Frequency Splitting Characteristics and Suppression Methods for Three-Coil Wireless Power Transfer Systems (Postprint)

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Abstract

First, based on Kirchhoff' s laws, the S-parameters, transmission efficiency, and frequency splitting phenomenon of the three-coil WPT system are analyzed. Frequency splitting is a phenomenon where over-coupling between coils leads to the system having more than one resonant frequency. Frequency splitting occurs only in the over-coupling region, where it is caused by the mismatch between internal resistance and system input impedance; this paper proposes an impedance matching method to suppress this phenomenon. By adding an L-type impedance matching network at the transmitter side and forming a conceptual impedance matching network at the receiver side, the objective of suppressing frequency splitting and increasing system efficiency is achieved. Finally, the validity of the proposed method is verified through comparative analysis of experiments and simulations.

Full Text

Frequency Splitting Characteristics and Suppression Method of Three-Coil Wireless Power Transmission Systems

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Abstract

Based on Kirchhoff' s laws, this paper analyzes the S-parameters, transmission efficiency, and frequency splitting phenomenon in three-coil wireless power transfer (WPT) systems. Frequency splitting is a phenomenon where over-coupling between coils causes the system to exhibit more than one resonant frequency. This phenomenon occurs exclusively in the over-coupled region and arises from the mismatch between the internal resistance and the system' s input impedance. To address this, we propose an impedance matching method to suppress frequency splitting. By adding an L-type impedance matching network at the transmitter and forming a conceptual impedance matching network at the receiver, we achieve simultaneous suppression of frequency splitting and enhancement of system efficiency. Finally, the validity of the proposed method is verified through comparative experimental and simulation analyses.

Keywords: Three-coil WPT system, magnetic resonance coupling, frequency splitting, impedance matching

1 Introduction

Currently, wired power transmission dominates human power delivery methods, yet conventional wired approaches suffer from numerous issues including spark generation, aging vulnerability, and inability to operate in special environments such as mines and underwater conditions. Consequently, achieving wireless power transmission has long been a major aspiration. Tesla first invented a wireless power transmission method in 1889 using alternating current and coils to illuminate a wireless incandescent lamp. However, due to distance limitations and other factors, wireless power transmission technology saw little breakthrough progress until 2007, when experiments at the Massachusetts Institute of Technology (MIT) reignited research enthusiasm. MIT researchers successfully lit a 60W bulb from a distance of 2 meters using electromagnetic resonance principles, achieving 40% efficiency. The MIT experiment' s advantages included flexible positioning of transmitter and receiver coils, longer transmission distances, and higher transfer efficiency [1-2].

Existing wireless power transfer methods fall into three categories: inductive wireless power transfer, microwave wireless power transfer, and magnetically-coupled resonant wireless power transfer. Inductive wireless power transfer operates only at very short distances and represents a relatively mature technology currently employed in mobile phone wireless charging. Microwave wireless power transfer, while capable of long-distance transmission, suffers from high transmission losses, directional constraints, and significant hazards to humans and living organisms, limiting its application scope. Magnetically-coupled resonant wireless power transfer utilizes resonance principles to transmit energy from transmitter to receiver. Compared with inductive wireless power transfer, two resonant coils operating at the same frequency can effectively exchange energy over longer operating distances with higher efficiency. In recent years, magnetically-coupled resonant wireless power transfer for mid-range transmission has attracted widespread attention due to its potential applications in biomedical implants, mobile device charging, and electric vehicle recharging [3].

However, resonant wireless power transfer in practical environments faces numerous challenges due to varying transmission distances and impedance conditions, including the frequency splitting phenomenon examined in this paper. The fundamental principle of magnetically-coupled resonant wireless power transfer is resonance. When the transmitter coil emits electromagnetic waves at a specific frequency, it induces resonance in the receiver circuit, maximizing its resonant amplitude and thereby absorbing most of the transmitted energy, which greatly enhances transmission power and efficiency. In a three-coil WPT system, the transmitter comprises a transmitting coil and a relay coil that transfer energy through direct coupling, while the receiver consists of a receiving coil. The transmitter and receiver achieve wireless power output through magnetic coupling resonance.

Since frequency splitting commonly occurs in magnetically-coupled resonant wireless power transfer systems, investigating this phenomenon is crucial for achieving efficient energy transmission between transmitter and receiver coils operating at resonant frequencies. Frequency splitting occurs in the over-coupled region, where excessive coupling between transmitter and receiver coils causes the system to exhibit multiple resonant frequencies. Numerous suppression methods have been proposed. For instance, references [4-6] established two-coil magnetically-coupled WPT systems, analyzed frequency splitting phenomena in detail, and proposed stabilizing system operation at a single resonant frequency by adjusting load resistance and capacitance ratios under different transmission distances—employing asynchronous transmitter and receiver coils to suppress frequency splitting and achieve uniform received power in WPT systems. Reference [7] analyzed frequency splitting in four-coil magnetically-coupled WPT systems and proposed suppressing frequency splitting by adjusting load resistance. While most literature focuses on two-coil and four-coil WPT systems, three-coil systems warrant investigation. For example, in electric vehicle wireless charging systems, adding an extra coil to the transmitter side to form a three-coil WPT system can improve efficiency without increasing the distance between transmitter and receiver coils. This paper focuses on frequency splitting phenomena and suppression methods in three-coil systems.

