Postprint: Model Predictive Control of Three-Phase PWM Rectifiers

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Abstract

This paper establishes and analyzes the mathematical model of a three-phase PWM rectifier and investigates the PWM rectifier control system based on model predictive control. Through analysis and simulation of conventional single-vector and double-vector model predictive control methods, the reasons for poor grid-side THD performance in conventional double-vector and single-vector model predictive control are studied. To improve the grid-side current THD performance of the model predictive control system, improved double-vector and improved single-vector model predictive control methods are proposed. The proposed method enhances the grid-side current THD performance while maintaining optimal dynamic performance of both current and voltage, and simulation results verify the feasibility and effectiveness of the method.

Full Text

Preamble

Research on Three-Phase PWM Rectifier Model Predictive Control

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Liu Congwei (male, born 1969) is an associate researcher and postdoctoral fellow whose research focuses on fundamental theories of power electronic converters, high-voltage high-power converters, motor control systems, and photovoltaic/wind power generation systems.

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Abstract

This paper establishes and analyzes the mathematical model of three-phase PWM rectifiers and investigates a model predictive control (MPC) system for PWM rectifier applications. Through analysis and simulation of conventional single-vector and double-vector MPC methods, we examine the reasons for poor total harmonic distortion (THD) performance of grid-side current in traditional approaches. To improve the THD performance of grid current under MPC, we propose modified double-vector and single-vector MPC methods. These methods enhance grid current THD performance while maintaining excellent dynamic performance for both current and voltage. Simulation results verify the feasibility and effectiveness of the proposed methods.

Keywords: PWM rectifier, model predictive control, target function, THD

1 Introduction

Three-phase PWM rectifiers achieve rectification through the switching operations of six fully-controlled semiconductor switches in the bridge arms. Compared with traditional diode rectifiers, PWM rectifiers offer superior control performance for grid-side current and DC-side voltage, along with bidirectional power flow capability. However, switching losses constitute a significant portion of the rectifier' s total losses. To improve the efficiency of three-phase PWM rectifiers, reducing switching losses while maintaining high control performance represents an important research direction.

Theoretical analysis and simulation verification reveal that when PWM rectifiers operate under classical closed-loop dual PI control, achieving satisfactory control performance requires high switching frequencies. Moreover, PI parameters must be tuned according to different switching frequencies, making the system sensitive to parameter selection and inconvenient to implement [1]. Direct Power Control (DPC) replaces the current inner loop with a power loop and achieves control by directly regulating active and reactive power. Although DPC provides fast response, its steady-state performance is inferior to dual-loop PI control [2-5]. Model Predictive Control (MPC) offers precise control based on the rectifier's mathematical model, enabling different control objectives through various weighting factors in the cost function. However, this flexibility necessitates modifications to the cost function depending on control purposes [6-8].

Literature [9] and [10] proposed single-vector and double-vector MPC methods. At the same sampling frequency, double-vector MPC achieves better THD performance than single-vector MPC. However, at low switching frequencies, the control performance is unsatisfactory, and at identical switching frequencies, double-vector control may perform worse than single-vector control.

Building upon traditional double-vector MPC [10], this paper proposes an improved algorithm that achieves superior THD performance at the same switching frequency while maintaining good dynamic performance. We also apply this improvement to traditional single-vector MPC [9]. Through computer simulations comparing classical dual-loop PI control, single-vector MPC, double-vector MPC, improved single-vector MPC, and improved double-vector MPC at identical switching frequencies, we verify the feasibility of the proposed methods.

2 Mathematical Model of Three-Phase PWM Rectifier

The main circuit of a three-phase PWM rectifier consists of grid-side three-phase power supplies, line equivalent inductance and resistance, the rectifier bridge, DC-side capacitors, and loads. The main circuit diagram is shown in [Figure 1: see original paper].

The mathematical equations for the rectifier main circuit can be derived as:

$$e_a = L\frac{di_a}{dt} + Ri_a + V_a + V_{no} \tag{1}$$

$$e_b = L\frac{di_b}{dt} + Ri_b + V_b + V_{no} \tag{2}$$

$$e_c = L\frac{di_c}{dt} + Ri_c + V_c + V_{no} \tag{3}$$

where e_a, e_b, e_c are the grid-side three-phase supply voltages; L and R are the equivalent line inductance and resistance; V_a, V_b, V_c are the equivalent rectifier bridge voltages.

