Postprint: Optimal Virtual Inertia for VSG-Controlled Microgrid Inverters

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Abstract

To support the voltage and frequency of the microgrid and improve its operational stability, this paper investigates microgrid inverters employing the Virtual Synchronous Generator (VSG) control strategy. Due to the presence of virtual inertia in VSG, abrupt changes in power dispatch commands cause oscillations in the frequency and output power of the microgrid inverter, which may in severe cases lead to inverter shutdown due to overcurrent protection. Based on root locus analysis of the power closed-loop transfer function to determine the parameter variation ranges affecting VSG stability and dynamic performance, this paper proposes an optimal virtual inertia control strategy using linear quadratic optimal control with power and frequency variations as constraints. This strategy solves the aforementioned problems through optimal configuration of VSG parameters. Matlab simulation and experimental results verify the correctness and effectiveness of the proposed control strategy.

Full Text

Preamble

Research on Optimal Virtual Inertia of Micro-Grid Inverter Based on VSG Control

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Abstract

To support microgrid voltage and frequency and improve operational stability, this paper investigates microgrid inverters employing Virtual Synchronous Generator (VSG) control strategies. Due to virtual inertia in VSG, frequency and output power oscillations occur when power dispatch commands change abruptly, which can cause inverter shutdown due to overcurrent protection in severe cases. Based on root locus analysis of the power closed-loop transfer function to quantify parameter variation effects on VSG stability and dynamic performance, this paper proposes an optimal virtual inertia control strategy using linear quadratic optimal control with power and frequency variations as constraints. This approach resolves the aforementioned issues through optimized VSG parameter configuration. Matlab simulations and experimental results verify the correctness and effectiveness of the proposed control strategy.

Keywords: microgrid, virtual synchronous generator, frequency control, power oscillation, optimal virtual inertia

1. Introduction

As a novel control scheme for microgrid inverters, Virtual Synchronous Generator (VSG) control has attracted increasing attention from researchers [1-3]. Compared with droop control, VSG not only exhibits steady-state power drooping characteristics but also emulates the stator electrical characteristics and rotor inertia of synchronous generators, enabling microgrid inverters to match synchronous generators in both external characteristics and operating mechanism.

However, since VSG-based microgrid inverters simulate primary frequency/voltage regulation and the rotor inertia/damping characteristics of synchronous generators, they exhibit oscillations in output frequency and power during power dispatch command changes or DC-side distributed generation disturbances, similar to synchronous generators. For inverters, transient disturbance rejection and overload capabilities are far inferior to those of synchronous generators. Oscillation-induced surge currents may trigger inverter protection shutdown, potentially jeopardizing power device safety and stable microgrid operation.

Based on a fourth-order small-signal model, reference [4] analyzed power oscillation issues that may arise from virtual inertia in dual-machine parallel operation. Constrained by synchronous generator inertia and damping characteristics, the vast majority of literature considers VSG inertia as a fixed positive constant. To address power oscillations, references [6-7] proposed an adaptive virtual inertia control strategy, introducing concepts of variable and negative virtual inertia. However, this scheme employs Bang-Bang control for virtual inertia values, and frequent inertia changes cause power jitter.

VSG implementation inevitably involves distributed generation, energy storage devices, and inverters. However, existing VSG research assumes sufficiently large energy storage capacity and focuses control strategies solely on the inverter itself. No literature has thoroughly investigated coordinated inverter control considering both distributed generation and energy storage devices.

Based on the above research status, this paper fully exploits the flexible configurability of VSG parameters. Through power closed-loop transfer function analysis of parameter variation effects on VSG stability and dynamic performance, and using inverter power and frequency variations as constraints with linear quadratic optimal control, an optimal virtual inertia control strategy is proposed to effectively resolve frequency and power oscillation issues.