We first investigate the theoretical foundation of three-coil magnetically-coupled resonant wireless power transfer systems, deriving the S-parameters and transmission efficiency and analyzing frequency splitting phenomena. Our analysis reveals that at equal distances, different internal resistances constitute an important factor affecting frequency splitting. Frequency splitting occurs only in the over-coupled region due to mismatch between internal resistance and system input impedance. Therefore, we propose an impedance matching method to suppress frequency splitting. By adding an L-type impedance matching network at the transmitter and forming a conceptual impedance matching network at the receiver, we achieve suppression of frequency splitting while increasing system efficiency. Finally, we verify the proposed method’ s validity through comparative experimental and simulation analyses.

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2 System Modeling

The three-coil magnetically-coupled resonant wireless power transfer system consists of circuit elements L, C, and R, with its circuit model shown in [Figure 1: see original paper].

[Figure 1: see original paper] Three-coil WPT system circuit model

In the circuit model, V_S represents the system' s high-frequency power source; R_S is the internal resistance of the high-frequency power source; R_{p1}, R_{p2}, and R_{p3} are the loss resistances of the transmitting, relay, and receiving coils, respectively. Since radiation resistance is much smaller than loss resistance, we ignore coil radiation resistance in this paper. C_1, C_2, and C_3 are the equivalent capacitances of the corresponding coils; L_1, L_2, and L_3 are the equivalent inductances of the corresponding coils; R_L is the load resistance; f_0 is the system resonant frequency. The mutual inductance between L_1 and L_2 is M_{12}, and between L_2 and L_3 is M_{23}. For calculation convenience, we ignore mutual inductance between the transmitting and receiving coils.

We analyze the three-coil WPT system using Kirchhoff' s laws:

$$\begin{cases} Z_1 I_1 - j\omega M_{12} I_2 = V_S \\ Z_2 I_2 + j\omega M_{12} I_1 - j\omega M_{23} I_3 = 0 \\ Z_3 I_3 - j\omega M_{23} I_2 = 0 \end{cases}$$

where the impedance calculations satisfy:

$$\begin{cases} Z_1 = R_S + R_{p1} + j\omega L_1 + \frac{1}{j\omega C_1} \\ Z_2 = R_{p2} + j\omega L_2 + \frac{1}{j\omega C_2} \\ Z_3 = R_L + R_{p3} + j\omega L_3 + \frac{1}{j\omega C_3} \end{cases}$$

The coupling coefficient satisfies:

$$k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}$$

The system' s S-parameters are:

$$S_{21} = \frac{2V_L}{V_S}$$

From equations (1), (2), and (4), we obtain the ratio of V_L to V_S for the three-coil system:

$$\frac{V_L}{V_S} = \frac{\omega^2 k_{12} k_{23} L_2 \sqrt{L_1 L_3} R_L}{Z_1 Z_2 Z_3 + \omega^2 (k_{12}^2 L_1 L_2 Z_3 + k_{23}^2 L_2 L_3 Z_1)}$$

The transmission efficiency of the three-coil WPT system is:

$$\eta = \frac{|S_{21}|^2}{1 - |S_{11}|^2}$$

where the quality factor Q represents the ratio of stored energy to dissipated energy: $Q_1 = \frac{\omega L_1}{R_{p1} + R_S}$ for the transmitting coil, $Q_2 = \frac{\omega L_2}{R_{p2}}$ for the relay coil, and $Q_3 = \frac{\omega L_3}{R_{p3} + R_L}$ for the receiving coil.

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3 Frequency Splitting Phenomenon

From equations (4) and (6), we derive the calculation formula for S_{21}. Using this formula and the parameter values in , we obtain the relationship between $|S_{21}|$, coupling coefficient k_{23}, and frequency f for the three-coil WPT system, as shown in [Figure 2: see original paper].

[Figure 2: see original paper] The relationship between the S-parameter, the coupling coefficient, and the frequency of the three-coil WPT system

Before analyzing [Figure 2: see original paper], we introduce the formula:

$$M_{23} = \frac{\mu_0 \pi N_2 N_3 r_2^2 r_3^2}{2 d_{23}^3}$$

where μ_0 is the permeability of free space; N_2 and N_3 are the numbers of turns in the relay and receiving coils; r_2 and r_3 are the radii of the relay and receiving coils; and d_{23} is the distance between them.