Since grid power supplies are generally three-phase sinusoidal, symmetrical, and stable, the rectifier main circuit can be equivalently represented as shown in [Figure 2: see original paper], where the rectifier bridge is directly equivalent to a rectifier bridge power source. From this, we derive the rectifier main circuit mathematical formula in complex vector form:

$$e = L\frac{di}{dt} + Ri + v$$

where $v = V_{a,b,c} + V_{no}$.

According to the complex power definition [1] and the relationship between power, current, and voltage, after coordinate transformation we obtain:

$$S = P + jQ = \frac{3}{2}ei^*$$

where i^* represents the conjugate of the current complex vector.

3 Model Predictive Control for PWM Rectifier

3.1 MPC Block Diagram

The block diagram of the PWM rectifier control system based on MPC is shown in [Figure 3: see original paper]. By detecting the AC-side grid voltage and current and the current DC-side voltage, we can obtain the present voltage, current, and power values. Based on the existing PWM rectifier model and the determined predictions of active and reactive power at the current moment, we can predict the impact of applying different switching vectors on the active and reactive power at the next moment. Through evaluation via the cost function, we can determine the optimal switching vector to apply at the current moment.

The cost function is defined as:

$$F = (p_{ref} - p_{k+1})^2 + (0 - q_{k+1})^2$$

Clearly, when the switching vector selected from the six non-zero vectors and zero vector is applied at the current k moment, the rectifier's active and reactive power after a single vector action can be predicted.

3.2 Single-Vector MPC Principle

Based on the PWM rectifier' s mathematical model in the rotating coordinate system, we can derive the power variation rates. Assuming the current moment is k, the impact of the presently applied switching vector on active and reactive power can be obtained through the MPC method:

$$\frac{dp}{dt} = \frac{|e|^2 - \mathrm{Re}(v^* e_k)}{L} - \frac{R}{L} p_k - \omega q_k - \frac{\mathrm{Im}(v^* e_k)}{L} q_k + \omega p_k$$

The active and reactive power after a single vector action can be predicted as:

$$p_{k+1} = p_k + \frac{dp}{dt} T_s q_{k+1} = q_k + \frac{dq}{dt} T_s$$

The evaluation is performed through the single-vector MPC cost function to select the switching vector that minimizes the error at the $k+1$ moment.

3.3 Double-Vector MPC Principle [10]

Compared with single-vector MPC, double-vector MPC can apply two switching vectors within each PWM period. Therefore, at the same sampling frequency, double-vector MPC achieves higher effective switching frequency and superior performance. However, at identical switching frequencies, its sampling frequency is lower than single-vector MPC. Moreover, double-vector MPC offers greater flexibility in selecting switching vector durations, theoretically yielding better control performance.

As shown in [Figure 4: see original paper], double-vector MPC applies two switching vectors within one PWM period (selected from six non-zero vectors and one zero vector). Assuming the first vector action ends at $k+1$ moment and the second at $k+2$ moment, with the first vector acting for time Δt and having active power impact dp_1/dt, and the second vector having impact dp_2/dt, we can derive:

At $k+1$ moment after the first vector action:

$$p_{k+1} = p_k + \frac{dp_1}{dt}\Delta t q_{k+1} = q_k + \frac{dq_1}{dt}\Delta t$$

At $k+2$ moment after the second vector action:

$$p_{k+2} = p_k + \frac{dp_1}{dt}\Delta t + \frac{dp_2}{dt}(T_{pwm} - \Delta t) q_{k+2} = q_k + \frac{dq_1}{dt}\Delta t + \frac{dq_2}{dt}(T_{pwm} - \Delta t)$$

The evaluation is performed through the cost function considering both vector actions at $k+2$ moment:

$$F = (p_{ref} - p_{k+2})^2 + (0 - q_{k+2})^2$$

4 Improved Model Predictive Control Methods

4.1.1 Improved Double-Vector MPC Principle

Conventional double-vector MPC optimizes the power deviation at the end of a PWM period ($k+2$ moment). However, if the active and reactive power deviations at $k+1$ moment after the first vector action are uncontrolled, the overall control performance remains poor. This explains why conventional double-vector MPC exhibits worse THD than single-vector MPC at the same switching frequency, as single-vector MPC has no uncontrolled intermediate moment.