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2. VSG-Based Microgrid Inverter

2.1 Main Circuit Topology and Control Structure

The main circuit topology and control structure of the VSG-based microgrid inverter studied in this paper are shown in Figure 1 [Figure 1: see original paper]. Where: U_{dc} is the DC-side voltage; L and C are the filter inductor and capacitor respectively; T is the isolation transformer; ST is the synchronous contactor; L_l and r_l are the coupling impedance between the microgrid inverter and common AC bus; i_{Lk}, i_{ok}, u_{ok} and e_k are the inductor current, output current, output voltage and grid voltage respectively; P_e and Q_e are the active and reactive power calculated after low-pass filtering; θ^* and U^* are the voltage magnitude and phase reference obtained through the VSG algorithm module; u_k is the modulation wave obtained through dual-loop control; where $k = a, b, c$.

The VSG-based microgrid inverter control structure consists of three layers: The first layer is the VSG controller, which introduces a synchronous generator second-order electromechanical transient model to emulate mechanical inertia and electrical characteristics to obtain the port voltage reference. The second layer is the inner-loop controller, typically a dual-loop of output voltage and capacitor current control, to track the reference output from the VSG controller. The third layer is the drive controller, including the SVPWM modulation module and IGBT drive module.

2.2 VSG Implementation

The VSG algorithm module includes active power-frequency control and reactive power-voltage control. VSG active power-frequency control emulates synchronous generator primary frequency regulation and rotor inertia/damping characteristics [8], comprising the governor equation and rotor motion equation, as shown in Figure 2 [Figure 2: see original paper].

The VSG governor equation is:

$$P_{ref} = P_m + m(\omega_{ref} - \omega_m)$$

The VSG rotor motion equation is:

$$P_m - P_e - D_0(\omega_m - \omega_g) = J_0 \frac{d\omega_m}{dt}$$

Where ω_{ref}, ω_m and ω_g are the VSG reference angular frequency, output angular frequency and grid angular frequency respectively; m is the active power droop coefficient; P_{ref}, P_m and P_e are the VSG reference power, mechanical power and output electromagnetic power respectively; J_0, D_0 are the VSG virtual inertia coefficient and virtual damping coefficient respectively; $J = J_0\omega_m$ is the virtual inertia, $D = D_0\omega_m$ is the virtual damping.

From Figure 2, the frequency increment $\Delta\omega$ with respect to active power increment ΔP is a first-order inertial element. The droop coefficient m and virtual damping D determine the steady-state characteristics of VSG output frequency, while dynamic performance can be optimized through virtual inertia J. VSG parameter configuration offers three controllable degrees of freedom, providing greater control flexibility to improve VSG stability and dynamic performance to some extent.

2.3 VSG Single-Machine Grid-Connected Mathematical Model

Considering that the inverter voltage-current inner-loop control bandwidth is much larger than the power outer-loop control bandwidth, this paper establishes only the active power-frequency closed-loop transfer function based on the power outer loop to study VSG stability and dynamic performance.

Figure 3 [Figure 3: see original paper] shows the equivalent circuit diagram of VSG grid-connected operation, where U and E are the inverter output voltage and grid voltage respectively, δ is the phase difference between U and E, and Z is the sum of inverter equivalent output impedance, transformer leakage inductance and line impedance, generally inductive.

The VSG apparent power output is:

$$S = P_e + jQ_e = \frac{UE}{Z}\sin\delta + j\frac{UE\cos\delta - E^2}{Z}$$

Since the power angle δ is generally small, $\sin\delta \approx \delta$, and the active power expression can be simplified as:

$$P_e = \frac{UE}{X}\delta$$

From Figure 2, we can obtain:

$$\frac{\Delta\omega(s)}{\Delta P(s)} = \frac{1}{Js + D + m}$$

Thus equation (2) can be written as:

$$P_m = P_e + D\frac{d\delta}{dt} + J\frac{d^2\delta}{dt^2}$$

After Laplace transformation:

$$P_m(s) = P_e(s) + Ds\delta(s) + Js^2\delta(s)$$

The corresponding control structure is shown in Figure 4 [Figure 4: see original paper].

It is evident that the transfer function between VSG input power and output power is a typical second-order system. Comparing with the standard second-order system closed-loop transfer function $G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, the VSG corresponds to:

$$\omega_n = \sqrt{\frac{UE}{JX}}$$

$$\xi = \frac{D}{2}\sqrt{\frac{X}{UEJ}}$$

Therefore, the root locus of equation (8)' s characteristic equation can be used to analyze VSG stability and dynamic performance, as shown in Figure 5 [Figure 5: see original paper].