We now analyze the frequency splitting phenomenon in magnetically-coupled resonant wireless power transfer systems. The operating region can be divided into under-coupled, critically-coupled, and over-coupled states based on the relationship between the actual coupling coefficient and the critical coupling coefficient. However, frequency splitting occurs only in the over-coupled region. From equation (8) and [Figure 2: see original paper], we observe that with constant three-coil parameters (number of turns, radii, etc.), the coupling coefficient k_{23} between resonant coils is inversely proportional to their distance. In the over-coupled region, as the distance d_{23} between relay and receiving coils decreases, the coupling degree between resonant coils gradually increases, making the frequency splitting phenomenon more pronounced. As distance d_{23} increases, the coupling degree gradually decreases, and the two branches of frequency splitting slowly converge until they meet at point f_0, the critically-coupled point where maximum energy transfer efficiency is achieved. The critically-coupled point marks the transition between under-coupled and over-coupled states. In the under-coupled state, wireless power transfer remains possible but with limited efficiency.

Since frequency splitting produces multiple resonant points, each achieving a local maximum in transfer efficiency, suppression measures are necessary. Before analyzing suppression methods, consider [Figure 3: see original paper], which compares system efficiency under the same internal resistance at different distances and under different internal resistances at the same distance. [Figure 3: see original paper] is obtained through simulation using data from and equation (7). Figures 3a and 3b show a distance of 20 cm with internal resistances R_S of 5Ω and 15Ω, respectively; figures 3c and 3d show a distance of 30 cm with R_S of 5Ω and 15Ω; figures 3e and 3f show a distance of 45 cm with R_S of 5Ω and 15Ω.

[Figure 3: see original paper] The relationship between efficiency and frequency

[Figure 3: see original paper] demonstrates that frequency splitting occurs only at short distances (the over-coupled region) and gradually disappears as distance increases. At 20 cm, frequency splitting appears, and the phenomenon is much more pronounced when $R_S = 15\Omega$ than when $R_S = 5\Omega$. At 30 cm, no frequency splitting occurs for $R_S = 5\Omega$, while $R_S = 15\Omega$ shows slight splitting. At 45 cm, frequency splitting no longer occurs. Thus, internal resistance significantly affects frequency splitting.

According to reference [9], in the over-coupled state with small distance between transmitting and receiving coils, the transmitter coil' s internal resistance substantially impacts WPT system efficiency. In over-coupled conditions, the input impedance at the original resonant point exhibits a large impedance angle and small magnitude, causing large source current that increases the difference between internal resistance and input impedance. However, below or above the original resonant point, the input impedance exhibits opposite characteristics: a small impedance angle and large magnitude. These opposing characteristics produce efficiency maxima at two resonant frequencies—the frequency splitting phenomenon. This confirms that mismatch between internal resistance and input impedance causes frequency splitting. To reduce internal resistance effects on frequency splitting, we employ an impedance matching network for suppression.

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4 Frequency Splitting Suppression Method

To suppress frequency splitting and enhance WPT system transmission efficiency, we implement impedance matching networks. Three main types exist: L-type, -type, and T-type. Due to its suitability for short transmission distances with large equivalent impedance and its simpler structure and easier implementation, we select the L-type matching circuit. As shown in [Figure 4: see original paper], we add an L-type impedance matching circuit at the source coil while employing a conceptual impedance matching method that combines the relay and receiving coils into another impedance matching network to improve WPT system transmission efficiency [10].

[Figure 4: see original paper] Impedance matching circuit

We first theoretically analyze the system with added impedance matching. According to the maximum power transfer principle, in resonant state, the input impedance Z_{in} of the transmitter-side matching network must have a real part equal to the source coil internal resistance R_S and zero imaginary part. Let Z_0 be the equivalent impedance of the WPT system excluding the transmitter-side matching circuit, as shown in [Figure 5: see original paper]. The equivalent impedance is:

$$Z_0 = Z_1 + \frac{\omega^2 M_{12}^2}{Z_2 + \frac{\omega^2 M_{23}^2}{Z_3}} - R_S$$

Setting the real part of Z_{in} equal to R_S and the imaginary part to zero yields the transmitter matching capacitance C_P and matching inductance L_S:

$$C_P = \frac{Q_1}{\omega_0 R_S}\sqrt{\frac{Z_0}{R_S} - 1}$$

$$L_S = \frac{R_S}{\omega_0 Q_1}\sqrt{\frac{Z_0}{R_S} - 1}$$

where ω_0 is the resonant angular frequency and Q_1 is the quality factor of the transmitter. Equation (12) applies when $R_S < Z_0$. Equation (12) shows that distance variations cause coupling coefficient changes, which alter the equivalent impedance Z_0, thereby changing the quality factor and consequently the matching capacitance and inductance values.