The proposed improvement includes two key aspects: 1. The optimization target combines power deviations at both $k+1$ and $k+2$ moments, eliminating the uncontrolled moment in double-vector control. 2. The improvement considers not only the deviations at these two moments but also the integral of deviations throughout the entire period from k to $k+2$. Geometrically, this equals the area between the reference and actual power values.

To minimize the integral deviation across the entire period, we propose a method that ensures opposite signs for deviations at $k+1$ and $k+2$ moments. This guarantees a smaller area between deviation and the time axis when deviation magnitudes are similar, thereby reducing the error integral value over the PWM period and lowering grid-side current THD.

As shown in [Figure 5a: see original paper], although switching vector v_1 yields the closest approach to the power reference at the end of a PWM period, vector v_2 produces a deviation area approximating a trapezoid near the reference, while v_1 creates two triangular areas distributed on both sides of the reference. Since the triangular areas distributed on both sides are theoretically about half the area on a single side, vector v_1 generates smaller power deviation area and consequently lower grid-side current THD.

The cost function F_1 considering both $k+1$ and $k+2$ moments is:

$$F_1 = (p_{ref} - p_{k+1})^2 + (0 - q_{k+1})^2 + (p_{ref} - p_{k+2})^2 + (0 - q_{k+2})^2$$

where Δt_1 is the first vector duration (t_1 in [Figure 5: see original paper]) and Δt_2 is the second vector duration (t_2), with $\Delta t_1 + \Delta t_2 = T_{pwm}$.

This cost function evaluates both the $k+2$ moment performance and the $k+1$ moment state, enabling selection of switching vectors that minimize voltage and current waveform distortion at both moments.

To further reduce the deviation integral across the period, we derive the optimal duration for the first vector (with the second vector duration being $T_{pwm} - \Delta t$):

$$\Delta t = \frac{(p_{ref} - p_k)(2\dot{p}_1 - \dot{p}_2) + (\dot{p}_2^2 - \dot{p}_1\dot{p}_2)T_{pwm} + (q_{ref} - q_k)(2\dot{q}_1 - \dot{q}_2) + (\dot{q}_2^2 - \dot{q}_1\dot{q}_2)T_{pwm}}{(\dot{q}_1 - \dot{q}_2)^2 + \dot{q}_1^2 + (\dot{p}_1 - \dot{p}_2)^2 + \dot{p}_1^2}$$

4.1.2 Switching Vectors and Their Durations

With two switching vectors selected from six non-zero vectors and one zero vector (treating both zero vectors as one effective zero vector), there are theoretically 49 combinations but only 42 effective ones, as 7 combinations duplicate others. Since F_1 has the greatest impact on steady-state performance, vector durations are calculated based on F_1.

4.2 Improved Single-Vector MPC Principle

To verify the improvement potential for other MPC algorithms, we apply the proposed improvement to conventional single-vector MPC. The principle ensures opposite signs for deviations at k and $k+1$ moments, whereas conventional methods only minimize deviation magnitude at each moment without considering sign changes. Similar to the double-vector improvement, this reduces the deviation area.

The additional term F_2 for sign change detection is:

$$F_2 = (p_k + p_{k+1} - 2p_{ref})^2 + (q_k + q_{k+1})^2$$

For single-vector MPC, the improved cost function combines F_1 and F_2 with weighting factors (1 for F_1 and 1/2 for F_2):

$$F = F_1 + \frac{1}{2}F_2 = (p_{ref} - p_{k+1})^2 + (0 - q_{k+1})^2 + \frac{1}{2}[(p_k + p_{k+1} - 2p_{ref})^2 + (q_k + q_{k+1})^2]$$

Since single-vector MPC applies only one switching vector per PWM period, no separate duration calculation is required.

