-time mathematical model of the current inner loop [11] is expressed as:

$$G_p(z) = \frac{I_{dq}(z)}{U_{dq}(z)} = \frac{1 - e^{-T_s/T_\sigma}}{R_s(z e^{j\omega_e T_s} - e^{-T_s/T_\sigma})}$$

where $T_\sigma = \sigma L_s / R_s$; T_s is the control period; z is the Z-transform operator; e is the natural logarithm base; ω_e is the synchronous electrical angular velocity; and j is the imaginary unit.

Considering the one-step delay compensation of the digital control system in the synchronous reference frame, the discrete-time mathematical model of the current inner loop for induction motors becomes:

$$G_{edq}(z) = \frac{I_{dq}(z)}{U_{dq}(z)} = \frac{1 - e^{-T_s/T_\sigma}}{R_s z e^{j\omega_e T_s}(z e^{j\omega_e T_s} - e^{-T_s/T_\sigma})}$$

Based on the current inner loop model in equation (15) and using a zero-pole cancellation design method, the discrete-time regulator $G_{cc}^{dq}(z)$ with adjustable parameter k is directly designed as shown in equation (16). The complete control block diagram is illustrated in Figure 2 [Figure 2: see original paper].

$$G_{cc}^{dq}(z) = \frac{U_{dq}(z)}{E(z)} = \frac{k R_s (e^{j\omega_e T_s} - z^{-1} e^{-T_s/T_\sigma})}{(1 - z^{-1}) e^{j\omega_e T_s}}$$

From the discrete-time model $G_{edq}(z)$ in equation (15) and the regulator $G_{cc}^{dq}(z)$ in equation (16), the open-loop transfer function is obtained:

$$G_{ol}^{dq}(z) = G_{edq}(z) G_{cc}^{dq}(z) = \frac{k(1 - e^{-T_s/T_\sigma})}{z(z-1)}$$

where $k_{con} = k(1 - e^{-T_s/T_\sigma})$. Since T_s is typically very small, the Pade approximation [21] is used to simplify equation (19):

$$G_{ol}^{dq}(z) \approx \frac{k_{con} e^{1.5 j\omega T_s}}{j\omega T_s}$$

From equation (20), the crossover frequency is $\omega_c = k_{con}/T_s$. Setting the phase margin to 45° yields:

$$\frac{\pi}{2} - 1.5\omega_c T_s = \frac{\pi}{4}$$

From equation (21), the maximum value of k is:

$$k_{max} = \frac{\pi}{6(1 - e^{-T_s/T_\sigma})}$$

Although parameter design using k_{max} provides fast rise time, it introduces significant overshoot, as shown in Figure 3 [Figure 3: see original paper]. Therefore, it is necessary to determine an optimal coefficient k_{opt}.

Based on the open-loop transfer function in equation (17), the closed-loop characteristic equation is:

$$z^2 - z + k(1 - e^{-T_s/T_\sigma}) = 0$$

According to automatic control theory, when both roots of the closed-loop characteristic equation lie on the real axis, the system exhibits no overshoot. This requires:

$$1 - 4k(1 - e^{-T_s/T_\sigma}) = 0$$

Thus, the adjustable parameter k for overshoot-free operation is:

$$k_{opt} = \frac{1}{4(1 - e^{-T_s/T_\sigma})}$$

Substituting the optimal value k_{opt} from equation (25) into the discrete-time regulator $G_{cc}^{dq}(z)$ in equation (16) yields the optimal discrete-time current regulator in the synchronous reference frame:

$$G_{ccopt}^{dq}(z) = \frac{zR_s(e^{j\omega_e T_s} - z^{-1}e^{-T_s/T_\sigma})}{4(1 - e^{-T_s/T_\sigma})(1 - z^{-1})e^{j\omega_e T_s}}$$

Simulation using the optimal coefficient k from equation (25) shows the closed-loop step response in Figure 3 [Figure 3: see original paper]. The system responds rapidly without overshoot, validating the effectiveness of the discrete-time regulator parameter design. Existing literature [9,20] has demonstrated through theoretical and experimental studies that discrete-time current regulators exhibit good stability and parameter robustness. However, they do not provide optimal design principles for the parameters. This section derives analytical expressions and optimal values for the current regulator parameter k

based on discrete-time design and automatic control theory. The following sections verify the effectiveness of this parameter design through simulation and experiments.

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5 Simulation and Experimental Results

5.1 Simulation Results

To validate the proposed algorithm, simulations and experiments were conducted on a 2.2 kW induction motor drive platform in the laboratory. The motor parameters are listed in the table below. The PWM period is 200 s. The traditional linear PI regulator coefficients were designed as $k_{op} = 1144$ according to optimal design principles. The discrete-time regulator gain coefficient was set to 7.54 based on equation (25).

Table: Simulation and Experimental Parameters

Parameter	Value
U_{dc}/V	
P_n/kW	
f_n/Hz	
T_n/N · m	
Ψ_s/Wb	

First, the effectiveness of the discrete-time current regulator parameter design was verified through simulation of a q-axis current step response. The motor initial speed was -1500 r/min, and torque was suddenly applied to accelerate the motor. The q-axis current response during acceleration is shown in Figure 3 [Figure 3: see original paper]. Using the optimally designed gain coefficient, the q-axis current tracks the reference value rapidly without overshoot. In contrast, using the parameter k_{max} designed for minimum phase margin results in shorter rise time but significant overshoot. The simulation results align well with theoretical analysis, indirectly verifying the correctness of the algorithm. For the discrete-time current regulator, the optimized gain parameter substantially reduces overshoot without sacrificing settling time.

