Ripple Current Suppression Method Based on Harmonic Injection PWM for High-Speed Permanent Magnet Synchronous Motors (Postprint)

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Abstract

In high-speed permanent magnet synchronous motor (PMSM) speed control systems, the low carrier ratio of PWM inverters and the small inductance of high-speed PMSMs lead to significant ripple currents in the motor windings. To effectively suppress these ripple currents, a harmonic injection PWM modulation scheme is proposed, wherein 3rd and 9th harmonics are injected into the SPWM sinusoidal signal wave. By optimizing the width of the PWM inverter supply voltage pulse sequence, the objective of reducing ripple currents in high-speed PMSMs is achieved. First, based on a regular-sampling high-speed PMSM control system, the principle and implementation method of harmonic injection PWM are discussed. Second, calculation formulas for ripple current harmonic components are derived. According to the characteristics of ripple current harmonic spectrum distribution, a ripple current harmonic group distortion rate is introduced to quantitatively describe the magnitude of ripple currents, thereby compensating for the limitations of the total ripple current distortion rate, and the ripple current suppression effect of harmonic injection PWM is analyzed through calculation. Finally, experimental verification of the harmonic injection PWM method is conducted. Experimental results demonstrate that the proposed harmonic injection PWM method can effectively suppress ripple currents in high-speed PMSMs.

Full Text

Preamble

High-Speed Permanent Magnet Synchronous Motor Ripple Current Suppression Based on Harmonic Injection PWM

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Abstract

In high-speed permanent magnet synchronous motor (HSPMSM) drive systems, the low carrier ratio of PWM inverters and the small inductance of HSPMSMs result in significant ripple current in motor windings. To effectively suppress this ripple current, this paper proposes a harmonic injection PWM modulation method that injects 3rd and 9th harmonics into the SPWM sinusoidal reference signal, optimizing the PWM inverter voltage pulse sequence width to reduce HSPMSM ripple current. First, based on a regular-sampling HSPMSM control system, the principle and implementation of harmonic injection PWM are discussed. Second, the calculation formula for ripple current harmonic components is derived. According to the spectral distribution characteristics of ripple current harmonics, a ripple current harmonic group distortion ratio is introduced to quantitatively describe ripple current magnitude, compensating for the limitations of total ripple current distortion ratio. The ripple current suppression effect of harmonic injection PWM is then analyzed through calculation. Finally, experimental verification demonstrates that the proposed harmonic injection PWM method can effectively suppress HSPMSM ripple current.

Keywords: High-speed permanent magnet synchronous motor, harmonic injection PWM, ripple current, harmonic calculation

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1 Introduction

Due to the use of rare-earth permanent magnet materials with high energy product, high-speed permanent magnet motors offer greater power density, higher reliability, and smaller size compared to conventional motors, attracting significant industrial interest and research in recent years [1-2]. These motors find numerous industrial applications, including compressors, vacuum pumps, turbine generators, flywheel energy storage systems, drilling tools, and friction welding equipment. In these high-speed industrial units, the elimination of gearbox intermediate stages through direct connection to the motor shaft not only reduces equipment volume but also improves transmission efficiency and system reliability [3-4].

Two-level sinusoidal PWM (SPWM) inverters are widely used in general low-voltage motor drives due to their simple structure and low cost. However, they face challenges in high-speed motor drive systems. Since the supply frequency is high (hundreds or even thousands of hertz), the inverter carrier frequency cannot be further increased due to power device switching losses, resulting in large-amplitude, low-frequency harmonic components in the output voltage. Combined with the small number of winding turns and low leakage inductance in low-voltage high-speed motors, this leads to significant current harmonics in motor windings, causing additional losses and thermal management difficulties (particularly in the rotor). Reducing inverter current harmonics is therefore an urgent problem in low-voltage high-speed motor drive systems.