4.3 Zero Vector Selection

If the predictive control selects one zero vector and one non-zero vector, the zero vector state (“000” or “111”) is chosen based on the non-zero vector: select “111” if the non-zero vector contains more “1” states, otherwise select “000” .

5 Simulation Results Analysis

Simulations were conducted comparing SVPWM control with conventional and improved single-vector and double-vector MPC methods. The PWM rectifier parameters are: AC grid voltage $e = 380$ V; inductance $L = 10$ mH; resistance $R = 0.3\ \Omega$; DC-side voltage $U_{dc} = 680$ V (with step change to $U_{dc}^* = 685$ V); load resistance $R_{load} = 97\ \Omega$; DC-side capacitance $C = 840$ F. The inner loop step response signal is 10,000 W for MPC methods. For SVPWM, which uses a current inner loop, the corresponding step signal is 14 A based on the current-power relationship. The SVPWM inner loop parameters are $P = 10$, $I = 1$, with switching frequency set to 2 kHz. The MPC simulation diagram is shown in [Figure 6: see original paper].

5.1 Dynamic Performance Comparison

To verify the dynamic performance of the improved double-vector MPC, we observed two dynamic responses: DC voltage reference step change and inner loop step response. The dynamic performance waveforms based on the above simulation parameters are shown in [Figure 7: see original paper].

[Figure 7: see original paper] displays the DC bus voltage response to a reference step change. At 2 kHz switching frequency, SVPWM shows the slowest response, while single-vector, double-vector, and improved MPC methods all stabilize within approximately 0.0053 s. However, single-vector MPC exhibits larger voltage overshoot than double-vector and improved double-vector MPC. The improved double-vector MPC demonstrates superior speed and the smallest overshoot among all MPC methods. The improved single-vector MPC shows minimal dynamic performance change, confirming that the improved MPC methods outperform the other three control algorithms.

[Figure 8: see original paper] shows the inner loop step response. SVPWM again exhibits slower performance. Although single-vector MPC reaches steady state slightly earlier than double-vector and improved methods, all have similar response times of about 0.00035 s. The improved double-vector MPC demonstrates the optimal steady-state power fluctuation performance.

5.2 Steady-State Performance Comparison

[Figure 9: see original paper] shows the grid-side current waveforms in steady state, while [Figure 10: see original paper] presents the FFT analysis spectra. The table below summarizes the steady-state THD metrics for various control methods at 2 kHz switching frequency.

Both conventional single-vector and double-vector MPC exhibit worse THD than SVPWM. However, after applying the improved methods, the improved single-vector MPC reduces THD by 14.6% compared to conventional single-vector MPC and by 9.2% compared to SVPWM. The improved double-vector MPC reduces THD by 32.6% compared to conventional double-vector MPC and by 18% compared to SVPWM. Among all methods, the improved double-vector MPC achieves the lowest THD of 8.08%, demonstrating the significant improvement in steady-state THD performance.

The FFT analysis reveals that SVPWM harmonics concentrate near the switching frequency or its multiples, while MPC methods, especially the improved double-vector approach, shift the spectrum toward higher frequencies, achieving superior control performance. This improvement method shows promise for extension to other MPC algorithms to enhance grid-side current THD performance.

Table: THD Comparison of Various Control Methods

Control Method	T_{pwm} (s)	THD (%)
SVPWM	-	9.86
Conventional Single-Vector MPC	-	10.45
Improved Single-Vector MPC	-	8.95
Conventional Double-Vector MPC	-	11.98
Improved Double-Vector MPC	-	8.08

6 Conclusion

Comparative data with conventional single-vector/double-vector MPC and SVPWM control methods demonstrate that the improved single-vector and double-vector MPC methods exhibit superior dynamic response at the same switching frequency, with smaller steady-state fluctuations than unimproved methods. Their steady-state THD metrics are significantly better than the other three control methods. The improved double-vector MPC achieves optimal dynamic and steady-state performance, followed by the improved single-vector MPC. Simulation results fully verify the feasibility of the proposed improvement methods for both double-vector and single-vector MPC algorithms and their excellent grid-side current THD performance.

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Note: Figure translations are in progress. See original paper for figures.