Figure 5 shows the family of characteristic root loci with virtual inertia J values of 5, 15, 30, 150 and 300 respectively, as virtual damping D varies from 0 to infinity. The system has a pair of conjugate complex roots with trends indicated by arrows.

For a specific virtual inertia J, as D increases, the conjugate complex roots lie on the left side of the complex plane with good dynamic characteristics but overshoot. As D increases further, the system transitions from underdamped oscillation to overdamped attenuation, reducing overshoot but slowing dynamic response. However, excessive D causes one characteristic root to approach zero, reducing system stability margin.

As virtual inertia J increases, the separation point of conjugate complex roots moves closer to the imaginary axis, slowing system response and degrading dynamic performance. However, excessive J causes severe overshoot and stability deterioration.

It is evident that VSG virtual inertia J determines the oscillation frequency during dynamic response, while virtual damping D determines the oscillation attenuation rate. Generally, during virtual damping tuning, the "Siemens second-order optimal system" method can be employed to achieve fast response speed and small overshoot by defining the system damping ratio at $\xi = 0.707$, i.e., $D = \sqrt{2J}$.

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3. Optimal Virtual Inertia Control Strategy

3.1 Single-Machine Grid-Connected Power Oscillation Problem

The above analysis shows that due to introduced inertia and damping characteristics, VSG-based microgrid inverters can effectively address adverse effects from DG randomness and uncontrollability, but also cause VSG output active power oscillations similar to synchronous generators during dynamic processes.

The oscillation process of microgrid inverter output frequency and power during power dispatch command 突变 is analyzed below. From equation (8), when VSG mechanical power P_m steps by ΔP_m, the output electromagnetic power P_e dynamic response is:

$$\Delta P_e = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\Delta P_m$$

In engineering applications, overdamped systems are generally undesirable due to slow response. Considering underdamped conditions ($0 < \xi < 1$), the system dynamic response curve is shown in Figure 6 [Figure 6: see original paper], with the corresponding VSG power-angle characteristic curve shown in Figure 7 [Figure 7: see original paper].

Initially, VSG operates stably at no-load with $P_e = P_m = 0$, $\omega_m = \omega_g$, and power angle $\delta = 0$ (point a). At a certain moment, input power or dispatch command suddenly increases, causing VSG output mechanical power P_m to increase, resulting in $P_m > P_e$. VSG begins to accelerate with $\omega_m > \omega_g$. The rotor motion equation generates frequency deviation, which after time integration further increases the power angle δ (a-b). Since $\omega_m > \omega_g$, δ continues to increase, P_e overshoots, ω_m decreases until $\omega_m = \omega_g$, where VSG output electromagnetic power P_e reaches maximum (b-c). Because $P_m < P_e$, VSG hasn' t reached power balance, ω_m continues to decrease, δ decreases, P_e decreases until $P_e = P_m$ (c-b). With $\omega_m < \omega_g$, δ continues to decrease, $P_m > P_e$, ω_m increases until $\omega_m = \omega_g$, where P_e reaches minimum (b-a).

Thus, under inertia and damping effects, after reciprocating dynamic oscillations, VSG is finally pulled into synchronization by the grid, entering a new steady-state operating point with output electromagnetic power $P_m = P_e$, output angular frequency $\omega_m = \omega_g$, and power angle $\delta = \delta_1$ (point b).