Currently, harmonic analysis primarily employs two methods: simulation [6-9] and analytical methods [10-11]. Simulation methods are highly applicable and provide intuitive harmonic content visualization, but their relationship with motor system parameters is not explicit. Analytical methods establish mathematical relationships with system parameters, revealing how harmonics vary with parameters, though the derivation is complex and only a few PWM modulation methods yield perfect analytical solutions for armature current harmonics. Simplified analytical calculations often focus on voltage harmonic analysis [12-14]. Sinusoidal PWM (SPWM) and space vector PWM (SVPWM) are the most widely used AC modulation methods in inverter systems, but neither is optimal for ripple current suppression. In terms of comparative effectiveness, two-level SVPWM outperforms SPWM [15-17], while multi-level PWM methods are superior to two-level PWM [18], though multi-level methods require a large number of vectors, complicating process analysis and optimization design [19].

This paper uses a two-level inverter as an example, introducing harmonics into the SPWM sinusoidal reference wave to investigate the influence of harmonic injection PWM on HSPMSM ripple current. Studies show that proper injection of 3rd and 9th harmonics can effectively suppress ripple current. The research first explains the basic concept and implementation of harmonic injection PWM based on regular sampling motor control systems. Second, it derives analytical expressions for ripple current harmonics, introduces the harmonic group distortion ratio to quantitatively describe ripple current, and analyzes the ripple current suppression effect of harmonic injection PWM. Finally, experiments validate the effectiveness of the proposed method.

2 Harmonic Injection PWM Method

2.1 Principle of Regular Sampling SPWM

[Figure 1: see original paper] shows the overall control block diagram of the HSPMSM control system used in this paper. The system employs id = 0 control with a speed outer loop and current inner loop, both using proportional-integral (PI) feedback control. The error between speed reference ref and feedback passes through the outer PI controller to generate q-axis current reference iqref.

The errors between d-q axis current references (idref, iqref) and feedbacks (id, iq) pass through inner PI controllers to generate d-q axis voltage references ud, uq. Combined with the detected rotor angle , these are transformed via dq/abc coordinate transformation to obtain stationary frame voltage references ua, ub, uc, which serve as the signal waves for regular sampling PWM. These are sinusoidal functions with 2 /3 phase shift, compared with triangular carrier g( ) to generate drive signals that control PWM inverter power switches, producing three-phase PWM voltage outputs to supply the HSPMSM.

PWM waveforms can be generated using natural sampling or regular sampling methods. Natural sampling is rarely used in practice due to heavy computational load and long calculation times. Regular sampling is a linear approximation simplification that significantly reduces computation, making it widely used in engineering applications. As shown in [Figure 2: see original paper], the length between two positive peaks of triangular carrier g( ) is taken as one sampling period c. During the k-th sampling period, the arbitrary signal wave h( ) is sampled at the negative peak moment k of the triangular carrier. A horizontal line through the sampling point intersects the triangular wave at two points, where power switching devices are controlled to generate a pulse-modulated waveform with width k.

In the k-th sampling period, the ratio of pulse width k to sampling period c is called the PWM duty ratio Dk, whose relationship with signal wave h( ) is:

$$D_k = \frac{1 + Mh(\theta_k)}{2}$$

where M is the modulation ratio and c = 2 /Nc is the sampling angle period, representing the carrier ratio (the ratio of triangular carrier g( ) frequency to signal wave h( ) frequency). Equation (1) shows that knowing the specific functional form of the regular sampling signal wave allows determination of the pulse voltage in any sampling period.

2.2 Principle of Harmonic Injection PWM

Both SPWM and SVPWM can be generated using regular sampling, differing only in their signal wave h( ). The SPWM signal wave is a sinusoidal function:

$$h(\theta) = \cos\theta$$

The SVPWM signal wave is a saddle wave function:

$$h(\theta) = \cos\theta + \frac{\sqrt{3}}{3}\cos 3\theta - \frac{\sqrt{3}}{45}\cos 9\theta + \cdots$$

Expanding the SVPWM signal wave h( ) in Fourier series yields:

$$h(\theta) = \cos\theta + \sum_{k=1}^{\infty} \frac{4}{[3(2k-1)]^2 - 1} \cos[3(2k-1)\theta]$$

Equation (4) shows that h( ) contains not only the fundamental but also 3k-order harmonics (where k is odd). [Figure 3: see original paper] plots the SVPWM signal wave and its partial harmonics, where h( ), h1( ), h3( ), and h9( ) represent the SVPWM signal wave and its fundamental, 3rd, and 9th harmonics, respectively.