To evaluate the dynamic performance of both current regulators, the flux current was fixed at 3.2 A while the torque current reference was set as a sawtooth wave. During this process, the motor speed varied between 750 and 1500 r/min. The corresponding simulation waveforms are shown in Figure 4 [Figure 4: see original paper]. Due to the delay term and filtered synchronous electrical angular velocity in simulation, the d-axis current is slightly affected during large dynamic transitions. For the discrete-time current regulator, the d-axis current quickly recovers to its reference value, whereas the d-axis current under the traditional linear PI regulator remains continuously affected by varying q-axis current. This occurs because the control delay prevents the traditional linear PI regulator from achieving complete decoupling between the two current loops through feedforward compensation.

Figure 5 [Figure 5: see original paper] shows simulation results for sudden load changes at 1500 r/min, comparing traditional PI and discrete-time regulators. The results demonstrate that during sudden load changes, the q-axis current rapidly tracks the command value, motor speed experiences slight variation before quickly stabilizing, and the d-axis current remains essentially constant, validating the effectiveness of the discrete-time current regulator design.

5.2 Experimental Results

In addition to simulation verification, experiments were conducted on a two-level AC drive test platform to validate the proposed discrete-time current regulator design. The experimental setup is shown in Figure 6 [Figure 6: see original paper]. The motor parameters and system sampling frequency are identical to those in simulation, as listed in the table above. A DSP TMS320F28335 was used to execute the main algorithm. Load torque was applied using a magnetic powder brake, and actual motor speed was obtained directly from an incremental encoder. Internal motor variables (except stator current) were output to an oscilloscope through a 12-bit DA chip on the control board, while stator current was measured using current probes. Channel 1 displays measured speed, Channel 2 shows estimated q-axis current, and Channel 3 shows estimated d-axis current. The scales for Channels 1-3 are marked on the figures, while Channel 4' s current scale is indicated in the oscilloscope screenshot.

Since this paper does not consider high-speed field-weakening operation, the performance difference between the two current loops is not significant in experimental tests. Therefore, only experimental results using the discrete-time regulator are presented. First, the regulator parameter design values were tested under motor standstill conditions. Figure 7 [Figure 7: see original paper] shows that when the regulator parameter is set to k_{max} from equation (22), the d-axis current tracks the command value within 4 ms but exhibits some overshoot. In contrast, using the optimally designed parameter k_{opt} from equation (25), overshoot is eliminated while the d-axis current still tracks the command value in approximately 4 ms. Unlike simulation, the motor parameters used in the experimental controller inevitably contain some errors, causing slight differences from the simulation results in Figure 4. This experiment validates the correctness of the previous analysis on optimal regulator parameter design.

Figures 8 [Figure 8: see original paper] and 9 [Figure 9: see original paper] show experimental results using the optimal coefficient k_{opt} for the discrete-time regulator in sensored vector control at 150 r/min and 1500 r/min under no-load and rated-load conditions, respectively. The results demonstrate that FOC operates well in both low- and high-speed ranges with smooth sinusoidal current waveforms. To further verify low-speed performance, rated-load experiments were conducted at 6 r/min. Figure 10 [Figure 10: see original paper] shows good system operation with smooth sinusoidal current waveforms, indicating excellent low-speed performance of the discrete-time current regulator vector control. These results confirm that the proposed discrete-time regulator design achieves good steady-state performance across the entire speed range.

In addition to steady-state performance verification, dynamic performance was tested as shown in Figures 11 [Figure 11: see original paper] and 12 [Figure 12: see original paper], presenting system starting and high-speed reversal test waveforms. To ensure sufficient load capacity during starting, DC pre-excitation was applied (q-axis current set to zero while constant DC current was injected into the d-axis for motor excitation). Starting was initiated when the air-gap flux reached the set value. Figure 11 shows rapid d- and q-axis current responses without overshoot during the dynamic process, validating the theoretical analysis of parameter design. Figure 12 shows reversal test waveforms from 1500 r/min to -1500 r/min. The q-axis current quickly increases to its maximum value without overshoot or oscillation, while the d-axis current remains stable at its reference value throughout the q-axis current variation, verifying that the discrete-time current regulator design achieves decoupled control of d- and q-axis currents during dynamic operation. Additionally, smooth speed transition through the zero-speed region indicates good dynamic performance of the overall system.

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6 Conclusion

This paper presents a detailed study of current loop regulator design methods in vector control, focusing on both traditional linear PI current regulators and discrete-time designed current regulators. Based on phase margin and automatic control theory, analytical expressions for discrete-time regulator parameters are derived, and the optimal value of coefficient k is theoretically determined to ensure overshoot-free operation while meeting dynamic response requirements. The effectiveness of both regulator designs is verified through simulation and experiments. Although the advantages of discrete-time regulators are most prominent in high-speed field-weakening and low carrier ratio conditions [18-19], and this paper does not consider high-speed field-weakening operation (resulting in similar performance between the two regulators in simulations and experiments), the discrete-time regulator offers a simpler structure by treating the current loops as an integrated system rather than separate entities. Furthermore, it eliminates the need for discretization required by traditional linear PI regulators in practical implementation, thereby reducing discretization errors. Therefore, the proposed direct discrete-time current regulator design effectively enhances the industrial applicability of vector control.

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Note: Figure translations are in progress. See original paper for figures.