The oscillation process can be summarized as: - a-b: $d\delta/dt > 0$ ($\omega_m - \omega_g > 0$) and $d\omega_m/dt > 0$ - b-c: $d\delta/dt > 0$ ($\omega_m - \omega_g > 0$) and $d\omega_m/dt < 0$ - c-b: $d\delta/dt < 0$ ($\omega_m - \omega_g < 0$) and $d\omega_m/dt < 0$ - b-a: $d\delta/dt < 0$ ($\omega_m - \omega_g < 0$) and $d\omega_m/dt > 0$

Power oscillations accompany frequency variations. The power angle change rate $d\delta/dt$ (i.e., angular frequency ω variation) and frequency change rate $d\omega/dt$ jointly determine the oscillation process, equivalent to the acceleration and deceleration processes of conventional synchronous generators during disturbances. Therefore, to suppress power oscillations during dynamic regulation and reduce frequency fluctuations, we can qualitatively conclude: virtual inertia J should be increased during acceleration processes (a-b and c-b) to reduce power overshoot and frequency deviation, while decreased during deceleration processes (b-c and b-a) to accelerate power reaching the new equilibrium point.

3.2 Optimal Virtual Inertia Control Strategy

Based on qualitative analysis, this paper employs linear quadratic optimal control to quantitatively configure virtual inertia and damping values. This method minimizes energy released from energy storage at the cost of small control energy, maintaining minimal dynamic power oscillations and frequency fluctuations to achieve comprehensive optimization of energy and error, thereby improving microgrid stability and economy [9]. The optimal virtual inertia control strategy is elaborated below.

From equation (2), the VSG small-signal model is:

$$J\frac{d\Delta\omega_m}{dt} + D\Delta\omega_m + \Delta P = 0$$

Let $a = -D/J$, $b = -1/J$, then equation (12) simplifies to the standard form:

$$\frac{d\Delta\omega_m}{dt} = a\Delta\omega_m + b\Delta P$$

Since $\Delta\omega_m = 0$ at steady state, the linear quadratic problem simplifies to a state regulator form with quadratic performance index:

$$I = \int_0^\infty (H\Delta\omega_m^2 + R\Delta P^2)dt$$

Equation (14) includes two energy components: $\int_0^\infty H\Delta\omega_m^2 dt$ is the process cost, limiting angular frequency error during dynamic regulation to ensure appropriate response speed (weight matrix $H = 1$). The other component $R\Delta P^2 dt$ is the control cost, limiting output power magnitude and smoothness to ensure stable operation. Additionally, DC-side energy storage should be considered as it importantly limits total energy output during the entire control process, ensuring appropriate energy efficiency and economy (weight matrix $R = m^2$).

To minimize I in equation (14), the corresponding optimal solution is:

$$\Delta P^* = -\frac{b}{R} F \Delta\omega_m$$

where F is the positive solution of the Riccati equation:

$$2aF - \frac{b^2}{R}F^2 + H = 0$$

Solving yields:

$$\Delta P^* = \frac{D + \sqrt{D^2 + m^{2J}}}{m^{2J}} \Delta\omega_m$$

The energy storage in VSG can be expressed as:

$$\Delta P = -\frac{dE}{dt} = -J\omega_m \frac{d\Delta\omega_m}{dt}$$

Comparing with equation (15), the optimal virtual inertia J^* corresponding to the optimal solution ΔP^* is:

$$J^* = \frac{2J^*}{D + \sqrt{D^2 + m^{2J^*}}} \frac{UE}{X} \Delta\omega_m$$

Substituting into equation (10) yields the positive analytical solution for J^* from the quadratic equation:

$$J^* = \frac{4AC^2 + C^2\sqrt{16A^{2C^2} + B}}{2B^2}$$

where $A = \frac{UE}{X}\Delta\omega_m$, $B = m^2$, and $C = \frac{d\Delta\omega_m}{dt}$.

When angular frequency deviation $\Delta\omega_m \in (-0.2\pi, 0.2\pi)$ rad/s and angular frequency change rate $\frac{d\Delta\omega_m}{dt} \in (-0.24\pi, 0.24\pi)$ rad/s^2, the optimal virtual inertia J^* values for the microgrid inverter are shown in Figure 8 [Figure 8: see original paper], with corresponding virtual damping D determined by equation (10). When angular frequency deviation and change rate are outside these ranges, virtual inertia J^* takes a constant value of 200, with virtual damping determined by equation (10).

Thus, the optimal virtual inertia control strategy organically combines the three control degrees of freedom (droop coefficient, virtual inertia, and virtual damping) in VSG, balancing system stability and dynamic performance indicators. It achieves optimized configuration of key VSG parameters while solving power oscillation problems introduced by virtual inertia.