Equation (4) also shows that SVPWM signal wave harmonic amplitudes decay with the square of harmonic order. The SVPWM signal wave can be approximated as:

$$h(\theta) \approx \cos\theta + \frac{1}{6}\cos 3\theta - \frac{1}{90}\cos 9\theta$$

Equation (5) reveals that the 3rd and 9th harmonic amplitudes in SVPWM are fixed. If these amplitudes are set as variables H3 and H9, a new PWM modulation method—harmonic injection PWM—is created, with signal wave function:

$$h(\theta) = \cos\theta + H_3\cos 3\theta + H_9\cos 9\theta$$

Compared with SPWM and SVPWM, harmonic injection PWM adds two degrees of freedom, enabling adjustment of the PWM voltage pulse sequence to optimize HSPMSM ripple current.

According to the trigonometric identity:

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

The 3rd harmonic cos3 in the harmonic injection PWM signal wave (6) can be indirectly obtained from the fundamental cos . Similarly, the 9th harmonic cos9 can be indirectly obtained from the 3rd harmonic cos3 . The acquisition method is detailed in [Figure 4: see original paper], where H(•) describes the mapping relationship from fundamental cos to the harmonic injection PWM signal wave h( ) in (6).

The three-phase sinusoidal voltage signal waves ua, ub, uc obtained through dq/abc transformation for SPWM modulation are processed through mapping H(•) in [Figure 4: see original paper] to obtain uha, uhb, uhc for harmonic injection PWM modulation. Let the SPWM voltage signal waves be:

$$\begin{cases} u_a = U_m\cos(\theta + \phi) \\ u_b = U_m\cos(\theta - 2\pi/3 + \phi) \\ u_c = U_m\cos(\theta - 4\pi/3 + \phi) \end{cases}$$

where Um and are amplitude and phase, respectively. Then the harmonic injection PWM voltage signal waves are:

$$\begin{cases} u_{ha} = U_m h(\theta + \phi) \\ u_{hb} = U_m h(\theta - 2\pi/3 + \phi) \\ u_{hc} = U_m h(\theta - 4\pi/3 + \phi) \end{cases}$$

where h( ) is the signal wave function from (6). This yields the HSPMSM control system block diagram based on harmonic injection PWM, shown in [Figure 5: see original paper].

3 Ripple Current Analysis

3.1 Harmonic Calculation of Ripple Current

The three-phase half-bridge PWM inverter is shown in [Figure 6: see original paper], where Ud is DC bus voltage; eA, eB, eC are PMSM three-phase back EMFs; iA, iB, iC are three-phase armature currents; L is phase inductance; R is phase resistance; O is DC bus neutral point; and N is motor three-phase winding neutral point.

The PWM voltage output from any inverter phase leg to the power supply midpoint can be expressed as a sequence of Nc pulse square waves. As shown in [Figure 7: see original paper], taking the k-th sampling period pulse as an example, the switching angles 2k and 2k-1 are given by:

$$\begin{cases} \theta_{2k-1} = k\theta_c - \frac{\theta_c}{2} D_k \\ \theta_{2k} = k\theta_c + \frac{\theta_c}{2} D_k \end{cases}$$

The PWM voltage ( ) from any phase leg to the power supply midpoint can be expressed as:

$$\delta(\theta) = U_d \sum_{k=1}^{N_c} [\varepsilon(\theta - \theta_{2k-1}) - \varepsilon(\theta - \theta_{2k})]$$

where Ud is DC bus voltage and (t) is the unit step function. Expanding in Fourier series yields:

$$\delta(\theta) = \frac{U_d}{2} + \frac{U_d}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sum_{k=1}^{N_c} [\sin n\theta_{2k} - \sin n\theta_{2k-1}] \cos n\theta - [\cos n\theta_{2k} - \cos n\theta_{2k-1}] \sin n\theta$$