[Figure 5: see original paper] Equivalent Circuit

Similarly, in resonant state, the output matching network makes the receiver output impedance Z_{out} conjugate-matched to the optimal load impedance $R_{L(opt)}$:

$$R_{L(opt)} = R_{p3}\frac{1 + k_{12}^2 Q_1 Q_2}{k_{23}^2 Q_2 Q_3}$$

The above analysis establishes the relationships between matching capacitance, matching inductance, input impedance, and output impedance in the WPT system with added impedance matching. For the complete WPT system, adding impedance matching networks must satisfy both input impedance matching to internal resistance and output impedance matching to load impedance to simultaneously suppress frequency splitting and improve efficiency. For the source coil matching network, the primary task is determining matching capacitance and inductance values; for the receiver matching network, the primary task is determining the coupling coefficient value. Both are distance-dependent factors, making distance the key factor affecting the proposed impedance matching system. We verify the proposed method’ s validity through experiments and simulations.

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5 Experimental and Simulation Analysis

Our experimental setup is shown in [Figure 6: see original paper]. The WPT power source uses an inverter that converts DC voltage to fixed-frequency AC at the magnetically-coupled resonant WPT system resonant frequency $f_0 =$ 10 MHz. The input voltage V_S is set to 10V to limit input current. The transmitter comprises transmitting and relay coils; the receiver comprises a receiving coil. All coils have 10 turns, with the transmitting coil radius of 8 cm and relay/receiving coil radii of 4 cm. The L-type matching network consists of air-core inductors and non-polarized capacitors. Detailed coil parameters are listed in .

[Figure 6: see original paper] Experimental device

** Details of the coil**

Parameter	Value
L_1 (H)	21.1
L_2, L_3 (H)	11.2
C_1 (pF)	24
C_2, C_3 (pF)	45
R_{p1} (Ω)	0.5
R_{p2}, R_{p3} (Ω)	0.3
R_S, R_L (Ω)	5
f_0 (MHz)	10

We apply the impedance matching method to suppress frequency splitting. Experiments yield matching capacitance C_P and matching inductance L_S values at different distances, shown in [Figure 7: see original paper]. The figure shows that matching capacitance and inductance values vary with distance because distance variations alter equivalent impedance, which changes the quality factor and consequently the matching component values. Since the transmitter-side L-type impedance matching network aims to suppress frequency splitting while the receiver-side matching network improves WPT system efficiency, presents the efficiency values of the WPT system at different distances. Efficiency 1 represents pre-matching network efficiency, while Efficiency 2 represents post-matching network efficiency. The results show that Efficiency 2 significantly increases in the over-coupled region because frequency splitting is suppressed. Additionally, system efficiency varies little across different distances in the over-coupled region, which is the purpose of adding impedance matching. In the under-coupled region, system efficiency also increases, primarily due to the receiver-side conceptual impedance matching network.

[Figure 7: see original paper] Value of the matching capacitance and matching inductance at different distances

** Efficiency values at different distances**

Distance (cm)	Efficiency 1 (%)	Efficiency 2 (%)
20	45.2	78.6
25	52.1	79.3
30	58.7	80.1
35	65.4	81.5
40	71.2	82.3

Analyzing the system with the matching network at $d = 20$ cm as an example, the system remains in the over-coupled state. Before adding the matching network, the system at $d = 20$ cm exhibited over-coupling with low efficiency due to frequency splitting. After adding the matching network, input impedance approximately equals system internal resistance, approaching critical coupling, while output impedance becomes conjugate-matched to load impedance. This suppresses frequency splitting and substantially improves system efficiency.

In summary, adding impedance matching networks suppresses frequency splitting in the over-coupled region while generally increasing system efficiency (including in the under-coupled region). Therefore, the proposed impedance matching method can suppress frequency splitting and improve system efficiency to a certain extent.

6 Conclusion

1. This paper focuses on frequency splitting characteristics in three-coil magnetically-coupled resonant WPT systems, deriving S-parameters

and efficiency while analyzing frequency splitting phenomena through relational diagrams.

2. The analysis reveals that different internal resistances at equal distances significantly affect frequency splitting. The mismatch between internal resistance and system input impedance causes frequency splitting, leading to the proposed impedance matching method.
3. In the over-coupled region, the impedance matching method yields different matching capacitance and inductance values for various distances. Meanwhile, the receiver-side conceptual impedance matching network suppresses frequency splitting while increasing system transmission efficiency. Experiments and simulations verify that the method suppresses frequency splitting and improves system efficiency.

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Note: Figure translations are in progress. See original paper for figures.