4. Simulation Results and Analysis

To verify the theoretical analysis, a Matlab simulation model of the microgrid inverter shown in Figure 1 was established. The inverter capacity is 100 kV · A, transformer connection group is Dy11, transformation ratio is 270V/400V, and main circuit parameters are listed in the table.

Table: VSG System Parameters - Rated power: 100 kV · A - Rated line voltage: 400 V - DC-side voltage: 600 V - Bridge arm inductance: 0.5 mH - Filter capacitor: 50 F - Power angle droop coefficient m: 0.01 Hz/kW - Reactive droop coefficient n: 0.1 V/kvar - Switching frequency: 10 kHz

Initially, the microgrid inverter operates stably at grid-connected no-load until a 50 kW step command is applied at 4s. For comparative analysis, Figure 9 [Figure 9: see original paper] shows output active power waveforms under different control strategies. With constant virtual inertia, power output has no overshoot (equivalent to underdamped state) but takes 2s to reach the power reference, resulting in excessively slow dynamic response and long settling time. With adaptive virtual inertia, dynamic response is very fast but exhibits severe power overshoot (instantaneous power reaches 80 kW) with continuous oscillations during regulation and long settling time. In contrast, with optimal virtual inertia, power dynamic response is relatively fast (reaching power reference in 0.75s) with slight overshoot, enabling quick steady-state entry without dynamic oscillations.

Figure 10 [Figure 10: see original paper] shows system frequency response waveforms. With constant virtual inertia, frequency overshoot is minimal but settling time is longest. With adaptive virtual inertia, frequency oscillates and barely reaches steady state. With the proposed optimal virtual inertia, frequency overshoot remains within allowable range and quickly reaches the steady-state operating point.

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5. Experimental Results

To further verify the proposed scheme, experiments were conducted on a 100 kV · A prototype. Grid line voltage RMS and frequency are 380V and 50Hz respectively, VSG DC bus voltage is 600V, and other system parameters match the simulation model.

For comparative analysis, Figure 11 [Figure 11: see original paper] shows an extreme case of J and D selection. After a step power command, output current increases and overshoots beyond instantaneous current protection value, causing inverter shutdown (Figure 11a). The corresponding output power waveform is shown in Figure 11b.

With constant virtual inertia control strategy, a 50 kW step command causes grid-connected current to increase slowly, reaching steady state after 1s (Figure

12a [Figure 12: see original paper]). Although power overshoot is avoided, the regulation time is excessively long, with power reaching reference after approximately 1s (Figure 12b), consistent with simulation analysis.

With adaptive virtual inertia control strategy, a 50 kW step command causes severe overshoot and oscillation in grid-connected current and output power, taking about 2s to reach steady state (Figure 13 [Figure 13: see original paper]). Due to Bang-Bang control, frequent virtual inertia mutations cause continuous oscillations, consistent with theoretical analysis.

With optimal virtual inertia control strategy, a 50 kW step command causes slight overshoot in grid-connected current and power, which quickly reach steady-state operating points with fast dynamic response and short settling time (Figure 14 [Figure 14: see original paper]). This solves power and frequency oscillation problems while ensuring dynamic performance.

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6. Conclusion

This paper investigated the main circuit topology, control structure, and mathematical model of VSG-based microgrid inverters. Through root locus analysis, the parameter variation range affecting VSG stability and dynamic performance was analyzed. Using power and frequency variations as constraints, an optimal virtual inertia control strategy was proposed. Through optimized VSG parameter configuration, frequency and power oscillation problems in VSG single-machine grid-connected operation were resolved. Both Matlab simulations and experimental results verified the correctness and effectiveness of the control strategy. The introduction of optimal virtual inertia significantly accelerates power exchange between VSG and grid during dynamic processes while balancing dynamic and steady-state performance. Whether the slowed current sharing and power sharing issues in multi-VSG parallel operation caused by inertia, damping, and frequency/voltage regulation can also be resolved through optimal virtual inertia requires further research.

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