Based on the 2 /3 phase shift characteristic of ABC three-phase inverter leg voltages to the power supply midpoint, uAO, uBO, uCO can be expressed as:

$$\begin{cases} u_{AO} = \delta(\theta) \\ u_{BO} = \delta(\theta - 2\pi/3) \\ u_{CO} = \delta(\theta - 4\pi/3) \end{cases}$$

The relationship between phase voltages to load neutral point (uAN, uBN, uCN) and phase voltages to power supply midpoint (uAO, uBO, uCO) is:

$$\begin{cases} u_{AN} = u_{AO} - \frac{u_{AO}+u_{BO}+u_{CO}}{3} \\ u_{BN} = u_{BO} - \frac{u_{AO}+u_{BO}+u_{CO}}{3} \\ u_{CN} = u_{CO} - \frac{u_{AO}+u_{BO}+u_{CO}}{3} \end{cases}$$

Since uAO, uBO, and uCO are symmetrically distributed, their 3k-order harmonics (k = 1,2,3,···) are in-phase. From (14), the DC components and 3k-order harmonics cancel out, leaving the harmonics that constitute the phase voltage to load neutral point, denoted simply as u:

$$u = \frac{2U_d}{3\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sum_{k=1}^{N_c} [\sin n\theta_{2k} - \sin n\theta_{2k-1}] \cos n\theta - [\cos n\theta_{2k} - \cos n\theta_{2k-1}] \sin n\theta$$

Assuming ideal sinusoidal back EMF for HSPMSM:

$$e = K\omega_1 \cos(\omega_1 t - \alpha)$$

where K is back EMF constant, 1 is fundamental angular frequency, and is power angle. Substituting (13) and (14) into the voltage equation u = Ri + L(di/dt) yields the HSPMSM armature current Fourier series:

$$i(\theta) = \sum_{n=1}^{\infty} (E_n \cos n\theta + F_n \sin n\theta)$$

where En and Fn are Fourier coefficients:

$$\begin{cases} E_n = \frac{R(A_n - C_n) - n\omega_1 L(B_n - D_n)}{R^2 + (n\omega_1 L)^2} \\ F_n = \frac{R(B_n - D_n) + n\omega_1 L(A_n - C_n)}{R^2 + (n\omega_1 L)^2} \end{cases}$$

with An, Bn, Cn, Dn being coefficients from the voltage expansion. For n 1, Cn = 0 and Dn = 0. The harmonic amplitudes In are:

$$I_n = \sqrt{E_n^2 + F_n^2}$$

[Figure 8: see original paper] shows HSPMSM ripple current waveforms with parameters: phase resistance R = 66mΩ, phase inductance L = 0.32mH, flux linkage f = 52mWb, frequency f = 533.33Hz, load T = 10N · m, DC bus voltage Ud = 540V, sampling period 125 s, and carrier ratio Nc = 15. [Figure 9: see original paper] presents the calculated ripple current harmonic spectrum, showing harmonic components clustered near integer multiples of the carrier ratio.

3.2 Quantification and Calculation of Ripple Current

In HSPMSM systems, small inductance and low carrier ratio result in large armature current ripple. The total harmonic distortion (THD)   is commonly used to represent ripple magnitude:

$$\eta = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \times 100\%$$

While THD provides a general reflection of ripple magnitude, it cannot reflect the contribution of harmonics around each integer multiple of the carrier ratio. To quantify these contributions, current harmonics are categorized by carrier ratio multiples. Harmonics in the interval [2, 0.5Nc] are defined as the 0th harmonic group, [0.5Nc, 1.5Nc] as the 1st group, [1.5Nc, 2.5Nc] as the 2nd group, and so on, as shown in [Figure 10: see original paper]. The k-th harmonic group distortion ratio k is defined as:

$$\eta_k = \frac{\sqrt{\sum_{n=\max\{2,(k-0.5)N_c\}}^{(k+0.5)N_c} I_n^2}}{I_1} \times 100\%$$

The harmonic group distortion ratio (22) effectively complements the total distortion ratio (21), enabling more detailed and comprehensive quantitative study of harmonic injection PWM' s effect on ripple current.

3.4 Double-Degree Freedom Harmonic Injection PWM Ripple Current Suppression

Following the principle of progressing from simple to complex, single-degree freedom injection of either 3rd or 9th harmonic was first analyzed. [Figure 11a: see original paper] shows the effect of 3rd harmonic amplitude H3 on total distortion ratio   and harmonic group distortion ratios k. As H3 increases, total distortion first decreases then increases, reaching minimum at H3 = 0.25, demonstrating that proper 3rd harmonic injection reduces armature current ripple. The 3rd harmonic primarily reduces ripple by suppressing the 1st carrier-ratio harmonic group.

[Figure 11b: see original paper] shows the effect of 9th harmonic amplitude H9. Both total distortion and harmonic group distortion ratios k vary monotonically with H9: 0th and 1st group distortions increase monotonically, while 2nd and 3rd group distortions decrease. Pure 9th harmonic injection can suppress higher-order harmonics but introduces lower-order harmonics.

This section investigates simultaneous injection of 3rd and 9th harmonics. [Figure 12: see original paper] plots the variation of total distortion ratio and harmonic group distortion ratios k with H3 and H9. [Figure 12a: see original paper] shows total distortion is minimized at H3 = 0.25, H9 = 0, indicating SVPWM is not the optimal ripple suppression method and harmonic injection PWM is needed for best performance.

[FIGURE:12b-e] show that injecting both harmonics increases 0th group distortion while creating complex patterns in other groups. The 3rd group changes minimally, but proper H3 and H9 settings can reduce 1st and 2nd group amplitudes while increasing the 3rd group amplitude, beneficial for reducing filter reactor capacity and achieving ideal ripple suppression.

4 Experimental Results

Experiments were conducted on a custom HSPMSM system to verify the feasibility and practicality of harmonic injection PWM for ripple current suppression. The experimental platform is shown in [Figure 13: see original paper]. The control system uses id = 0 rotor field-oriented vector control with harmonic injection PWM modulation and 2 s dead time.

First, the HSPMSM prototype back EMF was tested using a coupled drive method. The prototype was driven to steady speed by an adjustable prime mover, and three-phase back EMF waveforms were measured using a Tektronix oscilloscope. [Figure 14: see original paper] shows the measured back EMF, demonstrating good sinusoidal quality and validating the ideal sinusoidal back EMF assumption in (16).

[Figure 15: see original paper] shows measured current waveforms using SPWM modulation (H3 = 0, H9 = 0) at time scales of 2.50 ms/div and 250 s/div. The waveforms match calculated results, showing large ripple current. [Figure 16: see original paper] compares calculated and measured harmonic spectra, with good agreement verifying the ripple current harmonic calculation method. Harmonics concentrate near integer carrier ratio multiples, confirming the rationality of using harmonic groups.

[Figure 17: see original paper] shows measured current using harmonic injection PWM (H3 = 0.2, H9 = 0.02), which approximates SVPWM. Results demonstrate SVPWM' s advantage over SPWM in ripple suppression. [Figure 18: see original paper] shows current with H3 = 0.25, H9 = 0.0, representing optimized harmonic injection PWM. Measured results confirm that harmonic injection

PWM provides better ripple suppression than both SPWM and SVPWM, making it the optimal method among the three.

5 Conclusion

A harmonic injection PWM method is proposed to address large armature ripple current in PWM inverter-fed HSPMSM drive systems. By appropriately injecting 3rd and 9th harmonics, the method suppresses armature current harmonics in two-level SPWM inverter-fed high-speed permanent magnet motors. The method has broad application prospects and engineering value in two-level SPWM HSPMSM control systems, with experimental results proving its feasibility and practicality. Research demonstrates that harmonic injection PWM can significantly reduce HSPMSM ripple current, with suppression performance superior to both SPWM and SVPWM.

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Note: Figure translations are in progress. See original paper for figures.